THEORETICAL DETERMINATION OF MOISTURE AND HEAT TRANSFER TO LUMBER DURING PREHEATING

Zhu-Qiang Hou
Associate Professor

Xiao-Mei Jiang
Professor

Ning Guan
Professor
The Research Institute of Wood Industry
Chinese Academy of Forestry
Wan Shou Shan
Beijing 100091
P. R. of China

and

Zhangjing Chen†
Research Specialist
Department of Wood Science and Forest Products
Virginia Polytechnic Institute and State University
Blacksburg, VA 24061

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ABSTRACT

Two-dimensional heat and mass flux equations were used to describe preheating process during wood drying. Mathematical formulae of heat and moisture transfer to wood were developed. Based on the theoretical calculation, the increase in lumber moisture content (MC) was less than 8% during the preheating process. The calculated results also indicated that it takes about 3 h for the center of 4-cm-thick lumber to reach ambient temperature.

Keywords: Chemical potential, heat transfer, moisture transfer.

INTRODUCTION

Preheating is a necessary step in lumber drying during which the wood is heated to the required temperature (Alexiou et al. 1990 a,b). It is always of interest to know how much heat and moisture are transferred to wood during the preheating process. Nonequilibrium thermodynamics theory has been applied to the nonisothermal diffusion in wood. Skaar and Siau (1981) presented nonisothermal equations based on the MC gradient to describe steady-state thermal diffusion in wood. The equations were also used by Siau and Babiak (1983) to analyze moisture movement in wood. The theoretical calculation from the equations was in reasonable agreement with the experimental results. Skaar (1988) further discussed the nonisothermal heat and mass transfer in wood based on the irreversible thermodynamics. Nelson (1986) explained the mass and thermal diffusion of the bound water from the model developed by Siau (1980). Nelson (1989) also discussed the combined diffusion of bound water and water vapor in wood.

† Member of SWST.

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However, little work has been done on theoretical formulae related to lumber preheating during wood drying. The objective of this study was to develop the mathematical formulae that can be used to calculate the heat and moisture transfer to lumber during the preheating process.

HEAT AND MASS FLUX EQUATIONS

For the purpose of formulating the preheating process, the following assumptions were made:

1) Two-dimensional heat and mass transfer is considered,

2) Relative humidity in the kiln during preheating is 100%,

3) The lumber surface is at the ambient temperature when preheating starts,

4) The initial MC of wood is above the fiber saturation point (FSP).

The equations of heat fluxes and moisture fluxes were given (Degroot 1952; Skaar 1988):

$$J_{mx} = L_{mq}^{(x)} \frac{d}{dx}\left(\frac{1}{T}\right) - L_{mm}^{(x)} \frac{d}{dx}\left(\frac{\mu}{T}\right) \quad (1)$$

$$J_{qx} = L_{qq}^{(x)} \frac{d}{dx}\left(\frac{1}{T}\right) - L_{qm}^{(x)} \frac{d}{dx}\left(\frac{\mu}{T}\right) \quad (2)$$

$$J_{my} = L_{mq}^{(y)} \frac{d}{dy}\left(\frac{1}{T}\right) - L_{mm}^{(y)} \frac{d}{dy}\left(\frac{\mu}{T}\right) \quad (3)$$

$$J_{qy} = L_{qq}^{(y)} \frac{d}{dy}\left(\frac{1}{T}\right) - L_{qm}^{(y)} \frac{d}{dy}\left(\frac{\mu}{T}\right) \quad (4)$$

where X is the axis that represents tangential direction, Y is the axis that represents radial direction, J_{mx} and J_{my} Kg/(m^2·s) are the moisture flux in X and Y direction, J_{qx} and J_{qy} (J/m^2·s) are the heat flux in X and Y direction, T is the Kelvin temperature, $L_{mq}^{(x)}$, $L_{mm}^{(x)}$, $L_{qq}^{(x)}$, $L_{qm}^{(x)}$, $L_{qm}^{(y)}$, $L_{mm}^{(y)}$, $L_{qq}^{(y)}$, and $L_{qm}^{(y)}$ are phenomenological coefficients, μ is the chemical potential and

$$\mu = h - TS \quad (5)$$

where h (J/mol) is the molar enthalpy; S (J/mol·°K) is the molar entropy. According to Nelson (1986),

$$\mu = -9069.2 + 11.97T - 75.3T \ln(T/273.2) \quad \text{(J/mol)} \quad (6)$$

$$h = 11506 + 75.3(T - 273.2) \quad \text{(J/mol)} \quad (7)$$

$$S = 63.65 + 75.3 \ln(T/273.2) \quad \text{(J/mol·°K)} \quad (8)$$

Differentiating Eq. (5) with respect to temperature,

$$\frac{d\mu}{dT} = -S \quad \text{(J/mol·°K)} \quad (9)$$

According to Skaar (1988), the phenomenological coefficients can be given,

$$L_{mm}^{(x)} = TK_{\mu x} \quad (10)$$

$$L_{qq}^{(x)} = T^2 K_{qx} \quad (11)$$

$$L_{qm}^{(x)} = L_{mq}^{(x)} = (h + Q^*)L_{mm}^{(x)} \quad (12)$$

where $K_{\mu x}$ (mol/m·J·s) is the moisture diffusivity in X direction and a function of temperature and humidity; K_{qx} (J/m·°C·s) is the thermal conductivity in X direction, Q^* is the heat of transfer and can be written as (Halsopoulos and Keenan 1981):

$$Q^* = -\left[\frac{d\bar{Q}}{dm}\right]_T \quad (13)$$

where $d\bar{Q}$ and dm are the increase of heat and moisture. The heat of transfer relative to free water is (Nelson 1991),

$$Q^* = Q_v - \frac{T}{2}[C_P(T_i) + C_P(T)] \quad \text{(J)} \quad (14)$$

where Q_v (J) is the molar heat of free water vaporization, and

$$C_P(T) = 30.204 + 0.009933T + 0.00000112T^2 \quad \text{(J/mol)} \quad (15)$$

The equations of heat and mass fluxes in X-direction can be rewritten as,

$$J_{mx} = -K_{\mu x}\left[\left(S + \frac{Q^*}{T}\right)\frac{dT}{dx} + \frac{d\mu}{dx}\right] \quad (kg/m^2\cdot s) \qquad (16)$$

$$J_{qx} = -K_{qx}\frac{dT}{dx} - K_{\mu x}(Q^* + h)\left(\frac{d\mu}{dT} - \frac{\mu}{T}\right)\frac{dT}{dx} \quad (J/m^2\cdot s) \qquad (17)$$

Substituting Eq. (9) into Eq. (16),

$$J_{mx} = K_{\mu x}\frac{Q^*}{T}\frac{dT}{dx} \quad (kg/m^2\cdot s) \qquad (18)$$

So, heat flux in X direction can be expressed:

$$J_{qx} = -\left[K_{qx} + K_{\mu x}\frac{h}{T}(Q^* - h)\right]\frac{dT}{dx} \quad (J/m^2\cdot s) \qquad (19)$$

Similarly, moisture and heat fluxes in Y direction can be written as

$$J_{my} = K_{\mu y}\frac{Q^*}{T}\frac{dT}{dy} \quad (kg/m^2\cdot s) \qquad (20)$$

$$J_{qy} = -\left[K_{qy} + K_{\mu y}\frac{h}{T}(Q^* - h)\right]\frac{dT}{dy} \quad (J/m^2\cdot s) \qquad (21)$$

MC INCREASE DURING PREHEATING

The mass of moisture transferred into lumber per second through one side of the lumber perpendicular to X direction can be expressed as,

$$m_x = A_x J_{mx} \quad (kg/s) \qquad (22)$$

where $A_x = bL$ (m^2), b (m) and L (m) are the thickness and length of lumber.

The total moisture transfer to wood at the time t_0 can be calculated:

$$M_x = \int_0^{t_0} A_x J_{mx}\,dt = -\int_0^{t_0} A_x K_{\mu x}\frac{Q^*}{T}\frac{dT}{dx}\,dt \quad (kg) \qquad (23)$$

T can be written as (Kollman and Côté 1968),

$$T = T_1 + (T_0 - T_1)\frac{16}{\pi^2} \times \left\{\exp\left[-\pi^2\left(\frac{a_r}{b^2} + \frac{a_t}{w^2}\right)t\right] \times \sin\frac{\pi x}{w}\sin\frac{\pi y}{b} + \cdots\right\} \quad (^\circ C) \qquad (24)$$

where T_0 (°C) is the initial temperature of lumber prior to preheating, T_1 (°C) is the surface temperature of lumber during preheating, i.e., the preheating temperature, a_r and a_t (m^2/hr) are the diffusivity factor in radial and tangential direction.

Differentiating Eq. (24) with respect to x,

$$\left.\frac{dT}{dx}\right|_{x=0,w} = \pm(T_0 - T_1)\frac{16}{\pi w} \times \exp\left[-\pi^2\left(\frac{a_r}{b^2} + \frac{a_t}{w^2}\right)t\right]\sin\frac{\pi y}{b} \quad (^\circ K/m) \qquad (25)$$

The mean surface temperature gradient can be obtained:

$$\frac{dT}{dx} = \frac{1}{b}\int_0^b \left.\frac{\partial T}{\partial x}\right|_{x=0} dy = \frac{32(T_1 - T_0)}{\pi^2 w}\exp\left[-\pi^2\left(\frac{a_r}{b^2} + \frac{a_t}{w^2}\right)t\right] \quad (^\circ K/m) \qquad (26)$$

Similarly,

$$\frac{dT}{dy} = \frac{32(T_1 - T_0)}{\pi^2 b}\exp\left[-\pi^2\left(\frac{a_r}{b^2} + \frac{a_t}{w^2}\right)t\right] \qquad (27)$$

M_x and M_y can be written as,

$$M_x = \frac{2.304 A_x K_{\mu x} Q^*(T_1 - T_0)}{\pi^4 w T\left(\frac{a_r}{b^2} + \frac{a_t}{w^2}\right)} \times \left\{1 - \exp\left[-\pi^2\left(\frac{a_r}{b^2} + \frac{a_t}{w^2}\right)t_0\right]\right\} \times 10^2 \quad (kg) \qquad (28)$$

TABLE 1. *Constants used in the calculation.*

Constant	L (m)	w (m)	b (m)	$(a_r + a_t)/2$[a] (m^2/hr)	$K_{\mu r}/K_\mu$	$K_{\mu t}/K_\mu$	K_{qr}/K_q	K_{qt}/K_q
Value	2.00	0.30	0.04	0.0004	1.05	0.95	1.05	0.95
						Q_v[b] (J/mol)		
Constant	M (%)	T_0 (°C)	β[a] (%)	$(a_r/a_t)_t$[a] (m^2/hr)	ρ_0 (g/cm^3)	100°C	80°C	60°C
Value	100	20	$28\rho_0$	1.10	0.4	41237.5	41990.6	42743.7

[a] Values cited from Kollman and Côté (1968).
[b] Values cited from Skaar (1998).

$$M_y = \frac{2.304 A_y K_{\mu y} Q^*(T_1 - T_0)}{\pi^4 b T\left(\dfrac{a_r}{b^2} + \dfrac{a_t}{w^2}\right)} \times \left\{1 - \exp\left[-\pi^2\left(\frac{a_r}{b^2} + \frac{a_t}{w^2}\right)t_0\right]\right\} \times 10^2 \quad \text{(kg)} \tag{29}$$

where $A_y = wL$ (m^2); w (m) is the width of lumber; $K_{\mu y}$ (mol/m·J·s) is the moisture diffusivity of lumber in Y direction. The moisture increase during preheating is

$$M_w = 2(M_x + M_y) \quad \text{(kg)} \tag{30}$$

When MC is above 40%, K_q can be expressed as (Siau 1984)

$$K_q = 4.18[G(4.80 + 0.125M) + 0.57] \times 10^2 \quad (\text{J/m}\cdot{}^\circ\text{C}\cdot\text{s}) \tag{31}$$

where K_q is wood conductivity in transverse direction; G is the specific gravity of wood, M (%) is the MC of wood.

The relationship between K_μ and K_P was given by Siau (1984):

$$K_\mu = \frac{HP_0}{100RT}K_P \quad (\text{mol/m}\cdot\text{J}\cdot\text{s}) \tag{32}$$

TABLE 2. *Increase in lumber MC (%) with time at different ambient temperatures during preheating.*

Preheating temperature (°C)	Preheating time (h) 1.0	1.5	2.0	2.5
100	6.79%	7.24%	7.37%	7.41%
80	2.45%	2.62%	2.66%	2.68%
60	0.72%	0.77%	0.78%	0.79%

where H (%) is the relative humidity, P_0 (Pa) is the saturated vapor pressure, R (J/mol·°K) is the gas constant.

The relation between P_0 and temperature (Skaar 1988):

$$P_0 = 1.333 \exp\left[51.29 - \frac{6651}{T} - 4.531 \ln(T)\right] \times 10^2 \quad \text{(Pa)} \tag{33}$$

According to Bramhall (1979)

$$K_P = \frac{3.83 \exp\left(\dfrac{M}{17.92 - 2.533M} - 9.2\right)}{70.4 - 0.133T} \times 10^{-4} \quad (\text{mol/m}\cdot\text{Pa}\cdot\text{s}) \tag{34}$$

With the constants listed in Table 1, the MC increase with time during preheating process was calculated by using Eq. (30). The calculated results are presented in Table 2. The calculated MC increases vary little after 2.5 h preheating. The maximum MC increase in results calculated is about 7.41% at temperature of 100°C. Simpson (1976) found that the change of MC in northern red oak was not more than several percent for the same preheating period. The calculations from the formulae are in good agreement with Simpson's experimental results.

HEAT TRANSFER DURING PREHEATING

In the same way, the heat transfer equations can be developed as,

TABLE 3. *Heat (J) transferred to lumber with time at different ambient temperatures during preheating.*

Preheating temperature (°C)	Preheating time (hr)			
	1.0	1.5	2.0	2.5
100	2.10×10^6	2.24×10^6	2.28×10^6	2.29×10^6
80	1.59×10^6	1.60×10^6	1.63×10^6	1.64×10^6
60	9.76×10^5	1.03×10^6	1.05×10^6	1.06×10^6

$$Q = 2(Q_x + Q_y) \quad (J) \tag{35}$$

where Q (J) is the heat transferred into lumber during preheating.

Q_x and Q_y are the heat transferred into lumber through each side of lumber perpendicular to X and Y direction, respectively. They are in the forms:

$$Q_x = \frac{2.304A_x(T_1 - T_0)\left[K_{qx} + K_{\mu x}\frac{h}{T}(Q^* - h)\right]}{\pi^4 w\left(\frac{a_r}{b^2} + \frac{a_t}{w^2}\right)} \times \left\{1 - \exp\left[-\pi^2\left(\frac{a_r}{b^2} + \frac{a_t}{w^2}\right)t_0\right]\right\} \times 10^5 \quad (J) \tag{36}$$

$$Q_y = \frac{2.304A_y(T_1 - T_0)\left[K_{qy} + K_{\mu y}\frac{h}{T}(Q^* - h)\right]}{\pi^4 b\left(\frac{a_r}{b^2} + \frac{a_t}{w^2}\right)} \times \left\{1 - \exp\left[-\pi^2\left(\frac{a_r}{b^2} + \frac{a_t}{w^2}\right)t_0\right]\right\} \times 10^5 \quad (J) \tag{37}$$

where K_{qy} is the thermal conductivity of lumber in Y direction.

Equation (35) can be used to calculate the heat transferred to lumber with time at different temperatures during the preheating process. The calculated results are presented in Table 3, showing that it takes about 2.5 h for the center of the lumber to reach the ambient temperature. This is close agreement with 3 h determined experimentally by Gu and Garrahan (1984).

TABLE 4. *The ratio of J_{qmx} to J_{qx} (J_{qmy} to J_{qy}) (%).*

MC (%)	Preheating temperature (°C)		
	60	80	100
60	4.6	9.7	17.4
80	4.4	9.2	16.8
100	4.2	8.9	16.3

COUPLING OF MOISTURE AND HEAT FLUX

Equations (19) and (21) indicate that heat flux consists of two parts. The first ones, K_{qx} and K_{qy}, describe the heat flux due to temperature difference; the second part represents the heat flux due to moisture transfer.

Let

$$J_{qqy} = -K_{qx}\frac{dT}{dx} \quad (J/m^2 \cdot s) \tag{38}$$

$$J_{qmx} = K_{\mu x}\frac{h}{T}(Q^* - h)\frac{dT}{dx} \quad (kg/m^2 \cdot s) \tag{39}$$

$$K_{qmx} = \frac{K_{\mu x}(Q^* - h)h}{T} \quad (kg/m \cdot {}^\circ K \cdot s) \tag{40}$$

The ratio of J_{qmx} to J_{qx} can be given:

$$\frac{J_{qmx}}{J_{qx}} = \frac{K_{qmx}}{K_{qx} + K_{qmx}} \tag{41}$$

and correspondingly,

$$\frac{J_{qmy}}{J_{qy}} = \frac{K_{qmy}}{K_{qy} + K_{qmy}} \tag{42}$$

The ratio of J_{mqx} to J_{qx} or J_{mqy} to J_{qy} varies with the preheating temperature and MC (Table 4). The effect of moisture transfer on heat transfer is greater at the higher MC.

CONCLUSIONS

1) Mathematical formulae for the determination of heat and moisture transport to lumber during preheating were developed.

2) The calculated results showed that the increase of MC during the preheating process is less than 8%.

3) From theoretical calculation, it takes about 3 h for the center of 4-cm-thick lumber to reach the ambient temperature during preheating.

REFERENCES

ALEXIOU, P. N., A. P. WILKINS, AND J. HARTLEY. 1990a. Effect of pre-streaming on drying rate, wood anatomy, and shrinkage of regrowth *Eucalytus pilularis Sm.* Wood Sci. Technol. 24(1):103–110.

———, J. F. MARCHANT, AND K. W. GROVES. 1990b. Effect of pre-streaming on drying rate, wood anatomy, and shrinkage of regrowth *Eucalytus pilularis Sm.* Wood Sci. Technol. 24(2):201–209.

BRAMHALL, G. 1979. Mathematical model of lumber drying I. Principles involved. Wood Sci. 12(1):14–21.

DEGROOT, S. R. 1952. Thermodynamics of irreversible processes. North-Holland Publishing Co., Amsterdam, 38 pp.

GU, L., AND P. GARRAHAN. 1984. The temperature and MC in lumber during preheating and drying. Wood Sci. Technol. 18(2):121–135.

HALSOPOULOS, G. H., AND J. H. KEENAN. 1981. Principles of general thermodynamics. Robert Krieger Publishing Company, New York, NY. 628 pp.

KOLLMAN, F. F. P., AND W. A. CÔTÉ JR. 1968. Principles of wood science and technology I. Solid wood. Springer-Verlag, Berlin. 246 pp.

NELSON, R. M. JR. 1986. Diffusion of bound water in wood Part 3: A model for non-isothermal diffusion. Wood Sci. Technol. 20(4):309–328.

———. 1989. Nonisothermal diffusion of moisture in wood. Pages 357–377 *in* C. Schuerch. ed. Cellulose Conference, Syracuse. John Wiley and Sons, New York, NY.

———. 1991. Heats of transfer and activation energy for bound water diffusion in wood. Wood Sci. Technol. 25(3):193–202.

SIAU, J. F. 1980. Nonisothermal moisture movement in wood. Wood Sci. 13(1):11–13.

———. 1984. Transport processes in wood. Springer-Verlag, Berlin. 218 pp.

———, AND M. BABIAK. 1983. Experiment on nonisothermal moisture movement in wood. Wood Fiber 15(1):40–46.

SIMPSON, W. T. 1976. Effect of pre-steaming on moisture gradient of northern red oak during drying. Wood Sci. 8(4):272–278.

SKAAR, C. 1988. Wood water relations. Springer-Verlag, Berlin, Heideberg, New York. 199 pp.

———, AND J. F. SIAU. 1981. Thermal diffusion of bound water in wood. Wood Sci. Technol. 15(2):105–112.