ANALYTICAL MODELING OF RHEOLOGICAL POSTBUCKLING BEHAVIOR OF WOOD-BASED COMPOSITE PANELS UNDER CYCLIC HYGRO-LOADING

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ABSTRACT

This study was conducted to develop analytical models to predict postbuckling behavior of wood-based composite panels under cyclic humidity conditions. Both the Rayleigh method and von Karman theory of nonlinear plate with imperfection were used to obtain a closed form solution to the hygro-buckling and postbuckling. In addition, mechano-sorptive creep effects were also taken into account for the derivation of analytical models. The closed-form solutions derived for both isotropic and orthotropic materials showed a good agreement with the experimental results in terms of the center deformation of hardboard, especially in the case of the edge movements. The unrecovery deformation was much greater at the first cycle and then decreased as the number of cyclic hygro-loading increased.

Keywords: Wood-based composite panels, postbuckling, hardboard, creep, nonlinear plate, orthotropic material.

INTRODUCTION

Under natural conditions, wood and wood-based composite panels are subjected to great hygro-loads developed by moisture changes. Most of the hollow core components are made of thin wood-based composites such as fiberboard, particleboard (PB), and plywood that are fixed to studs, rafters, and joists. The out-of-plane deformation of these components is likely to occur as in-plane forces are developing due to dimensional changes, especially when their axially restrained edges are exposed to higher humidity conditions (Kang and Jung 2001a). Similarly, it could be a problem in these components because cumulative effects of lengthwise expansion of the siding may cause buckling in a localized area (McNatt 1973). In addition, flush garage doors also have the same problem as buildings are becoming more tightly constructed and door sizes increase (Marck 1972).

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It is expected that the furniture industry will use more fiberboard and PB than plywood because of the shortage of quality logs and smooth surfaces. One of the problems in using these wood-based composite panels is their greater dimensional change than that of plywood due to moisture change. As a result, the out-of-plane deformation due to hygro-buckling and postbuckling increases with an increase in the magnitude of the dimensional change. More accurate predictions of the hygro-buckling and postbuckling behavior could provide more efficient utilization and reliable designs with wood-based panels.

Many studies have been conducted on thermal buckling due to temperature changes in engineering materials such as steel and fiber reinforced plastic composites (Leissa 1987; Feldman and Aboudi 1995; Shen and Lin 1995). Basic theories that have been developed for the thermal buckling of these materials could also be employed to understand the hygro-buckling behavior of wood-based composite panels. However, direct application of these theories for hygro-buckling of wood-based composites is inappropriate since most of these theories are based on a numerical method for a simply supported condition. Nevertheless, some experimental studies have been done on the prediction models for postbuckling of wood-based panels (Spalt and Sutton 1968; Marck 1972; McNatt 1973). By contrast, little theoretical studies were undertaken until a closed form solution was recently obtained by Kang and Jung (2001b). Their result showed that the postbuckling deformation predicted by an elastic model was much larger than the experimental result, since the model used did not take rheological effects into account. In addition, they assumed that the surface of face panels attached to the core rail and stile is perfectly smooth. However, it is not the case for these panels in reality. The face panels fixed to the core rail and stile have initially curved surfaces to some extent because moisture content (MC) of the panel changes during manufacturing and the surfaces of core parts are also imperfectly flat.

Therefore, the objectives of this study were to develop analytical models to predict postbuckling behavior of wood-based composite panels under cyclic hygro-loading, and to optimize the design of hollow core type components by taking the space between the core rail and stile, geometrical shape, and material nonlinearity into consideration.

THEORETICAL MODELS

Constitutive equation

A constitutive equation of an elastic model is obtained with the total strain consisting of four strains, i.e., stress-induced elastic strain (ϵ_e), moisture-induced (free shrinkage-swelling) strain (ϵ_m), mechano-sorptive (MS) strain (ϵ_{ms}), and creep strain (ϵ_c).

$$\epsilon = \epsilon_e + \epsilon_m + \epsilon_{ms} + \epsilon_c \qquad (1)$$

However, it is difficult to derive the explicit constitutive equation if the ϵ_c is involved. Therefore, the ϵ_c was not considered in this work in order to derive the closed-form solutions, which might underestimate the total strain and overestimate the stress.

The strain for one-dimensional model may be expressed as

$$\epsilon = \frac{\sigma}{E} + \alpha\Delta M + \kappa\sigma\Delta M \qquad (2)$$

It should be noted that the unit and order of MS parameters were different among researchers (Mårtensson 1988, 1990; Dahblom et al. 1996). For example, the strains due to MS effect were expressed with different parameters such as $k\alpha\Delta M$, $k\sigma\alpha\Delta M$, and $k(\sigma/\sigma_u)\alpha\Delta M$. In order to predict drying stresses occurring in tree disk, Kang and Lee (2002) attempted to derive a two-dimensional constitutive equation without considering viscoelastic creep effect. In this study, those equations were extended for the prediction of postbuckling of wood-based panels. A two-dimensional constitutive equation can be derived as

$$\begin{bmatrix}\epsilon_x\\ \epsilon_y\\ \gamma_{xy}\end{bmatrix} = \begin{bmatrix}\dfrac{1}{E_x} & -\dfrac{\nu_{yx}}{E_y} & 0\\ -\dfrac{\nu_{xy}}{E_x} & \dfrac{1}{E_y} & 0\\ 0 & 0 & \dfrac{1}{G_{xy}}\end{bmatrix}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} + \begin{bmatrix}\alpha_x\\ \alpha_y\\ 0\end{bmatrix}\Delta M$$
$$+ \begin{bmatrix}\kappa_x & -\mu_{yx}\kappa_y & 0\\ -\mu_{xy}\kappa_x & \kappa_y & 0\\ 0 & 0 & \kappa_{xy}\end{bmatrix}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}\Delta M \quad (3)$$

Rearranging it with respect to stresses, the constitutive equation for the orthotropic plate becomes to

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\end{bmatrix} = \begin{bmatrix}C_{11} & C_{12} & 0\\ C_{21} & C_{22} & 0\\ 0 & 0 & C_{66}\end{bmatrix}\begin{bmatrix}\epsilon_x - \alpha_x\Delta M\\ \epsilon_y - \alpha_y\Delta M\\ \gamma_{xy}\end{bmatrix} \quad (4)$$

where C_{ij} are modified stiffnesses given as:

$$C_{11} = \frac{CE_x}{AC - BD}, \qquad C_{12} = \frac{BE_y}{AC - BD},$$
$$C_{21} = \frac{DE_x}{AC - BD}, \qquad C_{22} = \frac{AE_x}{AC - BD},$$
$$C_{66} = \frac{G_{xy}}{1 + G_{xy}\kappa_{xy}\Delta M} \quad (5)$$

in which A, B, C, and D terms including MS coefficients are given as

$$A = 1 + E_x\kappa_x\Delta M, \qquad B = \nu_{xy} + E_x\mu_{yx}\kappa_y\Delta M$$
$$C = 1 + E_y\kappa_y\Delta M, \qquad D = \nu_{yx} + E_y\mu_{xy}\kappa_x\Delta M \quad (6)$$

If the relation between MS parameters is given as $\mu_{xy}\kappa_x = \mu_{yx}\kappa_y$ and the relation of elastic modulus and Poisson's ratio is given as $\nu_{xy}/E_x = \nu_{yx}/E_y$, the stiffness matrix becomes symmetric (Dahblom et al. 1996). It should be noted that ΔM should be positive and the sign of α depends on only shrinkage and swelling in solving the above equations. If MS parameters are omitted, the above equation is equal to the classical constitutive equation.

Buckling and postbuckling models with imperfection

The strain energy of an orthotropic panel is given as:

$$U = \int_0^a\int_0^b\left\{\frac{1}{2}A_{11}\epsilon_x^2 + A_{12}\epsilon_x\epsilon_y + \frac{1}{2}A_{22}\epsilon_y^2 + \frac{1}{2}A_{66}\gamma_{xy}^2 + \frac{1}{2}D_{11}\left[\frac{\partial^2(w - w_0)}{\partial x^2}\right]^2 + D_{12}\frac{\partial^2(w - w_0)}{\partial x^2}\frac{\partial^2(w - w_0)}{\partial y^2} + \frac{1}{2}D_{22}\left[\frac{\partial^2(w - w_0)}{\partial y^2}\right]^2 + 2D_{66}\left[\frac{\partial^2(w - w_0)}{\partial x\partial y}\right]^2\right\}dx\,dy \quad (7)$$

in which the membrane rigidities and flexural rigidities are given by

$$(A_{ij}, D_{ij}) = \int_{-h/2}^{h/2} C_{ij}(1, z^2)\,dz \quad (8)$$

According to von Karman theory (Feldman and Aboudi 1995), nonlinear strains in the mid-plane of panels with geometrically imperfect flat surface (i.e., initial deflection, w_0) were defined as:

$$\epsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2$$
$$\epsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2$$
$$\gamma_{xy} = 2\epsilon_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} - \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \quad (9)$$

The work done due to a constant moisture differential was given as:

$$W = \int_0^a\int_0^b (N_x^M\epsilon_x + N_y^M\epsilon_y)\,dx\,dy \quad (10)$$

where the stress resultant is expressed as:

$$N_x^M = (C_{11}\alpha_x + C_{12}\alpha_y)h\Delta M$$
$$N_y^M = (C_{21}\alpha_x + C_{22}\alpha_y)h\Delta M \quad (11)$$

In order to obtain a closed form solution, the admissible displacement functions satisfying the boundary conditions for the panels with all edges clamped were assumed as follows (Kang and Jung 2001b):

$$u = u_t \sin\frac{\pi x}{a}\sin\frac{\pi y}{b}, \quad v = v_t \sin\frac{\pi x}{a}\sin\frac{\pi y}{b},$$
$$w = w_t\left(\sin\frac{\pi x}{a}\right)^2\left(\sin\frac{\pi y}{b}\right)^2,$$
$$w_0 = W_0\left(\sin\frac{\pi x}{a}\right)^2\left(\sin\frac{\pi y}{b}\right)^2, \qquad (12)$$

where u_t, v_t, and w_t are the undetermined coefficients.

Following the Rayleigh method that minimizes the total potential energy, the undetermined coefficients can be obtained.

$$\frac{\partial(U - W)}{\partial w_t} = 0 \qquad (13)$$

in which the coefficient w_t is determined by

$$\begin{aligned} w_t = \frac{1}{\sqrt{5}\pi} \times \{1/[(21b^4A_{11} + 21a^4A_{22} \\ + 10a^2b^2A_{12} \\ + 20a^2b^2A_{66})^{1/2}]\} \\ \times \{128[-\pi^2(12b^4D_{11} + 12a^4D_{22} \\ + 8D_{12}a^2b^2 + 16D_{66}a^2b^2) \\ + 3a^2b^4N_x^M + 3a^4b^2N_y^M] \\ + 50\pi^2W_0^2(21b^4A_{11} + 21a^4A_{22} \\ + 10a^2b^2A_{12} \\ + 20a^2b^2A_{66})\}^{1/2} \end{aligned} \qquad (14)$$

If the initial deflection is removed, the resultant equation is the same as one of our previous results (Kang and Jung 2001b). This result indicates that a small change in moisture content could induce buckling if there is initial deflection.

Edge movement due to MC change

The longitudinal shrinkage of normal wood from green to oven-dry condition ranged between 0.1 and 0.9% (Kollmann and Côté 1968). Therefore, longitudinal shrinkage of wood may be neglected as usual because it is much less than radial or tangential shrinkages. However, it should be taken into account to predict the dimensional change of wood composites because the shrinkage ratio of wood to wood-based panels ranged from 1/10 to 8/10.

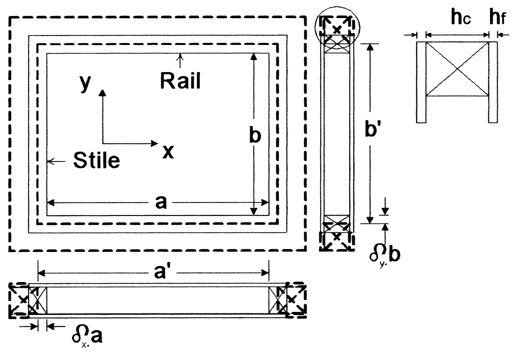

FIG. 1. Dimensional movement of edge boundary due to moisture change.

As shown in Fig. 1, a hollow core type of wooden assembly where a wood-based composite panel was fixed to stiles and rails experienced dimensional changes for the wood-based panel as well as core components (Sekino and Suematsu 2000). Therefore, dimensional changes of the assembly should be included in the predicting model. The dimensional change is expressed as Eq. (15) if it is assumed to follow an elastic behavior (Suchsland 1971).

$$\beta_{(x,y)} = \frac{\sum_{i=1}^{3} E_i h_i \alpha_i \Delta M_i/(1 + \alpha_i \Delta M_i)}{\sum_{i=1}^{3} E_i h_i/(1 + \alpha_i \Delta M_i)} \qquad (15)$$

In the case of three symmetric layers, both mechanical properties and thickness of two facing panels were assumed as the same.

$$\begin{aligned} \beta_{(x,y)} = [E_c h_c \alpha_c \Delta M_c/(1 + \alpha_c \Delta M_c) \\ + 2E_f h_f \alpha_f \Delta M_{fc}/(1 + \alpha_f \Delta M_{fc})] \\ \div [E_c h_c/(1 + \alpha_c \Delta M_c) \\ + 2E_f h_f/(1 + \alpha_f \Delta M_{fc})] \end{aligned}$$

$$= \alpha_f \Delta M_{fc} \frac{r(\alpha_c/\alpha_f)\Delta M_c/\Delta M_{fc} + 2}{r + 2} \quad (16)$$

$$r = \frac{1 + \alpha_f \Delta M_{fc}}{1 + \alpha_c \Delta M_c} \frac{E_c h_c}{E_f h_f} \cong \frac{E_c h_c}{E_f h_f} \quad (17)$$

in which ΔM_{fc} is not the same with ΔM_{ff} due to the difference of effective thickness or boundary conditions.

The rate of moisture content change of core and face panels depends on time and the difference of maximum moisture content. In general, there are differences in the rate of moisture content change because diffusion coefficients of wood and wood-based composites depend on density, moisture content, and temperature. In addition, layer thickness of the sandwiched panel component also affects diffusion coefficient.

$$\frac{\Delta M_c(t)}{\Delta M_{fc}(t)} = \frac{\Delta M_{max\_c}}{\Delta M_{max\_f}} \frac{f_c(\rho, M, T, h, t)}{f_f(\rho, M, T, h, t)} \quad (18)$$

in which $\Delta M_{max} = |M_e - M_0|$.

It should be noted that both β_x and β_y are affected only by rail and stile, respectively.

The incorporation of the edge movement into Eq. (11) gives the resultant hygro-stress in both x and y directions expressed as

$$N_x^M = [C_{11}(\alpha_x \Delta M - \beta_x) + C_{12}(\alpha_y \Delta M - \beta_y)]h$$
$$N_y^M = [C_{21}(\alpha_x \Delta M - \beta_x) + C_{22}(\alpha_y \Delta M - \beta_y)]h \quad (19)$$

EXPERIMENTAL PROCEDURES

Commercial hardboard (3 mm thick and with a density of 950 kg/m^3) bonded with urea-formaldehyde adhesive and manufactured by dry process was cut into two different sizes, i.e., 300 × 300 mm and 400 × 400 mm for buckling test specimens. Specimens for MOE, dimensional and moisture content changes were cut along the four edges. Wood species for the core panel was lauan (*Dipterocarpus* spp.) with the dimension of 60 mm × 30 mm and a density of 500 kg/m^3.

All the specimens were preconditioned in an environmental chamber at 25°C and 60% relative humidity (RH), which was slightly lower than the EMC indoors. The panels were assembled symmetrically with polyvinyl acetate (PVAc) adhesive as shown in Fig. 2, and were placed in a constant humidity chamber under 60% RH at 25°C. An 8-mm-diameter hole was made in the middle of core components to reduce an equilibrium difference between top and bottom face panels used for buckling and dimensional change of the specimens. Hollow core type assemblies were prepared with a pressure of 0.3 MPa for 3 h. For cyclic humidity, specimens were conditioned at two different humidity conditions (80% and 90% RH at 25°C) for 100 h and then kept at 60% RH for 100 h. These steps were regarded as one cycle, and were repeated for humidity cycling. Center deflections of hardboard panel and core bar were measured at some intervals by using a dial gauge. The change of Young's modulus was obtained from both the initial and equilibrium moisture contents.

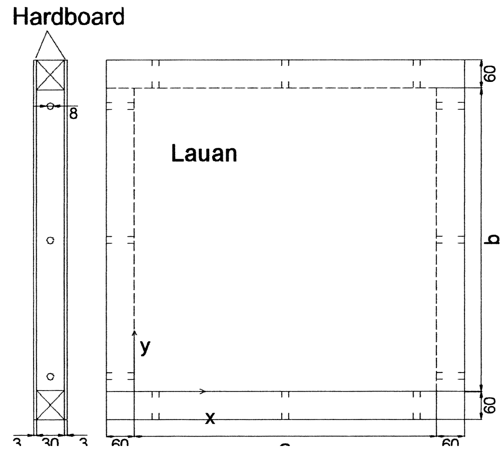

FIG. 2. Specimen for buckling and postbuckling test.

RESULTS AND DISCUSSION

Sorption and dimensional change

Figure 3 shows the changes of moisture content of a hardboard sample under two different cyclic humidity conditions. The results showed that the first cycle at both 60–80% RH and 60–90% RH produced much greater ΔM than other subsequent cycles. The equilibrium

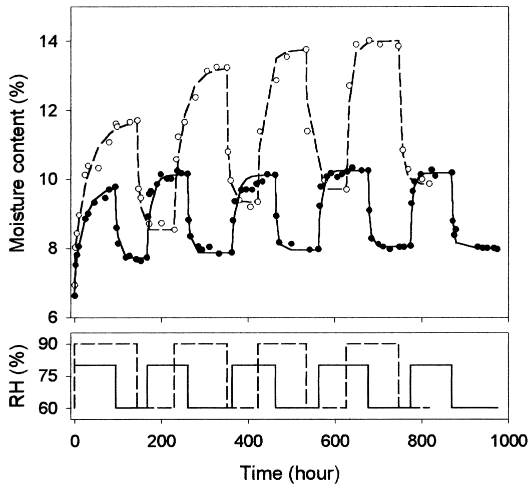

FIG. 3. Hardboard MC changes at two different cyclic conditions. Solid line: predicted MC for 60–80% RH; dashed line: predicted MC for 60–90% RH; Circle: measured MC.

moisture content (EMC) of adsorption and desorption was almost constant after the second cycle of 60–80% RH condition. However, the EMC gradually increased with an increase in the number of cycles for 60–90% RH condition. But, the difference in EMC was relatively small. This might be attributed to the presence of urea-formaldehyde (UF) resin as binder for the hardboard. As is well known, the UF resin degrades through hydrolysis under higher RH conditions (Dunky 1998). In consequence, the breakage of adhesive bond could produce voids in the board, which might also facilitate the diffusion of moisture under higher RH conditions.

Mårtensson (1988) assumed that the diffusion coefficient of moisture was 0.5×10^{-6} cm^2/s for hardboard manufactured by wet process. The diffusion coefficient of hardboard used for this study was approximately in that range (Table 1). ΔM was predicted by analytical equation to diffusion as shown in Fig. 3 (Crank 1975). The diffusion coefficient of the hardboard specimen depended on adsorption and desorption as well as the number of cycles. It was very close to the value of particleboard reported by Wu and Suchsland (1996). The diffusion coefficient was larger during desorption than during adsorption as in solid wood. But the ratio of adsorption to desorption diffusion coefficient was not constant, and was dependent on the number of cycles and ΔM. As the number of cycles increased, the diffusion coefficient also increased. This result might be due to spring-back, stress release, void increase in the panel, etc. Contrary to solid wood, the diffusion coefficient of hardboard was small for larger MC. This result showed an agreement with other results (Fig. 4).

Wu and Suchsland (1996) showed that diffusion coefficients of particleboard decreased with increasing MC. They attributed this to the difference in the moisture transfer mechanism between solid wood and particleboard. In other words, water-vapor diffusion through voids may be dominant in hardboard and particleboard, while bound water diffusion may play a more important role in solid wood.

Figure 5 shows dimensional changes of

TABLE 1. *Apparent diffusion coefficients of hardboard during adsorption and desorption.*

Parameter	No. of cycle				
	1	2	3	4	5
D_a	0.1	0.2	0.2	0.3	0.4
($\times 10^{-6}$ m^2/s)	(0.07)	(0.1)	(0.15)	(0.25)	(—)
D_d	0.4	0.5	0.6	0.6	0.6
($\times 10^{-6}$ cm^2/s)	(0.4)	(0.4)	(0.3)	(0.3)	(—)
MC	6.6–9.8–7.7	7.7–10.1–7.9	7.9–10.1–8.0	8.0–10.3–8.0	8.0–10.2–8.0
(%)	(6.9–11.7–8.5)	(8.5–13.2–9.3)	(9.3–13.8–9.7)	(9.7–14.0–9.8)	(—)
ΔM_{maxf}	3.2	2.4	2.2	2.3	2.2
(%)	(4.8)	(4.7)	(4.5)	(4.3)	(—)

The values in parentheses are measured values at 60–90% RH.

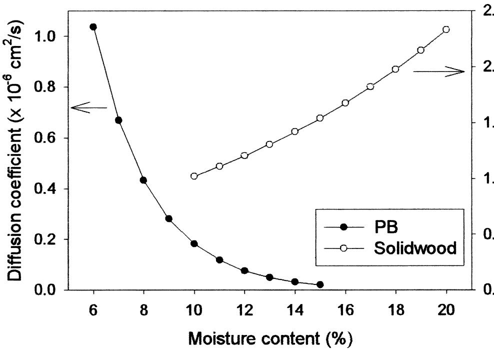

FIG. 4. Effect of moisture content on diffusion coefficients at 25°C. PB: $D = 1.42 \times 10^{-5} \exp(-0.437M)$ for adsorption (Wu and Suchsland 1996); solid wood: $\log_{10}D = 4.07 - 5.28\rho + 3.01\rho^2 - [2544/(T + 273)] + 0.036M$ (Sadoh and Yamazoe 1993).

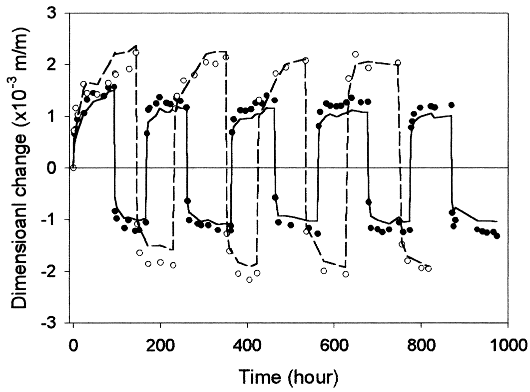

FIG. 5. Dimensional changes of hardboard at two different cyclic humidity conditions. Solid line: predicted for 60–80% RH; dashed line: predicted for 60–90% RH; Circle: measured.

hardboard at two different cyclic humidity conditions. The dimensional changes were constant for each cycle since the change of MC was constant. The rates of both dimensional changes and ΔM were almost same for each cycle. The prediction curves were calculated by using the coefficient of dimensional change that was obtained by oven-drying test samples. These prediction curves were pretty similar to the experimental results.

Postbuckling of panel without edge movement

In order to predict postbuckling behavior of panel, it was necessary to measure an initial flatness. However, it was hard to measure the initial flatness of board in practice. Therefore, some errors due to the imperfection was likely to be associated with the predicted results. In isotropic panels, Eq. (14) was modified to a simple equation as follows:

$$w_t = \frac{1}{\sqrt{5}\,\pi} \times \{[128(-A_D + 3N^M(a^2b^4 + a^4b^2)) + 50\pi^2 W_0^2 A_D]A_E\}^{1/2} \qquad (20)$$

in which

$$A_E = A_{11}[21b^4 + 21a^4 + 10va^2b^2 + 10(1 - v)a^2b^2]$$

$$A_D = \pi^2 D_{11}[12b^4 + 12a^4 + 8va^2b^2 + 8(1 - v)a^2b^2]$$

$$N^M = C_{11}\alpha_x(1 - v)h\Delta M$$

Postbuckling of panel with edge movement

In order to predict the edge movement and its rate of core rail and stile using Eq. (16), the adsorption rate of both core rail and hardboard attached to the core with adhesive was required. Adsorption diffusion coefficients of both core rail and face hardwood panel were assumed as 0.2×10^{-6} cm²/s and 1.0×10^{-6} cm²/s, respectively. The diffusion coefficient in the direction parallel to hardboard surface was assumed to be about ten times larger than that of the one perpendicular to the surface (Mårtensson 1988). Table 2 shows both EMC and ΔM_{max} of hardboard and core rail for each condition.

In one-dimensional analysis, numerical analysis has been done for PB and composite panel glued with high pressure laminate (HPL) by Wu and Suchsland (1996). To derive an analytical solution to moisture constant changes, the following equation was used with an

TABLE 2. *The EMC of hardboard and lauan used for calculation.*

Humidity level	Hardboard EMC (%)	Hardboard ΔMC_{max} (%)	Lauan EMC (%)	Lauan ΔMC_{max} (%)
60%	6.8	—	10.8	—
80%	9.3	2.5	15.8	5.0
90%	11.4	4.6	20.2	9.4

assumption that the diffusion coefficient was constant with MC.

$$\Delta M(t) = \Delta M_{max}[1 - \psi_x(D_x, w, t)\psi_y(D_y, h, t) \quad (21)$$

in which

$$\psi(D, h, t) = \frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\exp\left[-(2n+1)^2\pi^2\frac{Dt}{h^2}\right]$$

However, two-dimensional analysis for a composite beam goes through the following steps.

1. Find the effective diffusion coefficient in the y direction (Crank 1975).

$$D_{es} = \sum_{n=1}^{N} h_i \Bigg/ \sum_{n=1}^{N} \frac{h_i}{D_i} \quad (22)$$

In the case of symmetric three-layers, it becomes

$$\therefore \frac{D_{es}}{D_c} = D_f\frac{2h_f + h_c}{2h_f D_c + h_c D_f} = \frac{2h_f + h_c}{2h_f D_c/D_f + h_c} \quad (23)$$

2. Find the rate of MC change through both surfaces of hardboard.

$$\Delta M_{yf}(t) = \Delta M_{max\_f}[1 - \psi(D_f, 2h_f, t)] \quad (24)$$

3. Find the rate of MC change for core rail or stile using the relationship between the MC change rate and the effective diffusion coefficient in the y direction.

$$\frac{2\rho_f h_f \Delta M_{yf} + \rho_c h_c \Delta M_{yc}}{2\rho_f h_f \Delta M_{max\_f} + \rho_c h_c \Delta M_{max\_c}} = [1 - \psi(D_{es}, 2h_f + h_c, t)]$$

$$\Delta M_{yc}(t) = \frac{1}{\rho_c h_c}\{-2\rho_f h_f \Delta M_f(t) + (2\rho_f h_f \Delta M_{max\_f} + \rho_c h_c \Delta M_{max\_c}) \times [1 - \psi(D_{es}, 2h_f + h_c, t)]\} \quad (25)$$

4. Find the change rate of MC in both the x and y direction using the superposition principle.

$$\Delta M_f(t) = \Delta M_{max\_f}[1 - \psi_{xf}(10D_f, 2w, t) \times \psi_{yf}(D_f, 2h_f, t)]$$

$$\Delta M_c(t) = \Delta M_{max\_c}[1 - \psi_{xc}(D_c, w, t)\psi_{yc}] \quad (26)$$

in which

$$\psi_{yc} = 1 - \frac{\Delta M_{yc}}{\Delta M_{max\_c}}$$

Substituting Eq. (26) into Eq. (16) gives the edge movement, and then the resultant hygro-stress can be obtained from Eq. (27).

$$N^M = C_{11}h(1 + v)(\alpha\Delta M - \beta) \quad (27)$$

Table 3 shows the parameters used in the simulation of center deflection of hardboard panels. There was a large difference in MS behaviors between wood and wood-based composites, as well as among types of wood-based materials depending on the size of furnish and types of components in the panels. However, the k value, one of MS parameters, was obtained from the result of Mårtensson's work (1988). In this study, however, both κ_{xy} and μ_{xy} were assumed as in Table 3 since there were limited data available.

As shown in Fig. 6, the rate of core MC change was much slower than the one of hardboard and did not reach equilibrium even after 140 h. Figure 7 also shows the change rates of MC and dimension of hardboard at both the glued and non-glued locations. The rate of MC change of the face hardboard showed much greater differences between glued and non-glued conditions. The ratio of dimension

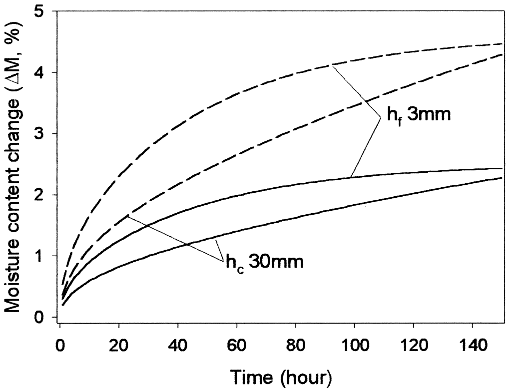

FIG. 6. Predicted moisture content change of hardboard and lauan assembly. Solid line: 60–80% RH; dashed line: 60–90% RH.

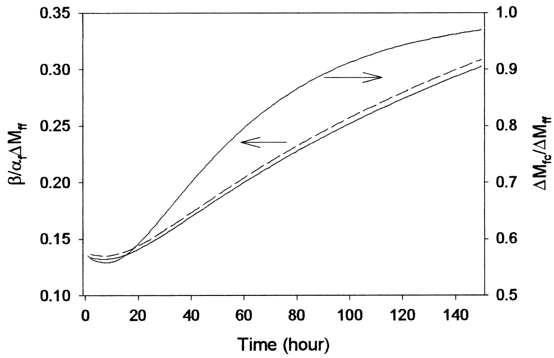

FIG. 7. The specific edge movement and the relative moisture content change of hardboards. Solid line: 60–80% RH; dashed line: 60–90% RH.

change rate to assembly and hardboard did not show significant differences between two RH conditions. But they slightly decreased at the initial stage and then increased at later stages. The reason might be attributed to a greater change of MC of hardboard glued to core components.

Three specimens were deflected inward and fifteen outwards. After the first cycle, the average residual deflection was 40% of the maximum deflection. The deflection was not proportional to hygro-load, and the extent of deflection decreased as the initial deflection increased at the same MC change. This might be attributed to the fact that the residual deflection not recovered and postbuckling increased as the number of cycles increased.

The elastic model did not take into consideration the MS creep effect while MS models did. However, both models did not consider edge movement. A complete model would be an MS model that takes the edge movement into account. To derive an analytical model, two assumptions were made: 1) the EMC of core components reached equilibrium after the second cycle for the prediction of center deflection, and 2) there was no residual stress in the hardboard.

As shown in Fig. 8, the complete model gave a maximum hygro-stress of 3.5 MPa, while the MS and elastic models gave, respectively, 4.7 MPa and 6.5 MPa. This result

TABLE 3. *Parameters used for simulation of center deflection of hardboard panels.*

Parameters	Units	Values	Reference
E_f	GPa	3.7 + 0.15 (6.8–MC)	This work
E_c	GPa	10.5 + 0.10 (10.8–MC)	
G	GPa	$E_f/2(1 + \nu)$	
ν	—	0.25	
h_f	Mm	2.95 − 0.03 (6.8–MC)	This work
h_c	Mm	30.0 − 0.05 (10.8–MC)	
α_f	—	0.475×10^{-3}	This work
α_c	—	0.134×10^{-3}	
κ	MPa^{-1}	0.06α	Mårtensson (1990)
κ_{xy}	MPa^{-1}	5.0κ	Assumed
μ_{xy}	—	0.0	Assumed

* Mechano-sorptive parameters were modified using the results of references cited.

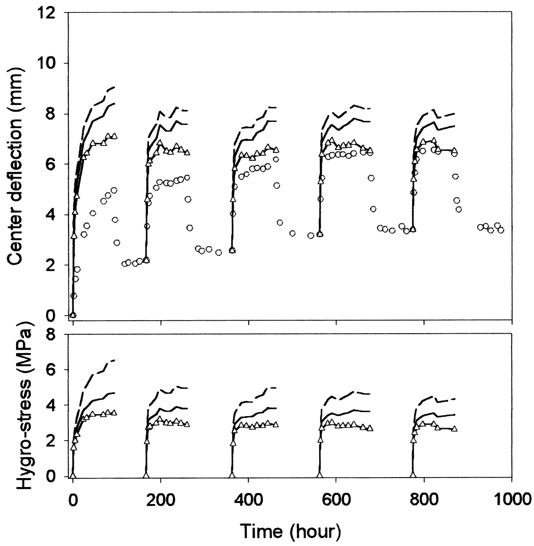

FIG. 8. Comparison between experimental and predicted center deflection, and predicted hygro-stress in midplane of hardboard at cyclic conditions, 60–80% RH. Solid line: MS model, dashed line: Elastic model, line and triangle: Complete model, circle: measured (400 × 400 mm).

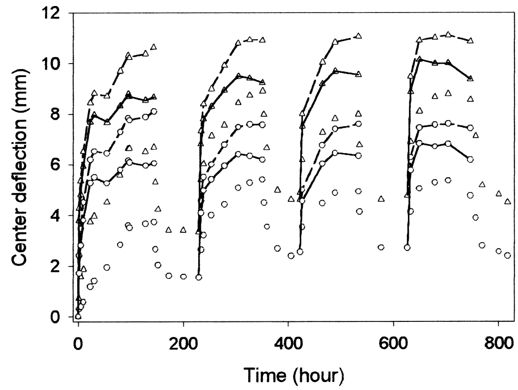

FIG. 9. Comparison between experimental and predicted center deflection of hardboard assembled at cyclic conditions, RH 60–90%. Solid line: MS model, line and triangle: Complete model, circle: measured (solid: 400 × 400 mm, white: 300 × 300 mm).

showed that hygro-stress of the complete model was much lower than that of MS and elastic models. Thus, this result indicated that the edge movement due to in-plane force might be much smaller than that of the previously reported result (Kang and Jung 2001b). Figures 8 and 9 were obtained from 60–80% and 60–90% RH, respectively. The prediction accuracy of three models were in the following order: complete model > MS model > elastic model. The measured values for deflection were much smaller for the adsorption of the first cycle than the predicted one. This result might be attributed to an increased moisture content by the water from polyvinyl acetate adhesives (non-volatile solid content: 40 wt%) as well as from the moisture adsorption in air. In other words, the increased moisture content could have caused tensile stress in hardboard during the initial stage of the test. The simulation result of the developed models showed better accuracy at 60–80% RH than at 60–90% RH condition. This result might be due to creep effect that was not accounted for in the models because the creep effect increased with increasing MC.

CONCLUSIONS

Analytical models to predict hygro-post-buckling behavior of wood-based composite panels under cyclic humidity condition were developed by using both the Rayleigh method and von Karman theory of nonlinear plate with imperfection. In addition, mechano-sorptive creep effects were also taken into account for the derivation of analytical models.

The elastic model without considering edge movement was simple because it required few parameters. However, this model gave an overestimated value that was about twice larger than the experimental one. Thus, it would be better to apply the MS model that took edge movement into account for the prediction of center deflection. In practice, it would be an option to increase the number of core components (i.e., rail and stile) to reduce buckling caused by changes of moisture content in wood-based panels. Another option would be to use the same material for core and face panel, which had similar physico-mechanical properties. In other words, it would be better to have greater dimensional changes for rail and stiles. In addition, the moisture content of hardboard should be adjusted to prevent hygro-buckling. It might be reasonable to adjust the MC slightly greater than the EMC at the time of manufacture. And its moisture content should be decreased during adhesive setting. This study was based on limited experiments and therefore further research is needed to improve the reliability of the analytical models developed. For example, the measurement of the rate of moisture content change of core components could provide a more accurate model for the prediction of buckling behavior prediction.

NOMENCLATURE

A_{ij} = membrane rigidities
D_{ij} = bending rigidities
E_x, E_y, G = elastic constants of the plate material
h = thickness
N_x^M, N_y^M = in-plane forces due to moisture changes
u, v, w = displacement in x, y, z directions, respectively
ΔM = moisture change below fiber saturation point
M_e, M_0 = equilibrium moisture content and initial moisture content, respectively
t = time
α = free shrinkage coefficient of panel
β = shrinkage of assembly
ϵ = strain
κ = mechano-sorptive coefficient
μ_{xy}, μ_{yx} = mechano-sorptive coupling coefficient
v = Poisson's ratio
ρ = oven-dry density
σ = residual stresse

Subscripts

x, y = Cartesian coordinates
c = core
f = face
fc = face glued on the core rail or stile
ff = face between rail and stile

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