TWO-DIMENSIONAL FINITE ELEMENT HEAT TRANSFER MODEL OF SOFTWOOD. PART II. MACROSTRUCTURAL EFFECTS

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ABSTRACT

A two-dimensional finite element model was used to study the effects of structural features on transient heat transfer in softwood lumber with various orientations. Transient core temperature was modeled for lumber samples “cut” from various locations within a simulated log. The effects of ring orientation, earlywood to latewood (E/L) ratio, and ring density were related to transient heat transfer. Quartersawn lumber was predicted to have a higher heat transfer rate than that of plainsawn lumber, and lumber containing pith may have a lower heat transfer rate than peripheral lumber. The model predicts that the denser the wood material, the slower the transient core temperature response. The effects of heat transfer and thermal storage (thermal diffusivity) are discussed. The two-dimensional finite element model is useful for studying transient heat flow in softwood lumber cut from any location in a log. The development of the model is described in Part I of this series. Part III addresses the effects of moisture content on thermal conductivity. Part IV addresses the effects of moisture content on transient heat transfer in lumber.

Keywords: Finite element analysis, transient heat transfer, ring orientation, earlywood/latewood ratio, density, thermal conductivity.

INTRODUCTION

Wood is an anisotropic, porous material with complicated cellular and macro-scale structural features and material properties. The structurally induced anisotropic effects on heat and mass transfer have significant implications for lumber drying, heating of logs in veneer mills, and hot-pressing of wood composites. In Part I of this series, a two-dimensional finite element model was developed to study fundamental thermal properties of the cellular structure of softwood. The model predicts effective thermal conductivity in the radial and tangential directions based on the properties of cell-wall substance, density (porosity), and cellular alignment. The softwood lumber model presented in this paper adapts this initial finite element model to a variety of cellular alignments from lumber cut from a log; variables include growth rate, percentage of latewood, and earlywood/latewood (E/L) ratio.

This paper focuses on using the model to study the effects of growth ring density, E/L ratio, and ring orientation configurations on

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transient heat transfer in softwood. Ring density relates to growth rate, which is defined as the number of annual growth rings per inch (25.4 mm) measured in the radial direction in lumber. The E/L ratio refers to the percentage of earlywood compared with latewood in an annual ring (Fig. 1). This ratio varies from ring to ring, but the model was built with a constant ratio applied to each growth ring. Ring orientation refers to the angular relationship between the ring and the wide surface of the lumber. The term plainsawn pertains to lumber with rings at angles of 0° to 45° to the wide surface and quartersawn to lumber with rings at angles of 45° to 90° to the wide surface (Forest Products Laboratory 1999).

Understanding the difference in heat transfer for different pieces of wood could be useful in a number of applications, which include sorting lumber before drying to achieve more balanced heating in the kiln for quicker and more uniform drying and improvements in moisture distribution, predicting heating time of logs in veneer mills, and predicting time of heat sterilization for killing insects in wood.

The significant effect of wood moisture content on heat transfer rate is well documented in the literature that (Forest Products Laboratory 1999). However, the scope of this initial investigation was intended to develop finite element models that can be used to study transient heat transfer effects in wood structure at constant moisture content conditions. These initial models assume zero moisture content in wood to study transient heat transfer differences from the perspective of the wood structure without the interaction of moisture-related mass transfer and its thermal flux. The effect of moisture content on effective thermal conductivity is presented in Part III and transient temperatures in lumber is presented in Part IV of this series of papers. The goal of the work reported here is to present a two-dimensional finite element model that was developed as an analytical tool to help understand fundamental transient heat transfer in softwood lumber.

PROCEDURE

Lumber can have a different number and orientation of rings depending on where the wood is cut from the log (Fig. 2). Within each growth ring, heat transfer properties are different in earlywood and latewood as a result of the different density (or porosity) of these regions (see Part I). A two-dimensional finite element softwood lumber model was developed to simulate a log of any size from which any size lumber could be

Fig. 1. Cross-section of ponderosa pine log showing growth rings: light bands are earlywood, dark bands latewood.

Fig. 2. ANSYS simulation of wood samples “cut” from a log per number and orientation of growth rings.
“cut” from any location (Fig. 2) and analyzed in transient heat transfer conditions. The ring width and E/L ratio in a log varies as a result of many factors, including growing conditions and wood species. The initial input parameters of the model are average softwood values for ring width of 7.11 mm and an E/L ratio of 5/2, but these values can be easily modified for any analysis. Earlywood density is assumed to be 370 kg/m\(^3\) and latewood density 1,263 kg/m\(^3\), based on assumed 76% and 18% porosity of earlywood and latewood, respectively. These density values are based on cell-wall substance density minus the porosity or lumen volume (see calculation in Part I). The equation for effective thermal conductivity, \(K_{\text{eff}}\), for earlywood and latewood softwood at room temperature was determined in Part I as

\[
K_{\text{eff}} = 5.135 \times 10^{-1} (\text{density})^3 - 1.681 \times 10^{-8} (\text{density})^2 + 1.555 \times 10^{-4} (\text{density}) + 2.195 \times 10^{-2} \quad (1)
\]

Thermal properties of wood, including effective thermal conductivity \(K\) and heat capacity \(c_p\), change with temperature (Hendricks 1962; Kuhlmann 1962; Skarr 1972). The relationship between thermal properties and temperature was documented by Siau (1995) and programmed into the model for transient heat analysis using the following equations:

\[
K = K_{30\degree C}[1 - 0.004(T - 30)]
\]

\[
c_{pw} = 1260[1 + 0.004 \times (T - 30)] \quad (2)
\]

where \(c_{pw}\) is specific heat of wood cellulose (J/kg K),

\(T\) temperature (°C) between 0° and 100°C, and

\(K_{30\degree C}\) thermal conductivity at 30°C, calculated from any model equation.

For the analyses in this paper, a rectangular piece of lumber 44.5 mm by 88.9 mm was generated at a number of pre-specified locations. The program created the lumber by retaining the inside area of the rectangle with assigned properties for the lumber inside the rectangle and discarded the areas outside the lumber. The finite element coordinate system was cylindrical and concurrent with the log pith. Boundary conditions were set up as convective heat flow simulating the cooling surface temperature condition in a wood drying operation. The convective heat transfer coefficients were 12.93 W/m\(^2\)·K for the side boundaries and 9.15 W/m\(^2\)·K for the top and bottom boundaries, calculated on the basis of equations for external flow over a flat plate (Incropera and DeWitt 1981). Details of the calculations can be found in Gu (2001). The higher coefficient is based on vertical orientation or higher flow across the sides as compared to the more stagnant flow across the horizontal orientation of the bottom and top.

A temperature of 100°C was applied to all outside boundaries, with the initial lumber temperature set at 20°C. A series of transient heat transfer analyses was conducted for simulated lumber “cut” from different locations in the log with different E/L ratios and ring densities. Temperature distribution on the cross-section or temperature rise at the core of each lumber sample was determined and plotted over time.

**ANALYSIS**

All the models used for this analysis were developed using ANSYS finite element software (ANSYS 2004). Input variables were chosen for easy model modification and analysis. ANSYS finite element type PLANE35, a two-dimensional 6-node triangular thermal solid element, was used for heat transfer analysis. The mathematical solution for conduction and heat transfer of this element is based on the first law of thermodynamics, the energy conservation law (Eq. (3)):

\[
\rho C_p \left( \frac{\partial T}{\partial t} \right) = k_{\text{eff},x} \left( \frac{\partial^2 T}{\partial x^2} \right) + k_{\text{eff},y} \left( \frac{\partial^2 T}{\partial y^2} \right) \quad (3)
\]

where

\(\rho\) is density of material,

\(C_p\) is heat capacity, and

\(k_{\text{eff},x}, k_{\text{eff},y}\) are effective thermal conductivities in \(x\) and \(y\) (radial and tangential) directions, respectively.
Transient heat flow in wood

The first analysis was of transient heat transfer in a lumber sample “cut” from the pith of the log (Fig. 2, sample C). Heat flow paths and temperature changes from the material effects of earlywood and latewood were examined. Figure 3 shows the heat flux vectors in the lumber when subjected to a conventional heating process. The vectors show the relative magnitude and direction of significant heat flux along the latewood rings. The effective thermal conductivity of latewood was significantly higher than that of earlywood as a result of the higher density (lower porosity) of latewood. Thermal energy was transferred transversely through the latewood ring, then radially across the earlywood ring from one latewood ring to the next, continuing toward the center of the lumber (see inset in Fig. 3). The higher heat flux in the latewood bands implies thermal expansion and higher stresses at the earlywood and latewood boundaries. Figure 3 clearly shows differential heating along the ring orientation. Therefore, different heating rates were anticipated for lumber with different ring orientations and densities.

Figure 4 shows contour temperature profiles of the same lumber (sample C) at several times during the heating process. The profiles show that heat is transferred faster into and through the thinner latewood rings than radially across the earlywood rings. This temperature distribution is symmetrical but not totally uniform because of ring orientation and heat flow along the rings. Figure 5 shows temperatures across the width and thickness of the lumber at various heating times. Surface temperatures on the side boundaries in the direction of lumber width were higher than those on the top and bottom boundaries in the direction of thickness; for this model, the convective heat transfer coefficient on the sides is higher than that on the top and bottom. The temperature gradient across the thick earlywood rings was steeper than that across the latewood rings. This indicates that the less dense material (earlywood) has a slower heat transfer rate.

Ring orientation effect

Figure 4 compares the contour temperature profiles for samples C and B4 (Fig. 2). Sample C was cut from the center of the log (pith) and sample B4 from the periphery, with rings characterized as perpendicular to the top and bottom faces (quartersawn). Heat flow was faster in sample B4 because the rings were oriented parallel to the shorter pathway (i.e., through the thickness of the lumber). The heat transfer rate was also higher in sample B4 because the latewood cells were able to transfer more energy from the surface boundaries to the core. In contrast, in sample C energy was required to pass

Fig. 3. Vector plot of heat flux for lumber cut from center of log: (a) entire plot; (b) relative high and low heat flux directions and magnitude in earlywood and latewood.
through the less thermally conductive earlywood rings. The temperature distribution of sample B4 at each time step (Fig. 4) was relatively uniform compared to that of sample C. This could be one reason why lumber of type C is often difficult to dry in terms of time.

Comparative analysis of lumber cut in a series starting from the pith to the bark was also used
to determine the effect of ring orientation on heat transfer (Fig. 6). The width of the lumber samples in series A was aligned tangentially (i.e., plainsawn) and that of the samples in series B was aligned radially (i.e., quartersawn). The heat transfer rates of the lumber samples in series A, except for sample A1, were only slightly different from each other and were grouped below that of the series B samples. Similarly, the B samples had almost identical heat transfer rates (upper lines in Fig. 6). The heat transfer rates of the B samples were higher than those of the A samples because the heat transfer paths in the latewood rings lie in the short distance (thickness) of the lumber. The highest heat transfer rate in series A occurred for sample A1 because of its ring orientation: the semi-circular latewood rings increased the rate of heat flow to the core of the lumber. Sample A1 was quartersawn whereas A5 was plainsawn. In general, the model can differentiate between lumber with slightly different ring orientations; in this case, plainsawn lumber (series A) had a lower heat transfer rate than quartersawn lumber (series B) because heat transfer followed the latewood rings.

To compare the effect of ring orientation on the rate of heat transfer, seven lumber samples were virtually cut from a log, as shown in Fig. 2. The samples had significantly different ring orientations. Figure 7 shows core temperature as a function of time. Sample C had the lowest heat transfer rate; samples B1 and B4 had the highest transfer rate due to the quartersawn ring orientation and the effect of latewood bands through the thickness of the lumber. Samples C and B4 showed the most difference in heat transfer rate. After 3,000-s, the core temperature of sample B4 was about 7% higher than that of sample C (Fig. 7). The heat transfer rate was similar in samples D and E, between that of samples B and A. The rings of samples D and E were oriented between the horizontal and vertical directions.

We conclude that the ring orientation of lumber taken from any location in a softwood log can have an important effect on heat flow.

**Growth rate effect**

The two-dimensional heat transfer lumber model also provided an opportunity to study the effects of ring width or growth rate and the E/L ratio (sometimes called latewood percentage) on heat transfer in oven-dry wood. Three plainsawn and three quartersawn samples with different ring widths were generated by the model (Fig. 8). The E/L ratio (5/2) remained the same for all analyses. Temperature rise values at the core of the six simulated oven-dry lumber samples were compared to examine the effect of growth rate. Fast growth produces more material in an annual ring than does slow growth, resulting in a wider ring. Three ring widths were simulated by the model: 3.55, 7.11, and 14.2 mm. As illustrated in Fig. 8, no significant difference in heat transfer
occurred among the different growth rates for either orientation. For a given E/L ratio, the ring width in a lumber had no effect on transferring heat from the surface to the core since the density of the lumber (plainsawn or quartersawn) was the same. The density of individual rings was the dominant factor in heat transfer.

Figure 8 reiterates the significant difference between lumber with differing ring orientations. The heat transfer rate was higher in quartersawn lumber samples compared with plainsawn.

**Earlywood/latewood ratio effect**

The previous growth rate effects were examined under the assumption that all three lumber samples had the same E/L ratio. We expected the E/L ratio to have an effect on the heat transfer of wood because of the significant difference between the thermal properties of earlywood and latewood. Four cases were simulated with ring orientation as shown for lumber sample B4 in Fig. 2. The E/L ratio was 5/2 for the first analy-
sis and the opposite (2/5) for the second analysis. Though the latter ratio does not exist in a real tree, it was used for comparison and clear demonstration of the trends. The assumptions for the remaining two analyses were lumber consisting of only earlywood or only latewood, again extreme cases.

The transient core temperatures of the four lumber simulations are compared in Fig. 9. The simulated lumber with all-latewood material had the lowest core temperature profile and the all-earlywood lumber the highest profile. The temperature profiles indicate that the more latewood material in a growth ring, the slower the change in transient core temperature for oven-dry softwood. The higher heat transfer coefficient of dense latewood material suggests that lumber with a higher percentage of latewood would have a high heat transfer rate and its core temperature would reach a higher temperature faster than would lumber with less latewood. However, the results from the model show the opposite (Fig. 9). Examination of the governing equation (Incropera and DeWitt 1981) for two-dimensional transient heat conduction (Eq. (4)) confirms the model prediction:

$$\rho C_p \left( \frac{\partial T}{\partial t} \right) = k_{\text{eff},x} \left( \frac{\partial^2 T}{\partial x^2} \right) + k_{\text{eff},y} \left( \frac{\partial^2 T}{\partial y^2} \right)$$

(4)

where

- $\rho$ is material density,
- $C_p$ is heat capacity, and
- $k_{\text{eff},x}$, $k_{\text{eff},y}$ are thermal conductivities in $x$ and $y$ (radial and tangential) directions.

If the term for volumetric heat capacity ($\rho C_p$) is switched to the right side of the equation and $k_{\text{eff},x}$ and $k_{\text{eff},y}$ are assumed to be equal, then Eq. (4) becomes

$$\frac{\partial T}{\partial t} = \frac{k_{\text{eff}}}{\rho C_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

(5)

The term ($k_{\text{eff}}/\rho C_p$) is called thermal diffusivity (a) and measures the ability of a material to conduct thermal energy relative to its ability to store thermal energy. The calculated thermal diffusivity for oven-dry latewood is higher than that for earlywood due to the higher density of latewood. For a given temperature change per time (rate) ($\partial T/\partial t$) and higher diffusivity, latewood has a lower temperature gradient ($\left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$) in the $x$ and $y$ (radial and tangential) directions and earlywood, which has low diffusivity, has a higher temperature gradient. A higher/lower ($\left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$) results in a faster/slower temperature change at the center of a piece of lumber. Therefore, the more dense the oven-dry softwood material, the slower the tran-
sient core temperature change, as predicted by the model (Fig. 9). The conduction heat flux, however, is still higher in latewood than in earlywood (Fig. 10) because of the higher thermal conductivity of dense material. Heat transfer took a longer time to reach the core in the all-latewood lumber because the greater density of latewood enables it to store more thermal energy compared to earlywood.

In a real case, balsa is a better insulator than oak because it is less dense and thus transfers less thermal energy as a result of low thermal conductivity. However, the temperature of balsa lumber may reach equilibrium faster than the temperature of oak because of the higher density of oak. This interaction of heat transfer with temperature may not be easily determined when first considering the heat transfer process in oven-dry wood. Therefore, the model is a good tool to help predict specific outcomes for different input conditions.

The authors believe the two-dimensional finite element model described here can be used toward understanding the transient heat transfer effects for the complicated interaction of ring orientation, E/L ratio, and ring density. For any size lumber cut from any location in a log, this two-dimensional finite element model realistically describes transient heat transfer effects in softwood lumber. Part III of this series addresses the effects of moisture content on transient heat transfer in softwood lumber.

CONCLUSIONS

A flexible two-dimensional finite element heat transfer model for softwood was applied to lumber with various macro-structural character-
istics to examine effects on transient heat transfer. Our analyses show that ring orientation, the ratio of earlywood to latewood (E/L ratio), and ring density have significant effects. Although this study examined only oven-dry wood with 0% moisture content, further analyses with increasing moisture contents are easily adapted and is reported in Part IV of this series.

This fundamental approach to studying heat transfer in wood has numerous practical applications, which include optimizing drying schedules for different cuts of lumber (plainsawn vs. quartersawn), determining heat sterilization times for killing insects, and determining heat curing times for solid wood and composites.

Specific conclusions are as follows:

- **Heat transfer in softwood is affected by ring orientation.** Heat flux is higher through latewood compared with earlywood.
- **The heat flow path, which follows ring orientation, determines the heat transfer rate in wood.** The rate of heat transfer to the core is higher in quartersawn than plainsawn lumber because of the shorter distance of the heat pathway through the thickness of the lumber via the latewood.
- **Lumber containing pith has a lower heat transfer rate to the core than does lumber cut from the peripheral area with rings parallel to the thickness direction (plainsawn lumber).** Energy transfer to the pith must pass through the earlywood, which has lower thermal conductivity than latewood.
- **Ring width or growth rate does not affect the heat transfer rate if the density and E/L ratio do not change.** Lumber density is the dominant factor in heat transfer. In most trees, when ring width or growth rate changes, the density and E/L ratio change as well. For example, suppressed growth trees usually have higher overall density with relatively narrow rings, which results in a higher heat transfer rate compared with that of faster grown trees, which have lower density and wider rings.
- **The ratio of E/L in a growth ring affects heat transfer in lumber.** Change of core temperature is dependent on the thermal diffusivity of the material. Diffusivity is a measure of the ability of a material to conduct thermal energy relative to its ability to store energy. Material with a low E/L ring ratio (denser material) stores more thermal energy than does less dense material, resulting in slower transient core temperatures.
- **This finite element heat transfer model can be easily modified for most softwood products subjected to various heat treatment conditions.**

**REFERENCES**


