MODELING THE NONLINEAR MOMENT-ROTATION RELATIONSHIP OF A NAIL PLATE CONNECTOR[1]

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ABSTRACT

This paper presents a mechanics-based procedure for modeling the nonlinear moment resistance of multiple-dowel wood connections. The lack of a consistent methodology for predicting rotational resistance of multiple-dowel joints was identified as one of the barriers to adoption of a ready-to-assemble (RTA) wood framing system. Integral to the RTA system is a nail plate connector (NPC) that consists of a metal plate with multiple dowel and is used to assemble RTA framing members into complete structural systems. The principle of energy conservation is used to derive the model. The proposed procedure is formulated such that the nonlinear response of the nails and plate bearing are explicitly included in the model to accurately predict the moment-rotation relationship over a wide range of deformations. Therefore, the model provides the information on both the ultimate strength and deformation capacity needed to establish safety margins and to perform serviceability checks, respectively. The method requires input of the lateral load-displacement relationships for an individual nail and plate bearing on the end grain of wood framing member. These relationships can be readily measured using lateral test procedures or determined analytically. The formulation showed excellent correlation with test results ($R^2 = 0.98$). The proposed model presents an engineering and research tool and has the potential to promote the use of timber frames assembled with multiple dowel joints.

Keywords: Timber connectors, nail plate connector, timber engineering, moment rotation, nonlinear modeling, wood, ready to assemble building system, multiple dowel joint.

INTRODUCTION

The moment resistance of multiple-dowel connections is the subject of this paper. The lack of a consistent methodology for predicting rotational resistance of multiple dowel joints was identified as one of the barriers to adoption of a ready-to-assemble (RTA) wood framing system introduced by Platt (1998) and extended by Kochkin (2000) and Loferski et al. (2000). Integral to the RTA system is a nail plate connector (NPC) that consists of a metal plate with multiple dowels (Fig. 1.a) and is used to assemble RTA framing members (Fig. 1.b) into structural systems.

The concept of the NPC was originally devel-

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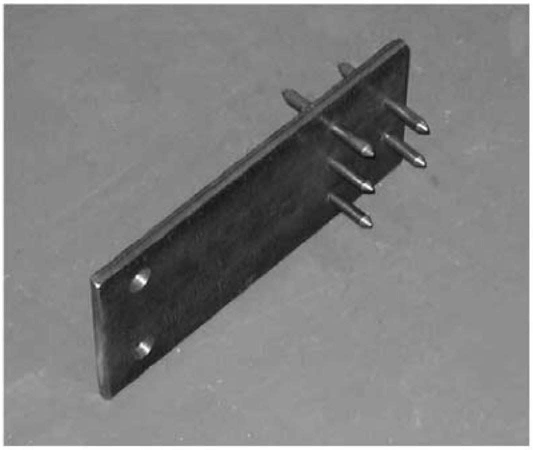
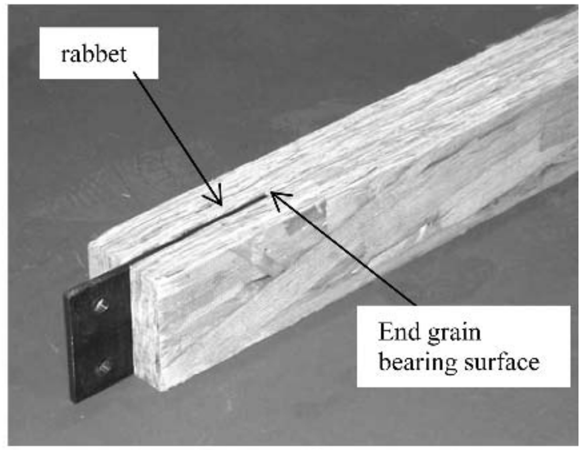


a. Nail plate connector



b. RTA framing member

FIG. 1. Key components of RTA framing system.

oped by Piskunov (1993) as a method for mechanical lamination of wood framing members as well as improved assembly of timber structures. Structures designed by Piskunov were assembled with pinned joints that did not impose rotational forces on the NPC. When used with the RTA system, the NPCs are assembled into moment-resistant joints to provide temporary bracing during the construction process. Therefore, engineering design of RTA frames requires a method for analysis of rotational resistance of the NPC.

To establish a consistent safety margin against collapse and to design the structure for an explicit failure mode, the connection capacity and entire load-deformation curve should be known. Moreover, the load-deformation relationship provides a useful method for determination of the deformation capacity of the connection and the global frame deflections that can be important for P-delta analyses and for limiting deformation demand on the finish materials. Preliminary testing indicated a nonlinear behavior of the NPC such that there was no direct correlation between the lateral resistance of an individual dowel and the moment resistance of a multiple dowel joint. This effect was in part due to the dowel placement pattern used in the NPC and in part due to the significant contribution of the connector plate bearing on the end grain of the rabbet of the RTA framing member (Fig. 1.b). Therefore, a more sophisticated model was necessary to predict the nonlinear load-deformation relationship of the NPC.

The objectives of this paper are to present a method for modeling a nonlinear moment-rotation relationship of a NPC and to validate the proposed method with test results. Although the analytical methodology presented in this paper is discussed in context of the NPC and RTA system, it is applicable to a variety of multiple dowel-type wood connections. While the focus of this paper is on analytical modeling, details on specimen configuration, manufacturing, and testing can be found in Loferski et al. (2000), Kochkin (2000), and Platt (1998).

MODEL FORMULATION

The model is formulated using the energy conservation principle, which states that the work done by the external forces, W_E, is equal to the work done by the internal forces, W_I:

$$W_E = W_I \tag{1}$$

The response of a NPC under moment loading is depicted in Fig. 2.

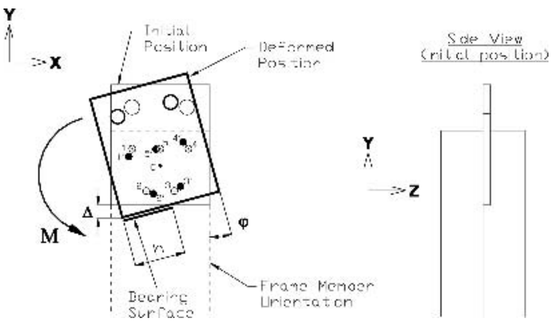


FIG. 2. Response of NPC to moment loading.

The external force acting on the NPC is moment, M, and the work done by this moment is a scalar product of the moment value and the rotation angle, φ. Because the moment is a nonlinear function of the rotation angle, the integral of this product is computed:

$$W_E = \int_0^{\varphi_{max}} M \, d\varphi \tag{2}$$

The internal work consists of the work done by the dowels, W_N, and the work done by the plate bearing on the end grain, W_b:

$$W_I = W_N + W_b \tag{3}$$

Solving this equation for the moment force and substituting into Eq. (1), the moment resisted by the NPC is found:

$$M = \frac{dW_E}{d\varphi} = \frac{d(W_N + W_b)}{d\varphi} = \frac{dW_N}{d\varphi} + \frac{dW_b}{d\varphi} \tag{4}$$

Moment resisted by the dowels

The work done by the dowels, W_N, is equal to the sum of the work done by each dowel. Work done by a dowel is a scalar product of the force, p_t, resisted by this dowel and its displacement, δ. Integral of this product is used because the dowel force is a nonlinear function of the displacement:

$$W_N = \sum_{nails} \left(\int_0^{\delta_{max}} p_t \, d\delta \right) \tag{5}$$

To enable this integration, the dowel load-displacement relationship should be described with a mathematical function. Initially, three functions were considered: power function, logarithmic function, and the Foschi equation (Foschi 1974). The power function is easy to integrate and would result in a simple final formulation, yet it significantly overestimates the initial stiffness. The logarithmic function proposed by McLain (1975) for modeling the dowel slip up to 0.1 in. overestimates stiffness at higher deformations. The model based on this function would have limited application for the connectors with complex dowel arrangements and would overestimate the connection capacity. The Foschi equation fits the data well throughout the entire range of displacements, but the complex formulation of this function leads to a very cumbersome solution of Eq. (4). A modified logarithmic function is proposed:

$$p_t = A \ln(1 + B\delta) - C\delta \tag{6}$$

This function predicts the load-slip relationship of the NPC consistently throughout the entire range of displacements and results in a relatively simple solution. The function was fit (Table 1) to the data on lateral resistance of the dowels measured by Platt (1998).

Substitution of Eq. (6) into Eq. (5) and integration of the latter for δ results in the formulation of the work done by the dowels in terms of the dowel displacement:

$$\begin{aligned} W_N &= \sum_{nails} \int_0^{\delta_{max}} (A \ln(1 + B\delta) - C\delta) \, d\delta \\ &= \sum_{nails} \left\{ -A\delta - 0.5\, C\delta^2 + \frac{A}{B} \ln(1 + B\delta) \right. \\ &\quad \left. + A\delta \ln(1 + b\delta) \right\} \end{aligned} \tag{7}$$

TABLE 1. *Estimates of parameters of Eq. (6) for a single dowel.*

Parameter	Estimate
A	692
B	394
C	3,650

To use Eq. (7) with Eq. (4), δ is expressed in terms of φ based on the premise that the tangent of a small angle ($\varphi < 15°$) is equal to the value of this angle:

$$\delta = r\varphi \tag{8}$$

The moment resisted by the dowels, M_N, is determined as follows:

$$\begin{aligned} M_N &= \frac{dW_N}{d\varphi} \\ &= \frac{1}{d\varphi}\sum_{\text{nails}}\Big\{-Ar_i\varphi - 0.5\,Cr_i^2 + \frac{A}{B}\ln(1 \\ &\quad + Br_i\varphi) + Ar_i\varphi\ln(1 + Br_i\varphi)\Big\} \\ &= \sum_{\text{nails}}\Big\{-Ar_i - Cr_i^2\varphi + \frac{Ar_i}{1 + Br_i\varphi} + \frac{ABr_i^2\varphi}{1 + Br_i\varphi} \\ &\quad + Ar_i\ln(1 + Br_i\varphi)\Big\} \end{aligned} \tag{9}$$

Moment resisted by the plate

In addition to the dowels, the moment resistance is provided by the NPC plate bearing on the end grain of the rabbet of the framing member (Fig. 1 and Fig. 2). The work done by the plate bearing on the end grain, W_b, is a scalar product of the resultant bearing force and the displacement of the point at which the resultant force is applied. Both the value of the resultant force and its position are nonlinear functions of the rotation angle of the plate. Figure 3 shows a diagram of the plate bearing on the end grain where h corresponds to the height of the bearing surface in Fig 2. To simplify the derivation, it is assumed that the plate rotates around the middle of its side (point C on the diagram). The bearing force, p_b, changes from zero at point C to its maximum value at the corner of the plate according to an unknown function. The bearing surface is divided into n segments with a finite length ΔZ. It is further assumed that the bearing force within each segment is constant for any given rotation angle.

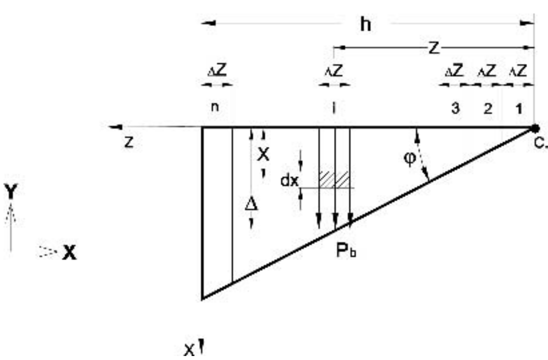


FIG. 3. Diagram of the plate bearing on the end grain.

The total work done by the bearing force is equal to the sum of the work done by the bearing forces in each segment. The work done by the bearing force within a segment is the product of the bearing force value and displacement, x. Because p_b is a nonlinear function of x, the integral from zero to the maximum displacement, Δ, is used. Furthermore, p_b has units of force per length and is multiplied by ΔZ:

$$W_b = \sum_{i=1}^{n}\int_0^{\Delta} p_b\,\Delta Z\,dx \tag{10}$$

Using the rules of integration, Eq. (10) can be modified as follows:

$$W_b = \sum_{i=1}^{n}\int_0^{\Delta} p_b\,dx\,\Delta Z = \int_0^{h}\int_0^{\Delta} p_b\,dx\,dz \tag{11}$$

The relationship between the bearing force, p_b, and plate displacement, x, was measured experimentally (Kochkin 2000) and modeled with a bilinear elastic-plastic function (Fig. 4):

$$\begin{aligned} p_b &= k_L\,x \quad \text{if } x \geq x_1 \\ p_b &= p_y \quad\;\; \text{if } x > x_1 \end{aligned} \tag{12}$$

where:

- p_b = bearing force, lb;
- x = lateral displacement of the plate, inch;
- k_L = linear slope, lb/in;
- p_y = yield load, lb; and,
- x_1 = coordinate of the yield load, inch.

The parameters of the elastic-plastic model were determined using TableCurve™ 2D v4 (SPSS Inc. 1996) software package for the plate

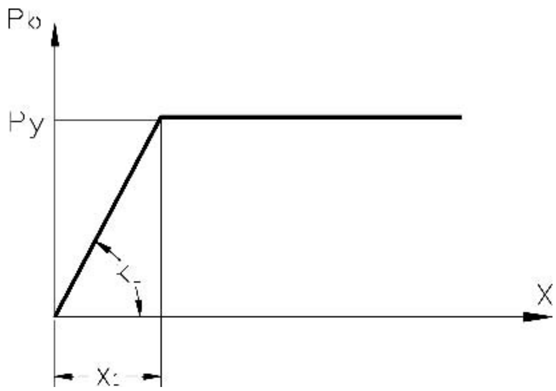


FIG. 4. Elastic plastic model.

TABLE 2. *Parameters of the elastic-plastic model.*

Function Parameters	Estimate
K_L, lb/inch	97,570
P_y, lb	4,800
X_1, inch	0.049

bearing conditions of the connection configuration depicted in Fig. 1.

After substitution of Eq. (12) into Eq. (11), the inner integral is computed as a sum of two integrals and parameters K_L and P_y are notated D and E, respectively:

$$\int_0^{\Delta} p_b \, dx = \int_0^{x_1} Dx \, dx + \int_{x_1}^{\Delta} E \, dx = \frac{Dx_1^2}{2} + E(\Delta - x_1) \quad (13)$$

The work done by the plate is computed as follows:

$$W_b = \int_0^{x_1/\varphi} \frac{D(\varphi z)^2}{2} dz + \int_{x_1/\varphi}^{h} \left(\frac{Dx_1^2}{2} + E(\varphi z - x_1) \right) dz = \frac{Dx_1^3}{6\varphi} + \frac{Dx_1^2}{2} \left(h = \frac{x_1}{\varphi} \right) + \frac{E\varphi}{2} \left(h^2 - \frac{x_1^2}{\varphi^2} \right) - Ex_1 \left(h - \frac{x_1}{\varphi} \right) \quad (14)$$

The moment, M_b, resisted by the bearing of the plate can be also determined:

$$M_b = \frac{dW_b}{d\varphi} = \frac{Dx_1^3}{3\varphi^2} + \frac{Eh^2}{2} - \frac{Ex_1^2}{2\varphi^2}$$

$$\text{for} \quad \varphi \leqslant \frac{x_1}{h} \; E = 0 \text{ and } x_1 = h\varphi \quad (15)$$

The method used to derive Eq. (15) was validated numerically. A computer code was developed using Visual Basic for Applications language to perform the numerical analysis. The error between the closed-form and numerical solutions was less than 0.1% (Kochkin 2000).

Derivation of Eq. (9) was based on the assumption that the NPC rotates around the geometrical center of the dowel pattern. The derivation of Eq. (15) was based on the assumption that the NPC rotates around the middle of the plate edge. In fact, the NPC neither rotates around the center of the rabbet nor around the geometrical center of the dowel pattern. Because the centroid moves as the NPC rotates, it has to be determined using the vector analysis for every loading step. To simplify the modeling, it is assumed that the NPC rotates around geometrical center of the dowel pattern because the dowels contribute about 70% of the connector strength and stiffness and the end grain bearing contributes the remainder. Moreover, visual examination of test specimens support this assumption. Therefore, Eq. (15) should be adjusted to account for the centroid position. Initially (i.e., $\varphi = 0$), the height of the bearing surface, h, equals to the half-depth of the member, h_0 (Fig. 5). As rotation progresses, the height of the bearing surface decreases according to Eq. (16).

$$h = h_0 - a \tan \frac{\varphi}{2} \quad (16)$$

where:

h_0 = initial bearing height;
h = bearing height after rotation by angle φ;
a = distance from the centroid to the center of the plate side.

Because the small angle theory was used to derive the solution, Eq. (16) can not be used

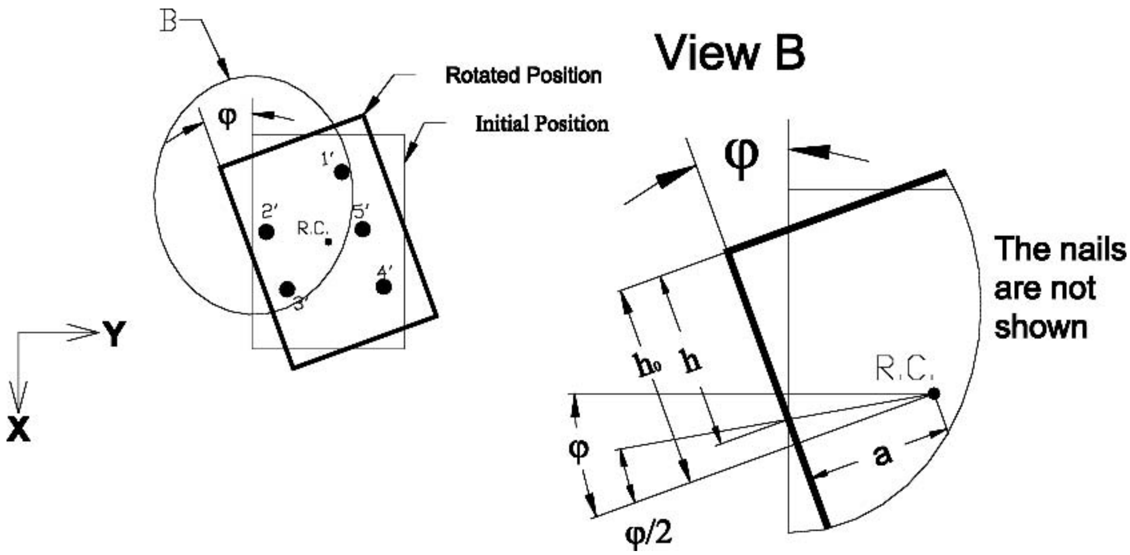


FIG. 5. Adjustment of the bearing height.

directly with the equations for M_b. In general, if a distributed load is applied to the element, the moment is a second power function of the element length. Therefore, the equations for M_b can be adjusted by factor k_h:

$$k_h = \left(\frac{h}{h_0}\right)^2 \tag{17}$$

Thus, the adjusted moment can be found:

$$M_b^{adj} = k_h\, M_b \tag{18}$$

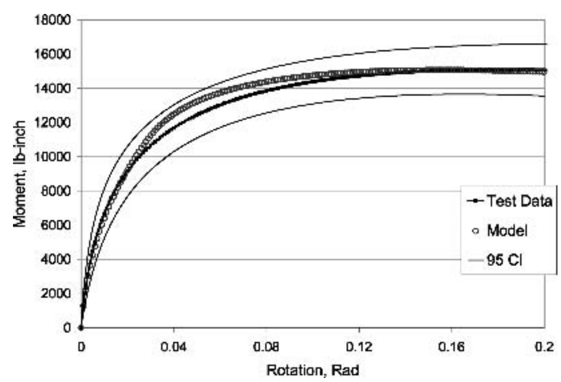


FIG. 6. Comparison of model and experimental results.

VALIDATION

The developed formulation was validated using the experimental results of the moment-rotation test on the NPC [2]. Figure 6 compares the average curve fit to the experimental data and the curve computed using the model in the range of rotational deformations up to 0.2 rad. Results of the model are within the 95% confidence interval for the experimental data. The model overestimates the moment resistance of the NPC in a range of 0.3–1.0 in with the maximum relative error of 6.7%. A regression analysis resulted in an R^2 value of 0.98. This degree of accuracy was accepted as sufficient for engineering applications.

Figure 7 shows the total moment resistance de-aggregated by the contribution provided by the dowels and by the plate bearing on the end grain. The dowels and plate contribute 75.7 and 24.3% to the maximum moment resistance, respectively. Therefore, a NPC of an identical configuration without a rabbet would provide a resistance of about 75% of that with the rabbet.

SUMMARY AND CONCLUSIONS

A mechanics-based methodology for modeling the moment-rotation relationship of a multiple dowel connections is presented and exem-

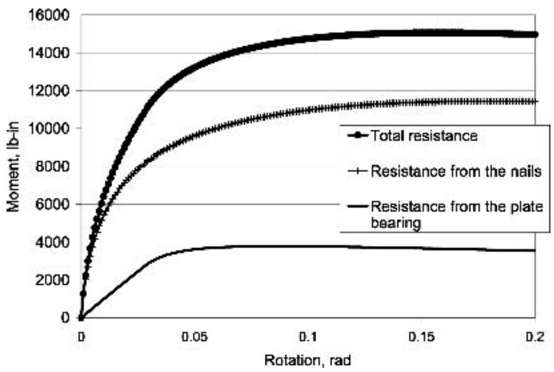


FIG. 7. The moment resistance of the NPC.

plified for the NPC. The model includes the resistance of dowels in bending and plate bearing on the end grain of the rabbet of the frame member. An R^2 value of 0.98 indicates excellent correlation with empirical results for rotational deformations up to 0.2 radian. The method requires input of the lateral load-displacement relationships for an individual dowel and plate bearing on the end grain of wood framing member. These relationships can be readily measured using lateral test procedures or determined analytically using models such as developed by Heine and Dolan (2001) for dowel slip.

The complexity of the final formulation depends on the functions selected to simulate the input load-displacement relationships. If the post-peak performance of the connection is important, the input models should include the dowel response beyond the peak load. If the plate bearing is not included in the design procedures, the model can be further simplified to account only for the contribution of dowels to the moment resistance.

The model can be used to predict linear and post-linear stiffness of moment-resistant connections. Furthermore, connection capacity and ductility can be accurately predicted. The model presents a tool for analysis of moment-resistant connections and should promote the use of timber frames assembled with multiple dowel joints. As one potential application, the moment-resistant frames can be used to provide a secondary lateral load path for shear buildings in seismic hazard regions.

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