A METHOD FOR PREDICTING NON-SHEAR COMPLIANCES IN THE RT PLANE OF WOOD[1]

Alan Sliker

Professor
Forestry Department
Michigan State University
East Lansing, MI 48824

(Received November 1986)

ABSTRACT

Equations were obtained relating the non-shear compliances in the radial-tangential, RT, plane to each other for a variety of hardwood and softwood species that were at 10 to 12% moisture content. From loadings made on short columns in the tangential, T, direction $S_{RT} = -0.255S_{TT} + 0.659 \times 10^{-6}$; for loadings made on columns in the radial, R, direction $S_{TR} = -0.887S_{RR} - 1.260 \times 10^{-6}$. In the compliance term S_{ij}, "i" signifies the direction of the observed strain and "j" the direction of the applied stress; i.e., S_{RT} relates strain in the R direction to stress in the T direction. Since $S_{RT} = S_{TR}$ for orthotropic materials, it follows that $S_{RR} = 0.291S_{TT} - 2.188 \times 10^{-6}$. S_{RR} and S_{TT} are entered as negative quantities in the above equations to indicate compression. Units for strain are inches per inch and for stress are pounds per square inch. These equations should be useful for finite element studies in mechanics and for studies on strains developed during wood drying.

Keywords: Compliance, Young's modulus, Poisson's ratio, stress, strain.

INTRODUCTION

There are twelve elastic constants and related compliances that could be used in the engineering design of wood structures. However, only one of these, Young's modulus in the longitudinal (L) direction, is readily available for the majority of commonly used species. The others, Young's moduli in the radial (R) and tangential (T) directions, the six Poisson's ratios, and the three shear moduli associated with the three major orthotropic planes, have not been thoroughly examined for a number of reasons. One of these is the difficulty of making appropriate measurements. Fortunately, recent advances in scientific equipment have helped to diminish this problem. Another consideration is that in the past, engineering problems could not be solved with as great precision as can be done currently with computers. Finally, extensive research activities with synthetic composites have stimulated research with natural composites such as wood.

In order to make up for the lack of engineering data on a large number of species, attempts have been made to predict elastic constants from well-known physical properties. Two of the most easily measured physical properties for wood are specific gravity (G) and Young's modulus in the longitudinal direction (E_L). Both of these quantities are available for major species in technical publications; they can also be obtained on a piece by piece basis from nondestructive testing.

One of the more detailed studies on predicting elastic constants was done by Bodig and Goodman (1973). They used their own data from plate and column

[1] This research was funded in part through the McIntire-Stennis Forest Research Act, Michigan Agri. Expt. Sta. J. Article No. 12159.

tests and data collected by Hearmon (1948) from a number of sources including the Forest Products Laboratory (Doyle et al. 1945–1946). Bodig and Goodman derived equations relating shear moduli and Young's moduli to specify gravity and to E_L. Coefficients of variation for their equations were on the order of 20%. Part of the reason for this large a variation was the result of using data from so many sources. Poisson's ratios could not be predicted as functions of G or E_L, so they proposed that the average of the values of Poisson's ratios from the data be treated as constants.

Work by Sliker (1985) suggested that equations for predicting compliances other than those for shear can be obtained from compression loading of short columns. He measured strains in the L, R, and T directions while loading short columns in the L direction. The slope of a strain versus stress plot for the data taken below the elastic limit was the value of a compliance. For instance, strain in the longitudinal direction (ϵ_L) versus stress in the longitudinal directions (σ_L) yields the compliance $S_{LL} = 1/E_L$. Similarly, ϵ_T versus σ_L and ϵ_R versus σ_L yield the compliances $S_{TL} = -\nu_{LT}/E_L$ and $S_{RL} = -\nu_{LR}/E_R$. In general terms, the compliance S_{ij} correlates the strain in the i direction with the stress acting in the j direction. E_i is the Young's modulus in the i direction, ν_{ji} is Poisson's ratio—the absolute value of the ratio of strain in the i direction to that in the j direction for loading in the j direction. The subscripts "j" and "i" can be R, L, or T except for Poisson's ratios where $j \neq i$. For an orthotropic material there exist three reciprocal relationships among compliances of the form $S_{ji} = S_{ij}$ where $i \neq j$ (Bodig and Jayne 1982). In Sliker's paper, compliances S_{RL} and S_{TL} were shown to be strongly correlated with S_{LL} from the testing of eighteen columns made from eight species. If similar relationships exist from loading columns in the R and T directions, it would be possible to predict all nine compliances of the form S_{ij} as functions of one compliance such as S_{LL}. This could be done since there are six equations relating the compliances from the three types of column loadings and three equations available from the reciprocal relationships. One of the difficulties in doing the testing required is that the compliances S_{LR} and S_{LT} are very difficult to determine experimentally because of the small strains that occur in the L direction from loading in the R and T directions. An example of measurements made of S_{LT} is given in Sliker (1972).

The objective of this research is to show that predicting the compliances S_{ij} as a function of S_{LL} is feasible for a large grouping of wood species. This will be demonstrated by making measurements on a variety of species in the RT plane where the limitation of measuring excessively small strains is not a problem. It will be attempted to show that S_{TR} can be expressed as a function of S_{RR}, that S_{RT} can be expressed as a function of S_{TT}, and that S_{RR} can be expressed as a function of S_{TT}. A future step would be to find similar relationships among the other compliances.

EXPERIMENTAL PROCEDURE

Wood selected for the experiment was material that had been in the laboratory for a period of 8 months or more. The species used are listed in Tables 1 and 2. No attempt was made to match the samples for the tangential loadings with those for the radial loadings. However, radial and tangential specimens for the following species were made either from the same board or the same log: the first sugar

TABLE 1. *Slopes of stress and strain combinations from load and strain data for members loaded in compression in the T direction.*

Species	ϵ_T versus σ_T (1/psi)	ϵ_R versus σ_T (1/psi)	$E_T = \lvert\sigma_T/\epsilon_T\rvert$ (psi)	$\nu_{TR} = \lvert\epsilon_R/\epsilon_T\rvert$	EMC @ test %	Specific gravity @ test
N. white cedar (*Thuja occidentalis* L.)	-22.27×10^{-6}	6.04×10^{-6}	44,900	0.271	11.8	0.30
Sugar pine (*Pinus lambertiana* Dougl.)	−22.53	6.22	44,400	0.276	11.7	0.31
Sugar pine (*Pinus lambertiana* Dougl.)	−14.76	4.15	67,700	0.281	10.7	0.34
Norway spruce (*Picea abies* L.)	−12.31	5.23	81,200	0.425	11.7	0.38
Douglas-fir (*Pseudotsuga menziesii* (*Mirb.*) Franco)	−5.23	2.29	191,100	0.438	11.2	0.58
Basswood (*Tilia americana* L.)	−28.19	8.24	35,500	0.292	9.9	0.39
Yellow-poplar (*Liriodendron tulipifera* L.)	−11.52	3.97	86,800	0.344	10.8	0.44
Black cherry (*Prunus serotina* Ehrh.)	−9.35	3.11	107,000	0.333	9.9	0.52
Red oak (*Quercus* species)	−8.68	2.51	115,300	0.290	10.5	0.62
Black locust (*Robinia pseudoacacia* L.)	−5.37	1.98	186,300	0.368	11.6	0.64

* Based on ovendry weight and volume at EMC at time of test.

TABLE 2. *Slopes of stress and strain combinations from load and strain data for test members loaded in compression in the R direction.*

Species	ϵ_R versus σ_R (1/psi)	ϵ_T versus σ_R (1/psi)	$E_R = \lvert\sigma_R/\epsilon_R\rvert$ (psi)	$\nu_{RT} = \lvert\epsilon_T/\epsilon_R\rvert$	EMC @ test %	Specific* gravity @ test
N. white cedar (*Thuja occidentalis* L.)	-7.25×10^{-6}	5.77×10^{-6}	138,000	0.795	11.5	0.28
Sugar pine (*Pinus lambertiana* Dougl.)	−8.56	5.44	116,800	0.635	11.4	0.32
Sugar pine (*Pinus lambertiana* Dougl.)	−7.32	3.87	136,700	0.529	11.0	0.33
White pine (*Pinus strobus* L.)	−9.53	7.54	105,000	0.791	10.4	0.33
Norway spruce (*Picea abies* L.)	−7.89	3.41	126,800	0.433	11.7	0.41
Douglas-fir (*Pseudotsuga menziesii* (*Mirb.*) Franco)	−4.30	2.14	232,700	0.497	10.8	0.57
Basswood (*Tilia americana* L.)	−10.83	8.86	92,400	0.818	10.9	0.40
Yellow-poplar (*Liriodendron tulipifera* L.)	−6.31	3.71	158,500	0.588	11.1	0.44
Soft maple (*Acer* species)	−5.27	3.68	189,800	0.699	11.7	0.51
Black cherry (*Prunus serotina* Ehrh.)	−4.33	3.19	231,200	0.737	9.9	0.52
Red oak (*Quercus* species)	−4.35	2.62	230,000	0.602	11.3	0.54
Black locust (*Robinia pseudoacacia* L.)	−3.37	1.95	296,600	0.578	11.7	0.65

* Based on ovendry weight and volume at EMC at time of test.

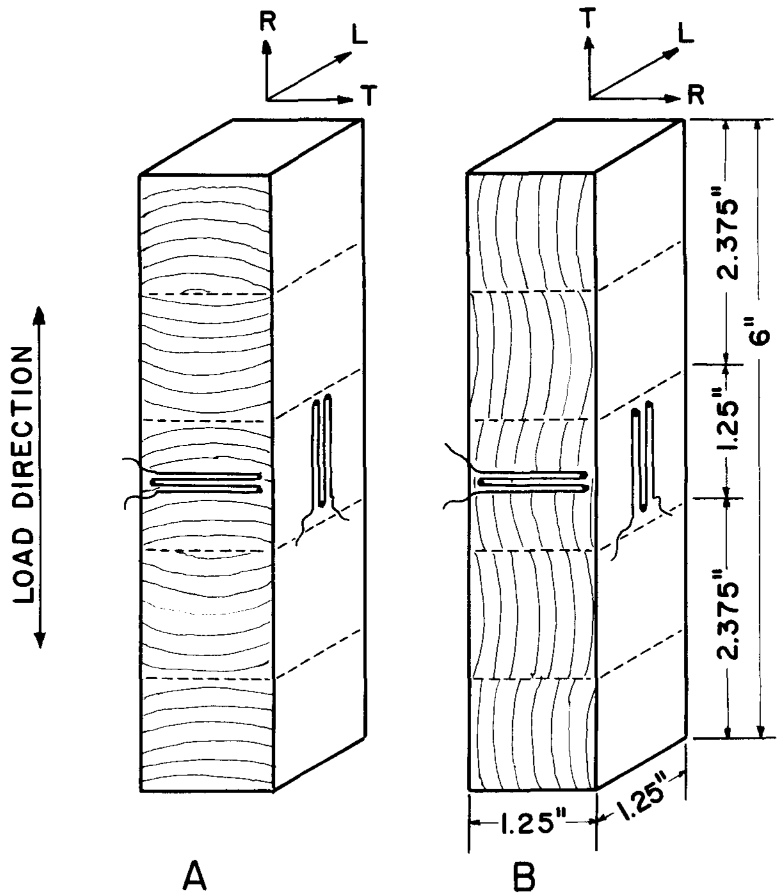

FIG. 1. Laminates for loading in the radial, A, and tangential, B, directions. Gages for measuring strain are shown oriented parallel and perpendicular to the specimen length on the middle lamina.

pine listed in each table, Douglas-fir, basswood, yellow-poplar, black cherry, and black locust. Each set of data in Tables 1 and 2 is the result of a single test on a single specimen. Except for the Norway spruce, black locust, and eastern white pine, the drying histories of the wood specimens were not known. Wood from these three species was dried directly from the green condition and attained a higher equilibrium moisture content (EMC) in a room maintained at 68 F and 65% relative humidity than did the others. These specimens, which were in excess of 12% moisture content, were subsequently placed in a room at 65 F and 50% relative humidity to reduce their moisture contents so that all specimens reached an EMC between 9.5 and 12%. Matched blocks of wood were used to monitor the moisture contents of test specimens after gages had been installed on them.

Two types of column specimens were made as shown in Fig. 1: one specimen type was made for loading in the R direction; the other for loading in the T direction. The technique for making individual specimens was to take a board and cut from it five pieces measuring 1.5 inches by 1.25 inches by 12 inches with the 12-inch dimension being in the L direction and the 1.25-inch dimension being in either the R or T direction depending on the specimen type to be made. These five pieces were then laminated with polyvinyl acetate adhesive into blanks mea-

FIG. 2. Load is applied to test specimen in compression cage by placing 10-pound weights on hanger suspended from bottom of cage. Two digital strain indicators (not shown) record strain in gages mounted parallel and perpendicular to specimen length.

suring 1.5 inches by 6.25 inches by 12 inches. After the blanks were machined to a thickness of 1.25 inches, specimens were made by cutting in the direction of the 6.25-inch dimension at 1.25-inch intervals in the L direction. The final length of each specimen was about 6 inches, as a slight trim was taken from each end to square it up.

Free-filament strain gages (Sliker 1967, 1971) made from four inches of strain-sensitive wire were applied to the specimens with thinned Duco Cement in the pattern shown in Fig. 1. Free-filament gages were used because the high modulus backing on commercial gages restrains movement of the substrate when making strain measurements on low modulus materials such as on wood perpendicular to the grain (Sliker 1971). Four gages were installed per specimen with gages on opposite faces being connected in series to eliminate the recording of bending strains. Gages were only on the center member of each five-layer laminate.

Loads were applied to the specimens by hanging 10-pound weights from a compression cage as shown in Fig. 2. A key feature of the compression cage was the placement of a spherical bearing between the top and bottom sections of the compression cage and the blocks which bore on the ends of the test specimen (Bodig and Goodman 1969). This allowed rotation of the bearing blocks so that

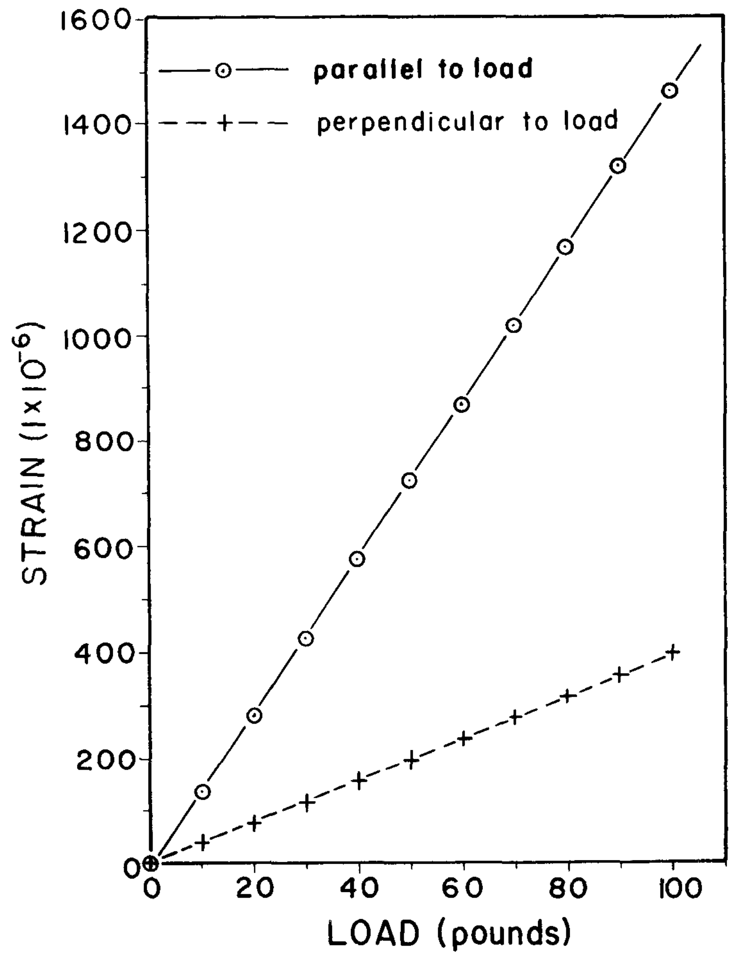

FIG. 3. Plots of strain in the T direction and strain in the R direction versus load for the northern white cedar specimen loaded in compression in the T direction. Strain in the load direction is compressive strain (negative), while that perpendicular to the load direction is tensile strain (positive).

equal pressure would be applied over the ends of the specimens. There were also universal joints connecting the compression cage to a support frame and a hanger to the compression cage. Ten 10-pound weights were applied in quick succession to the hanger suspended from the compression cage. Strains parallel and perpendicular to the load axis were read from digital strain indicators at zero load and after the application of each 10-pound weight. Strains were read to the nearest microstrain (10^{-6} inches per inch). Total loading time per specimen was less than 2 minutes.

RESULTS

Compliances S_{RT}, S_{TT}, S_{TR}, and S_{RR} were obtained from the slopes of strain versus load curves. An example of one of these curves obtained from loading in the tangential direction is given in Fig. 3. Curves for other specimens were also linear. Multiplying the slope of each curve by the cross-sectional area of the test specimen gives the compliance for the specified direction. Strain per unit stress

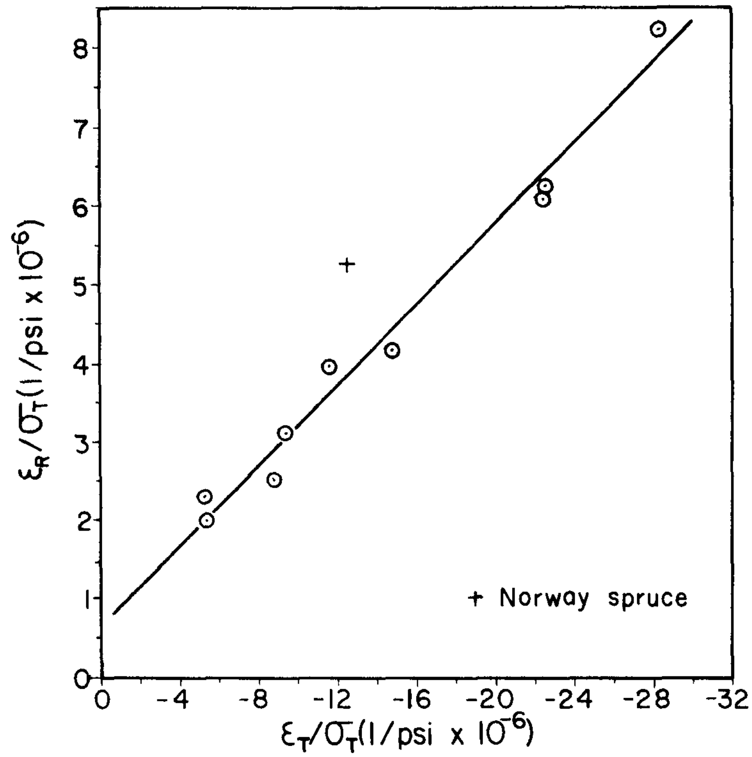

FIG. 4. Strain per unit stress perpendicular to the load direction is plotted as a function of strain per unit stress in the load direction for specimens loaded in compression in the T direction. Norway spruce was not included in the derivation of the regression line.

values (compliances) obtained from the test samples are presented in Tables 1 and 2. Minus signs are used to indicate compressive strains.

The compliance S_{RT} ($=\epsilon_R/\sigma_T$) was plotted as a linear function of S_{TT} ($=\epsilon_T/\sigma_T$) for all the specimens loaded in the T direction with the exception of Norway spruce. A graph of this is shown in Fig. 4. The equation obtained for the straight line generated from the data by least squares linear regression is:

$$S_{RT} = -0.255S_{TT} + 0.659 \times 10^{-6} \qquad (1)$$

Units for S_{RT} and S_{TT} are strain (inches per inch) per psi. Since S_{TT} is a negative number, indicating compressive strain per unit stress, S_{RT} will be positive. The correlation coefficient R for this equation is 0.990 and the standard error of estimate is 0.320×10^{-6}.

Similarly the compliance S_{TR} ($=\epsilon_T/\sigma_R$) could be plotted as a function of S_{RR} ($=\epsilon_R/\sigma_R$) for the specimens loaded in the R direction except for Norway spruce. Figure 5 is a graph of these data. The equation that results from linear regression analysis is:

$$S_{TR} = -0.887S_{RR} - 1.260 \times 10^{-6} \qquad (2)$$

Units for S_{TR} and S_{RR} are strain (inches per inch) per psi. S_{RR} is to be entered as a negative number to indicate a compressive strain per unit stress. The correlation coefficient for this equation is 0.954, and the standard error of estimate is 0.699×10^{-6}.

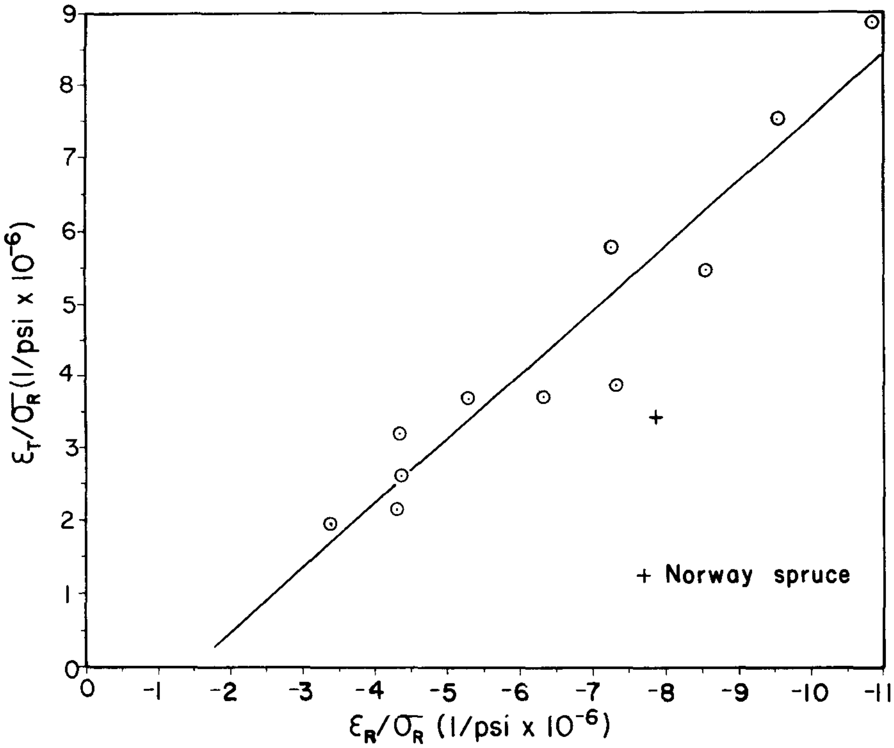

FIG. 5. Strain per unit stress perpendicular to the load direction is plotted as a function of strain per unit stress in the load direction for specimens loaded in the R direction. Norway spruce was not included in the derivation of the regression line.

The decision to leave the Norway spruce out of the calculations was twofold. First, the compliances for spruce deviated more from the regression lines than did those for any of the other test specimens for both radial and tangential loadings. Second, it appeared that the spruce samples were obtained from juvenile wood. They were made from quartersawn timbers that were 2 inches by 8 inches in cross section and which contained the pith of the tree at about the center of their cross section. Material for the specimens was cut from 5 to 16 rings from the pith where growth ring width averaged ¼ inch. In retrospect it was a poor selection of test material.

Since S_{TR} is supposed to equal S_{RT} for an orthotropic material, an expression relating S_{RR} to S_{TT} can be formed by combining Eqs. 1 and 2. The resultant equation is:

$$S_{RR} = 0.291 S_{TT} - 2.188 \times 10^{-6} \quad (3)$$

In Fig. 6, column data from Goodman and Bodig's research (1971) and data for specimens obtained from the same log or board in this report are compared with a plot of Eq. (3). There seems to be a close correspondence between the equation and the data points for the hardwoods from Goodman and Bodig's report but not for the softwoods. The larger divergences of the softwood data points from the equation occur for samples where E_T nearly equals E_R such as for the three Douglas-fir samples and one of the Englemann spruce samples, which

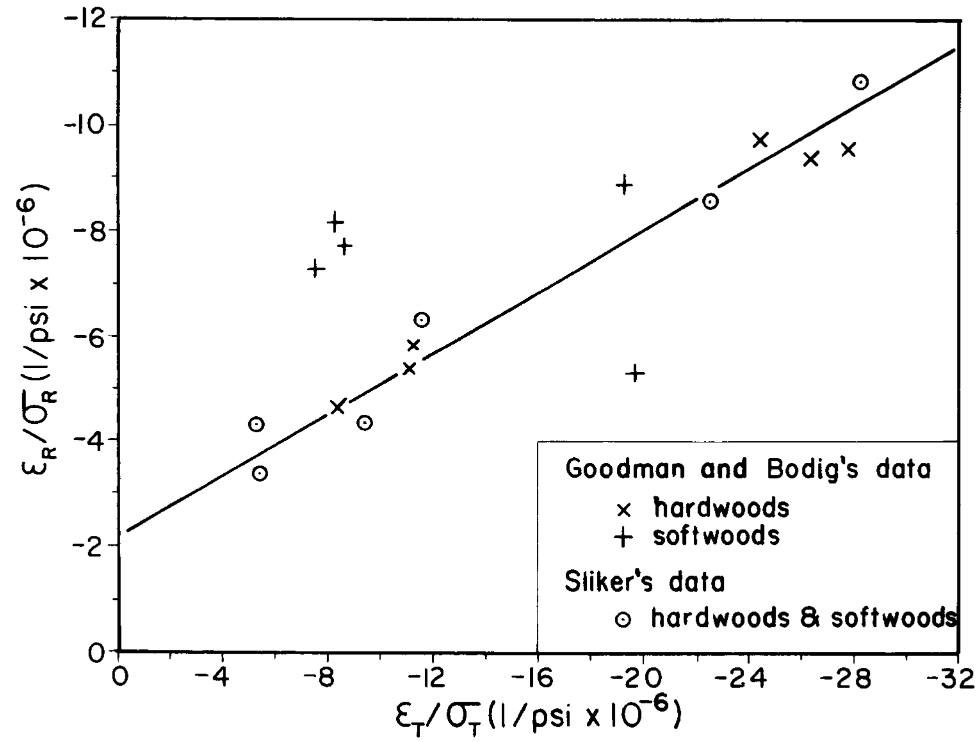

FIG. 6. Equation 3 expressing ϵ_R/σ_R as a linear function of ϵ_T/σ_T is shown as a solid straight line. Plotted points show relationships between these same two quantities for matched specimens from Goodman and Bodig's data (1971) and from this report, first sugar pine listed in Tables 1 and 2, Douglas-fir, basswood, yellow-poplar, black cherry, and black locust.

is not shown in Fig. 6 because it is off scale. Data from other sources (Doyle et al. 1945–1946; Hearmon 1948) indicate that E_R is significantly larger than E_T, so these may not be typical results.

Deviations of the measured compliances around the regression lines in Figs. 4 and 5 can be partially explained by the nature of woody tissue and by errors made in measurement. That these deviations were not greater may be attributable to the similarity in chemical composition of most woods. The data were more variable for the loadings in the radial direction than for those in the tangential. A probable explanation for this is that past studies have shown ray cells to be stiffer in the radial direction (Beery et al. 1983; Kennedy 1968; Schniewind 1959) than the cells around them and that there exists considerable variation in ray volume among the different species tested. Other anatomical reasons for deviations of the data around the regression lines are due to the nonhomogeneity of wood with regard to cell type and the presence of bands of earlywood and latewood tissues, which are greatly different in mechanical properties. Also to be considered is that wood pieces are not truly orthotropic because of growth ring curvature. Additional features that might lead to errors in measurement could be the presence of interlocking grain, reaction wood, or juvenile wood (Panshin and DeZeeuw 1980). In addition, as it was observed in this experiment, not all specimens attain the same moisture content at a given set of EMC conditions. Strain gage measuring errors could be attributable to misalignment of gages with respect to orthotropic axes, restraint of the substrate by the gages, and to noise and drift in the instrumentation. There was a little creep between some of the measurements. Exactly how this affected the data is not known.

Strong correlations are also evident between Young's modulus in the radial direction and specific gravity and between Young's modulus in the tangential direction and specific gravity. However, these correlations were not as good as those between compliances. This may be partly explained by the fact that compliances change proportionately with change in moisture content, whereas Young's moduli and specific gravity most likely do not change at the same rate with variation in moisture content. As a result, relationships among compliances should not be as sensitive to moisture content change as would relationships between Young's moduli and specific gravity. In some cases, differences among moisture contents of the test samples were large enough to produce significant differences in Young's moduli from those that would be found at a reference moisture content. There were poor correlations between Poisson's ratios and specific gravity.

CONCLUSIONS

General equations relating non-shear compliances in the RT plane of wood were obtained from the loading in the R and T directions of short columns made from a variety of species. The equations applied to all of the species tested with the exception of the Norway spruce, which was suspected to contain juvenile wood. Greater deviations around the regression line were observed for the specimens loaded in the R direction than for those loaded in the T direction. This was probably due to the stiffening effect of rays and the relative ray volume in the different species. Strong correlations were also found between the two Young's moduli, E_R and E_T, in the RT plane and specific gravity at test but were not evident between the Poisson's ratios, ν_{RT} and ν_{TR}, and specific gravity. The correlations between elastic constants and specific gravity were probably affected more by moisture content variations among the test samples than were the correlations between compliances.

Future research would be to determine general equations relating all the non-shear compliances to each other by making measurements in the LR and LT planes similar to those made in the RT plane for this report. A useful result would be to have all the compliances related to $S_{LL} = 1/E_L$ as this quantity is easily measured and typical values for it are readily available in the literature. A greater number of species should be tested over a wide range of moisture contents. The results could be applied in finite element studies of stress and strain distributions in mechanically stressed wood members and in studies of the strains encountered in drying wood.

REFERENCES

BEERY, W. H., G. IFJU, AND T. E. MCLAIN. 1983. Quantitative wood anatomy—Relating anatomy to transverse tensile strength. Wood Fiber Sci. 15(4):395–407.

BODIG, J., AND J. R. GOODMAN. 1969. A new apparatus for compression testing of wood. Wood Fiber 1(2):146–153.

———, AND ———. 1973. Prediction of elastic parameters of wood. Wood Sci. 5(4):249–264.

———, AND B. A. JAYNE. 1982. Mechanics of wood and wood composites. Van Nostrand Reinhold Co. Inc., New York. 712 pp.

DOYLE, D. V., J. T. DROW, AND R. S. MCBURNEY. 1945–1946. The elastic properties of wood. USDA Forest Service Report No. 1528; 1528 A-H. Forest Prod. Lab., Madison, WI.

GOODMAN, J. R., AND J. BODIG. 1971. Orthotropic elastic properties of wood. J. Struct. Div., ASCE 96 (ST 11):2301–2319.

HEARMON, R. F. S. 1948. Elasticity of wood and plywood. Forest Prod. Res. Spec. Report No. 7. His Majesty's Stationery Office, London. 87 pp.

KENNEDY, R. W. 1968. Wood in transverse compression: Influence of some anatomical variables and density on behavior. For. Prod. J. 18(3):36–40.

PANSHIN, A. J., AND C. DEZEEUW. 1980. Textbook of wood technology. I. 4th ed. McGraw-Hill, New York.

SLIKER, A. 1967. Making bonded wire electrical resistance strain gages for use on wood. Forest Prod. J. 17(4):53–55.

———. 1971. Resistance strain gages and adhesives for wood. Forest Prod. J. 21(12):40–43.

———. 1972. Measuring Poisson's ratios in wood. Experimental Mechanics 12(5):239–42.

———. 1985. Orthotropic strains in compression parallel to grain tests. Forest Prod. J. 35(11/12): 19–26.

SCHNIEWIND, A. P. 1959. Transverse anisotropy of wood: A function of gross anatomic structure. For. Prod. J. 9(10):350–360.