A STATISTICAL CHARACTERIZATION OF THE HORIZONTAL DENSITY DISTRIBUTION IN FLAKEBOARD

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ABSTRACT

Studies reported on the horizontal density distribution (HDD) of wood composites and paper materials are reviewed. The literature suggests that for wood composites a concept for quantifying the HDD has yet to be established. The variation of HDD is shown to decrease as the specimen size increases. When determined within the less sensitive range, the horizontal density variation is likely a reflection of the forming nonuniformity, while the variation exhibited at smaller specimen sizes is caused mainly by voids present in the board structure. A model is established to relate the magnitude of HDD, as quantified by the standard deviation, to the specimen size based on a density variation phenomenon and statistical considerations.

Keywords: Density distribution, structure, particleboard, wood composites.

INTRODUCTION

Present wood composite technology encompasses a wide variety of products. A substantial body of experimental knowledge has been accumulated concerning the manufacture of short fiber wood composites. While a large number of process or production variables associated with product properties have been identified, little quantitative information has been reported about the structure of non-veneer composite products. It is apparent that a more rigorous, analytical approach based on the knowledge of composite structure is needed to better understand the influence of production variables on composite properties.

Ultimately, a three-dimensional density distribution can define a composite structure. This can be further subdivided into vertical and horizontal components. The vertical component, which is a measure of the density variation between horizontal layers in the thickness direction, has been extensively studied (Kelly 1977). The vertical density profile is now universally accepted as a parameter for characterizing wood composite pressing schedules. The parallel concept of horizontal density distribution (HDD), which defines the nonuniformity of horizontal density in the plane of the board, has yet to be characterized in short fiber wood composites. The relationship of HDD to raw material characteristics, board formation methods, and board properties is not well understood.

This paper provides a literature review and develops the background for subsequent studies on the origin and implications of HDD to

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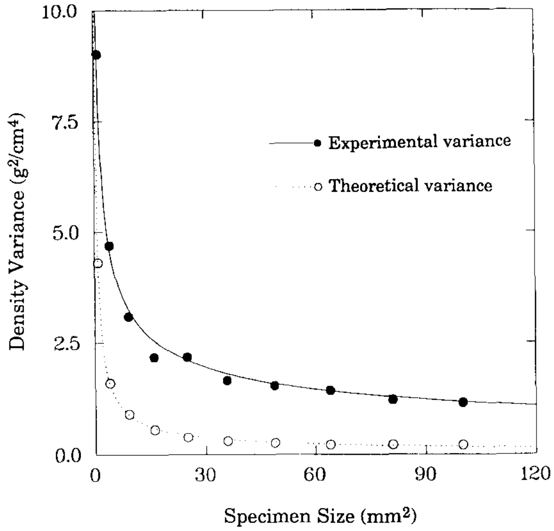

FIG. 1. Variation of DMD of one machine made paper as a function of specimen size (Data from Corte 1970).

flake- or particle-board type composite systems.

LITERATURE REVIEW

Paper structure

Paper materials could be viewed as a fibrous network with a structure exhibiting local variations of the areal mass density in the direction of the plane. The term "distribution of the mass density (DMD)" was coined to describe this nonuniformity in horizontal density (Corte 1969). The use of geometrical probability to study random paper networks was initiated as early as 1953 (Le Cacheux 1953). For random networks, the probability of finding r fiber centers in a square is given by the Poisson equation (Kallmes and Corte 1960)

$$p(r) = e^{-R}R^r/r! \tag{1}$$

where, R is the average number of fiber centers in a square. The variance of DMD, Var(d) is therefore given by (Corte 1969)

$$\mathrm{Var}(d) = glDk/a^2 \tag{2}$$

in which d is the variable areal mass density, g is the weight per unit length of the fiber, l is the fiber length, D is the average of d, a^2 is the specimen size, and k is a factor related to the size of fiber and specimen. The theoretical variance of DMD, calculated from Eq. (2) for one commercial paper sample at several specimen sizes, is presented by the dotted curve in Fig. 1.

Development of the β-ray absorption technique in the 1960s made the actual measurement of DMD possible. A comprehensive series of measurements comparing 24 different machine-made papers were published in 1970 (Corte 1970). Significant differences in DMD were found among these papers. The variance of DMD of the same paper sample that was used to calculate the theoretical variance was also measured by the β-ray method (Fig. 1). The much lower theoretical variance of DMD suggests that the paper uniformity can be enhanced significantly through improved formation.

No systematic studies on the effects of raw material and processing variables on DMD in paper have been reported. However, a few researchers have indicated that many properties of paper are related to DMD (Corte 1982; Seth 1990; Soszynski and Seth 1985). According to Seth (1993), increases in variance of DMD in paper structure always result in a negative effect on paper properties.

Particleboard structure

In particleboard, the concept of HDD was first proposed by Suchsland (1959). He considered an idealized particleboard structure that was made up of several discontinuous layers of particles, with voids existing between adjacent particles in any layer. Based on this simplification, a model consisting of a stack of veneers each containing equal numbers of randomly distributed holes was developed to characterize the density variation of particleboard in the plane of the board. In this model, the distribution of the sums of the veneer's thickness, as well as the density, over any small area follows a binomial distribution defined as

$$\phi(m) = \binom{n}{m} p^m (1 - p)^{n-m} \tag{3}$$

where $\phi(m)$ = fraction of total area over which the number of solid veneer elements is equal to m; n = total number of veneer layers; p = relative wood volume of each veneer layer. Because of this density distribution, the relative portion of the compressed wood particles was considered more important than the average board density for developing composite bending strength (Suchsland 1959).

Another model, consisting of narrow veneer strips arranged in mutually perpendicular layers whereby the number of veneer overlaps within a matrix element, represents the variation in the amount of wood material was also developed (Suchsland and Xu 1989). Direct measurements of internal bond and thickness swelling in these matrix elements were used to study the effect of nonuniformity of particleboard structure and other processing variables (Suchsland and Xu 1989, 1991a).

In investigating the influence of the particle size on the structural and strength properties of particle materials, another researcher also realized the importance of the horizontal density variation (Kusian 1968). He studied a geometrical model that involved localized deposition of randomly oriented particles within the boundary circular areas. First, the number of particles that were deposited in one layer was calculated. Then, the overlapping and crossing of particles between layers were considered for the multi-layer system. Since this model was based on the plane projections, both particle length and width effects were analyzed. By using the term "probability of HDD" f, Kusian showed that the relationship between f and the particle size could be expressed as

$$f = W_{mat}\sqrt{h^2 + 1}/(LtD_wF_{mat}) \qquad (4)$$

where h is the aspect ratio between particle length L and width w, W_{mat} and F_{mat} are the weight and surface area of the particle mat respectively, while t is the particle thickness, and D_w is the wood density. This analysis predicts a particleboard structure with decreasing horizontal density variation as the particle length and width increase, as graphically shown in Fig. 2. Although his analysis was mainly

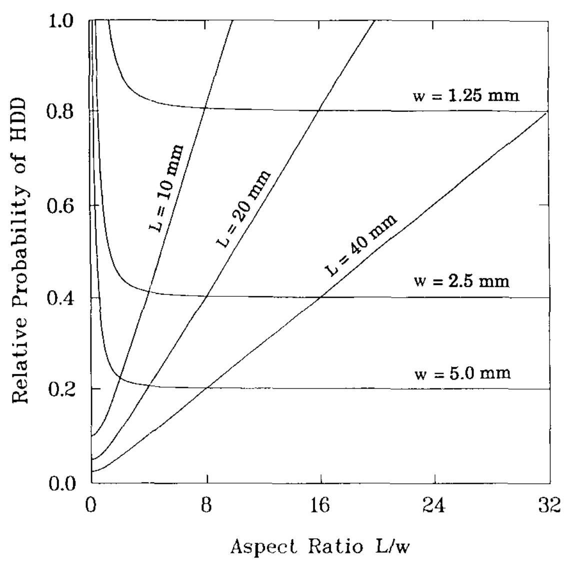

FIG. 2. Relationship between the probability of HDD, as defined in Eq. 4, and the particle dimensions, where L = length and w = width (adapted from Kusian 1968).

mathematical and based on certain particle arrangement, it is useful in understanding the possible significance of the particle size (length and width) in terms of the internal structure of a composite board.

Recently, the geometrical probability theory used for characterizing random fiber structure of paper has been extended to model and simulate composite spatial structure of a randomly distributed wood flake system (Dai and Steiner 1993, 1994). These efforts also identified the importance of the particle geometry on HDD and indicated how this information could be used to model compression behavior in flake mats.

This present study examines the phenomenon of HDD based on one commercial composite product and reports on the statistical relationship between HDD and specimen size.

MATERIAL AND METHODS

Four commercial waferboard panels, 122 cm × 244 cm each, from adjacent press loads were procured. The board density was 0.67 g/cm^3 with a nominal thickness of 11 mm. For the initial study, one panel section of 57 cm × 57

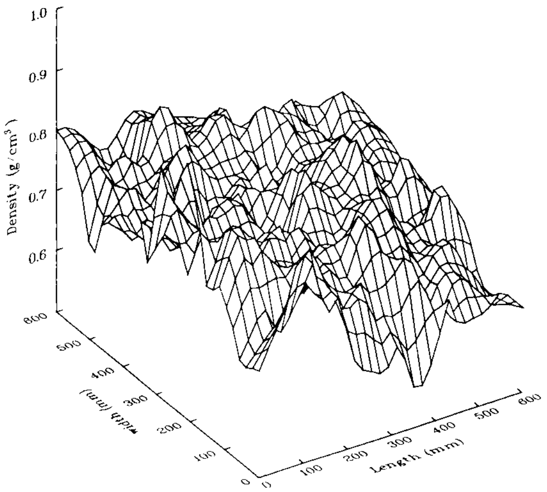

FIG. 3. Horizontal density variation in waferboard determined using a specimen size of approximately 4 cm^2.

cm was completely cut into small specimens of 2 cm × 2 cm to examine the density variation phenomenon.

For other HDD analysis, a more random process of specimen selection was applied. First, the commercial waferboard panels were cut into sections of approximately 25.2 cm × 25.1 cm, with 40 of these being randomly selected. Density and standard deviation of density at this specimen size were determined. These sections were then cut in half to give 78 specimens of 25.2 cm × 11.7 cm, for which the density and standard deviation of density were also determined. This partitioning process and the density analysis continued again to yield 153 specimens of 11.7 cm × 11.6 cm. At this stage, 30 of these latter-sized specimens were randomly selected and were in turn cut into 58 specimens of 11.6 cm × 5.4 cm and 116 specimens of 5.4 cm × 5.4 cm; and the density variations were analyzed.

A drilling technique was used on 40 randomly selected specimens of 11.7 cm × 11.6 cm to determine the density variations at smaller sizes. The specimen cross-sectional area is established by the size of the drill bit used, and the weight is taken as the weight loss of the specimen from before to after drilling. Using a random selection process (Xu 1993), about 65 holes were drilled at respective cross-sectional areas of 5.07 cm^2, 1.27 cm^2, 0.71 cm^2, and 0.31 cm^2. Standard deviation of density was calculated for each set.

RESULTS AND DISCUSSION

Phenomenon of HDD

Figure 3 presents the density variation map for a waferboard sample of approximately 57 cm × 57 cm. Individual density is measured at a specimen size of approximately 2 cm × 2 cm. Gaps of 0.3 cm, equivalent to the saw blade kerf, exist between adjacent specimens. As density varies from point to point in the two-dimensional plane, the horizontal density could be viewed as a random field. Furthermore, this density approximates a normal distribution. Figure 4 shows the distribution histogram of the density determined at a specimen size of 0.31 cm^2, together with a normal curve fitting and the statistical analysis. This approximation is expected since the individual density measured at any specimen size could always be taken as the average of several smaller-sized densities. The central limit theorem supports this approximation (Fisher 1950). This normal fitting of the horizontal density was also reported in another model investigation (Suchsland and Xu 1991b). The approximation of normal distribution suggests that the horizontal density can also be viewed as a Gaussian random field, and the standard deviation (S) of density can be used to quantify HDD to a certain extent, as S fully defines a normal distribution once the mean is known.

Within a random field, the variation of a measurement decreases as the measuring size (window) increases (Vanmarcke 1984). The relationship between the specimen size (A) and standard deviation (S) of density measured on these waferboard panels is presented in Fig. 5, which agrees with the random field theory since S decreases as A increases. This specimen size dependence has also been suggested by others (Suchsland and Xu 1991b).

Three factors contribute to HDD: variation

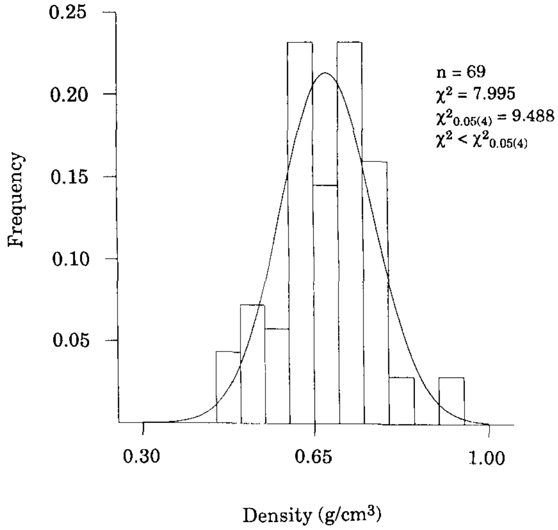

FIG. 4. Distribution characteristic of the horizontal density in waferboard at a specimen size of 0.31 cm^2. Normal fitting is justified as the calculated Pearson chi-squared statistic χ^2 is less than the corresponding critical value at a significance level of 0.05.

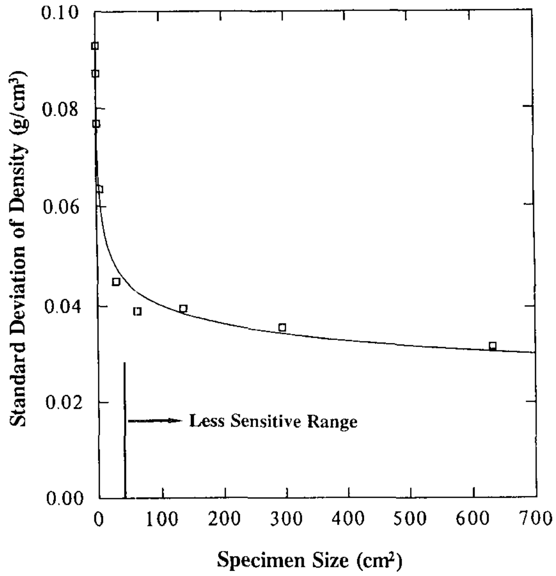

FIG. 5. Standard deviation of density vs. specimen size for a waferboard panel.

in particle density, nonuniformity in the forming process, and the existence of voids (Suchsland and Xu 1989). It can be seen from Fig. 5 that the sensitivity of the density variation in relation to the change of specimen size decreases dramatically as the specimen size exceeds 25–50 cm^2, which is referred to as the less sensitive range. As suggested by Suchsland and Xu (1991b), the variation of density in the less sensitive range is believed to be a reflection of nonuniformity in the forming process. The influences of voids and the variation of particle density that affect the HDD are likely minimized at these relatively large specimen sizes. The significantly smaller variation of the density in the less sensitive range suggests that it is also beneficial to determine the board properties at this specimen size range.

In the case of waferboard, when specimen size is less than 25 cm^2, all of the three factors contributing to HDD are probably interactive. However, the effect due to the variation of particle density decreases as the board thickness increases, because the number of particle layers increases and the variation of the average density of these layers decreases. Furthermore, it is believed that the contribution of the variation of particle density to HDD is less profound compared to that of voids. If the forming process is quite uniform, the density variation determined at relatively small specimen sizes could be considered to be caused mainly by voids. The measurement of the density variation caused by voids may reveal the influence of controllable raw material characteristics, and board lay-up (formation) methods on composite structure and board properties.

Relationship between standard deviation of density and specimen size

Let A_a and A_b represent the sizes of density sets A and B, D_a and D_b the density variables, and $Var(D_a)$ and $Var(D_b)$ the variances of densities associated with the density sets A and B. Then, if the densities of individual points are independent, the variance is inversely related to the specimen size, and the following relationship exists (Xu 1993),

$$Var(D_a)/Var(D_b) = A_b/A_a \qquad (5)$$

Taking the square root on both sides of Eq. (5), we have

$$S(D_a)/S(D_b) = \sqrt{A_b/A_a} \quad (6)$$

where S stands for the standard deviation.

Rearranging Eq. (6) and letting the equality equal a constant c, we also have

$$S(D_a)\sqrt{A_a} = S(D_b)\sqrt{A_b} = c \quad (7)$$

which could be generalized as

$$S = c(1/\sqrt{A}) \quad (8)$$

Equation (8) predicts a linear relationship between S and $1/\sqrt{A}$. However, the scatter plot of S to $1/\sqrt{A}$ for the measured waferboards as shown in Fig. 6 clearly deviates from a straight line. One possible explanation for this nonlinear trend is that the densities of the samples of various sizes are correlated. For the purpose of curve fitting, a curvelinear relationship of S to A is chosen as

$$S = c(1/\sqrt{A})^{b_1} \quad (9)$$

or

$$S = c(1/A)^b \quad (10)$$

where $b_1 = 2b$, and both b and b_1 are parameters to be determined by regression.

The form of Eq. (10) and the possible meaning and range of parameter b can be explained by considering statistical concepts. If wood composite production is under statistical control (quality control), the density at every point should vary within a certain limit around its mean (average board density). This density variation can then be considered as a stationary process (Bendat and Piersol 1980), and the measurement within this process must be positively correlated. In other words, the coefficient of correlation ρ is larger than or equal to zero.

Again consider two specimen sizes A_a and A_b, but specifically the case of $A_b = 2A_a$, and let Cov(*,*) stand for covariance. Since densities are positively correlated, we have (Xu 1993)

$$D_b = (D_a + D_a)/2 \quad (11)$$

and

$$\begin{aligned} \mathrm{Var}(D_b) &= [\mathrm{Var}(D_a) + \mathrm{Var}(D_a) + \\ &\quad + 2\ \mathrm{Cov}(D_a, D_a)]/4 \\ &= [2\ \mathrm{Var}(D_a) + 2\rho\ \mathrm{Var}(D_a)]/4 \\ &= [(1 + \rho)\mathrm{Var}(D_a)]/2 \end{aligned} \quad (12)$$

or

$$\mathrm{Var}(D_b)/\mathrm{Var}(D_a) = (1 + \rho)/2 \quad (13)$$

By using Eq. (10), we also have

$$\mathrm{Var}(D_b)/(D_a) = (1/2)^{2b} \quad (14)$$

Comparing Eqs. (13) and (14), the following relationship is established,

$$b = [\ln\{(1 + \rho)/2\}/\ln(1/2)]/2 \quad (15)$$

Therefore, parameter b is related to ρ, the coefficient of correlation between adjacent specimens. Numerically, when $\rho = 0$, $b = 1/2$ and $b_1 = 1$, according to Eq. (15). This corresponds to Eq. (8), in which no correlation exists between the density measurements. When $\rho = 1$, $b = b_1 = 0$, the variance is not a function of the specimen size. This corresponds to the situation where boards are perfectly homogeneous, i.e., the density is constant and therefore the densities between adjacent points are completely correlated. Naturally, the density variation is independent of the specimen size. Thus, parameter b possesses a physical meaning that indicates the level of correlation among the density points. As b increases, the strength of the correlation decreases.

It is interesting to note that b could not exceed 0.5. If $b > 0.5$, then $\rho < 0$ according to Eq. (15). This violates the property of the stationary process. As the nonstationary process can usually be converted to the stationary process for analysis (Bendat and Piersol 1980), the case of $b > 0.5$ can be safely excluded from the real world. Furthermore, b can not be negative. If $b < 0$, $\rho > 1$, according to Eq. (15), which does not agree with the definition of the correlation coefficient. In fact, if $b < 0$, Eq. (10) predicts an increase in the standard de-

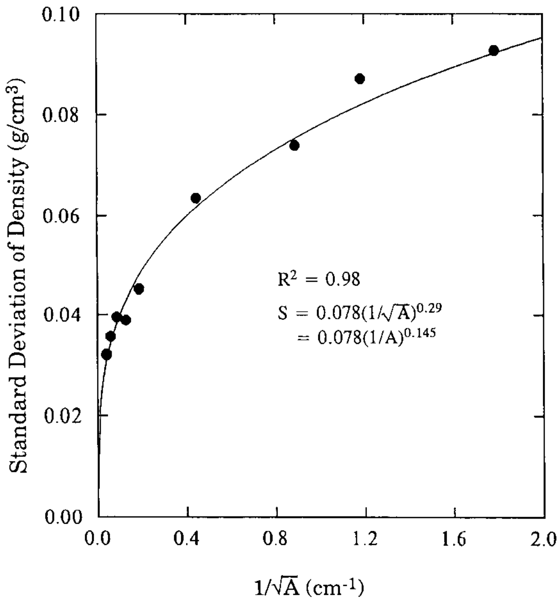

FIG. 6. Standard deviation of density vs. $1/\sqrt{A}$ where A is specimen size.

viation as the specimen size increases, which is difficult to imagine. An estimation of the S to 1/A relationship for Fig. 6 by the regression analysis results in 0.145 for parameter b, which falls within this boundary (Fig. 6).

It should be mentioned that the exact same expression as that of Eq. (10) was used to relate the variance of the crop yield per unit area to the plot size (Smith 1938). This relationship has been widely accepted and applied to predict agricultural crop yields (Kuehl and Kittock 1969; Nelson 1981). Recent comparison between geostatistics and Smith's work indicated the appropriateness of the latter, and a range between 0 and 1 was also recognized for parameter b_1 (Zhang et al. 1990).

SUMMARY AND CONCLUSIONS

A review of the literature has shown that structural nonuniformity in both wood composites and paper materials was probably recognized about 35 years ago. Since that time, only limited progress has been reported in this area for wood composites. This study has shown that the gravimetric method which includes a drilling technique is capable of detecting the micro-horizontal density variation in composites. As expected, the horizontal density variation can be viewed as a stationary Gaussian random field with the magnitude of the density variation decreasing as the specimen size increases. The Equation $S = a(1/A)^b$ is valid to model this relationship between the standard deviation of density S and the specimen size A. This knowledge will be used to further investigate the influence of raw material characteristics on HDD and board properties.

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