INVESTIGATING THE LINEARITY ASSUMPTION BETWEEN LUMBER GRADE MIX AND YIELD USING DESIGN OF EXPERIMENTS (DOE)

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ABSTRACT

Solving the least-cost lumber grade mix problem allows dimension mills to minimize the cost of dimension part production. This problem, due to its economic importance, has attracted much attention from researchers and industry in the past. Most solutions used linear programming models and assumed that a simple linear relationship existed between lumber grade mix and yield. However, this assumption has never been verified or rejected with scientific evidence. The objective of this study was to examine whether a linear relationship exists between yield and two- and three-grade lumber combinations using the USDA Forest Service's ROMI-RIP rough mill simulator and a cutting bill created by Buehlmann. The results showed that a simple linear relationship between grade mix and yield exists only for some grade combinations, but not for others. These findings were confirmed by repeating the tests using actual cutting bills from industry. It was observed that cutting bill characteristics, especially part length requirements and the lumber grades involved, are influential in causing a simple linear or nonlinear relationship between grade mix and yield.

Keywords: Lumber grade mix, least-cost lumber grade mix, simple linearity, mixture design.

INTRODUCTION

The search for a reliable method for solving the least-cost lumber grade mix problem has attracted significant interest from industry and academia (Englerth and Schumann 1969; Hanover et al. 1973; Martens and Nevel 1985; Timson and Martens 1990; Lawson et al. 1996; Steele et al. 1990). The least-cost lumber grade mix problem refers to the opportunity to minimize raw material and processing costs when producing dimension parts in rough mills by employing the optimum lumber grade or grade mix to produce the requirements of a given cutting bill. Such cost minimization creates competitive advantages by reducing raw material and processing costs without incurring additional expenses for implementing the lowest-cost grade mix solution. However, determining the lowest-cost lumber grade mix for a specific cutting bill is challenging since the interactions between cutting bills and different lumber qualities are complex (Thomas 1962; Hanover et al. 1973; BC Wood Specialties Group 1996; Buehlmann 1998).

The National Hardwood Lumber Association's (NHLA) lumber grading rules (NHLA 1998) differentiate six standard quality classes (grades) for hardwood lumber based on lumber size, minimum clear cutting sizes, basic yield, and maximum number of cuts. These six classes are, in decreasing order of quality, FIRST and SECOND (FAS), FAS ONE FACE (F1F), SELECTS (SEL), 1 Common, 2 Common (which is further differentiated into 2A Common and 2B Common), and 3 Common (3A Common and 3B Common). Higher grade lumber is more expensive but is easier to process and results in higher numbers of large parts and higher lumber yield. In contrast, lower grade lumber is less costly but yields significantly fewer and smaller parts per unit input. Also, the decreased yield obtained from the lower grades reduces rough mill productivity, since more material needs to be processed in order to produce the same amount of dimension parts.

Finding the lowest-cost grade mix requires knowledge about expected yields from different lumber grades and different lumber grade mixes for specific cutting bills. Expected yield[1] is an important component of a cost function to find the minimum total cost. Numerous studies have been conducted to solve the yield estimation problem in the past. Thomas (1962, 1965) first generated a set of yield prediction tables utilizing estimated yield results derived by computer simulation. Schumann and Englerth (1967) and Englerth and Schumann (1969) created a series of yield charts based on the YIELD simulation algorithm (Wodzinski and Hahm 1966) to calculate yield for hard maple lumber in crosscut-first mills. These results were then incorporated into yield nomograms. Later, this technique was used to build charts for black walnut and alder (Schumann 1971, 1972). In 1980, the nomograms were extended to predict the yield for rip-first processes (Hallock 1980). These nomograms were widely employed to estimate yields to solve the least-cost grade mix problem (Englerth and Schumann 1969; Hanover et al. 1973; Martens and Nevel 1985; Timson and Martens 1990; Lawson et al. 1996). Most if not all of these predictive yield charts were created using crosscut-first rough mill technology, although these models then were employed for crosscut-first and rip-first mills.

Starting in the 1980s, with the increase in computing power and programming capabilities coupled with easier-to-use interfaces, computer simulation programs such as CORY by Brunner et al. (1989), AGARIS (Thomas et al. 1994), ROMI-RIP (Thomas 1996a, 1999), ROMI-CROSS (Thomas 1998), and RIP-X (Harding 1991) were employed to calculate yields for cutting bills using a specified lumber grade or grade mix. These programs allow real-time simulation of the lumber cut-up and calculate the resulting yield. More accurate yield data are obtained from these programs than from the nomograms (Hoff 2000).

The basic idea for solving the least-cost lumber grade mix problem was to determine the optimal grade combination that minimizes the total lumber cost to fulfill a specific cutting order. In some instances, processing costs were included in these calculations (Harding 1991; Suter and Calloway 1994). To solve the optimization problem, estimated yields from either nomograms (Martens and Nevel 1985; Timson and Martens 1990, Lawson et al. 1996) or simulation programs (Harding 1991) were used. Linear programming was widely adopted to search for the most cost-efficient grade or grade combination (Hanover et al. 1973; Martens and Nevel 1985; Timson and Martens 1990; Harding 1991; Fortney 1994; Lawson et al. 1996).

Linear programming is a technique to maximize or minimize (i.e., optimize) the objective variable by providing optimal combinations for constraint variables from a series of simple linear functions (Winston 1994). Simple linear functions are functions where no higher order terms greater than one are significant in describ-

[1] Yield is defined as: Output part area/input lumber area $\times$ 100 percent

ing the dependent variable. The primary requirement for applying linear programming is that both objective function and constraint functions be simple linear. The wood products industry was one of the early users of linear programming technology. Linear programming was first introduced into the wood products industry in the late 1950s. Early applications were formulated to solve planning and distribution problems for the plywood industry (Bethel and Harrell 1957; Koenigsberg 1960; Raming 1968). Later, linear programming technology also was employed for sawmill planning and inventory problems (McKillop and Nielson 1968), as well as machine loading and production problems for furniture companies (Penick 1968; Fasick and Lawrence 1971). It was Hanover et al. (1973) who first employed linear programming to solve the least-cost grade mix problem for hardwood dimension manufacturers.

Based on Hanover et al.'s (1973) idea, several models were built in the following years (Martens and Nevel 1985; Timson and Martens 1990; Harding 1991; Fortney 1994; Suter and Calloway 1994; Lawson et al. 1996) and some of them were applied in software (Martens and Nevel 1985; Timson and Martens 1990; Lawson et al. 1996; Harding and Steele 1997). OPTIGRAMI was one of the early programs that employed linear programming to solve the least-cost grade mix program (Martens and Nevel 1985). In OPTIGRAMI, yield predictions are based on the hard maple nomograms developed in the late 1960s by Englerth and Shumann (1969). These charts, although derived from hard maple, were used for yield estimation for most hardwood species graded under standard NHLA rules. To make the program more user-friendly, OPTIGRAMI for the PC was developed in 1990 (Timson and Martens 1990). However, the continuing use of Englerth and Shumann's (1969) yield nomograms in OPTIGRAMI 1.0 reduced the accuracy of the least-cost calculations because these yield charts were based on the longer and wider lumber processed over 30 years ago. In 1996, OPTIGRAMI (Lawson et al. 1996) was modified to employ the updated yield charts for yellow-poplar and black walnut (Martens 1986a,b). For other hardwood species, Englerth and Schumann's yield nomograms (1969) are still in use.

In 1991, Harding (1991) used Brunner et al.'s (1989) rough mill simulator, CORY, for estimating yield in his least-cost grade mix optimization program called RIP-X. Fortney (1994) created a more advanced least-cost lumber grade mix tool, RIP-RIGHT, which incorporated the interactions between part sizes and part quantities into the constraint functions when computing the requirements for each grade. In the same year, Suter and Calloway (1994) created ROMGOP, a program that incorporates cost and other objectives such as budget and schedules into the calculations. Since this approach required solving a problem that contains a collection of goals, they used goal programming, a modified linear programming method.

All of these models are based on linear programming technology, where lumber grades and related yields were functioned as constraints assuming a simple linear relationship between yield and grade mix. However, this assumption has never been verified or rejected scientifically. Thus, the objective of this study was to investigate the validity of the assumed simple linear relationship between yield and lumber grade mix in a rip-first operation.

METHODS

The study employed lumber cut-up simulation software, lumber data from the USDA Forest Service, and cutting bills from academia and industry to investigate the relationship between yield and lumber grade mix in a rip-first rough mill.

Lumber cut-up simulation

The USDA Forest Service's ROMI-RIP 2.0 (RR2) simulation software (Thomas 1999) was employed to collect simulated yield information from the cut-up of lumber in a rip-first rough mill. To avoid confounding of the main effects sought in this study, no strips for glued panels were produced. The settings employed are listed below:

- All-blades–movable arbor type
- Salvage cut to primary lengths and widths
- Total yield used consists of primary and salvage yield (e.g., no excess salvage yield)
- Complex dynamic exponential part prioritization
- No random-width nor random-length parts
- Continuous update of part counts
- $^1/_4$-in. end and side trim

Lumber data

All the lumber data used in this research were from the 1998 Data Bank for Kiln-Dried Red Oak Lumber (Gatchell et al. 1998). The lumber grades included in the data bank are FAS, F1F, SEL, 1 Common, 2A Common, and 3A Common. However, the grading requirements for F1F and SEL are very similar except for lumber size and wane. In fact, in practice, F1F is often included in FAS and then called FAS or FAS/1F. However, to avoid confounding effects, F1F was not included in this study as a stand-alone grade. The five grades used in this study were FAS, SEL, 1 Common, 2A Common, and 3A Common lumber. Additionally, a SELECTS& BETTER (SEL&BETR) grade, which consisted of 34.8% FAS, 27.2% F1F, and 38.6% SELECTS, was tested in this study (Wiedenbeck et al. 2003). For each grade combination, three lumber samples each with 1000 board feet were randomly selected and composed from the Red Oak Data Bank using the MAKEFILE tool which is part of RR2 (Thomas 1999). If 1000 board feet was not enough lumber to satisfy the part quantity requirements of a cutting bill, the same original sample of digital boards was reprocessed until all cutting-bill requirements were met. This can easily be done in RR2 by copying the digital board data without biasing the yield results as confirmed by tests performed prior to this study (Buehlmann 1998).

Cutting bill

A cutting bill created by Buehlmann (1998) was used for this research. This cutting bill (Table 1) represents the "average" cutting bill used by the wood products industry and researchers with respect to part sizes and quantities as defined by Buehlmann (1998). However, for this study, the quantity requirements were adjusted so that at least 150 boards were processed to satisfy the part quantities required by the cutting bill. When at least 150 boards are processed, yield is no longer influenced by the amount of lumber processed (Buehlmann 1998).

For verification purposes, a published set of cutting bills by Thomas (1996b) and Wengert and Lamb (1994) was employed to compare the findings obtained from the Buehlmann cutting bill. However, one cutting bill, the most difficult one according to Thomas (1996b), was not used because all of its required part widths were between 4 in. and 6 in. wide. Such wide parts, if not produced from glued-up stock, are difficult to obtain from low-grade lumber such as 3A Common. Thus, a total of 10 cutting bills, 9 from Thomas (cutting bills A, B, C, D, F, G, H, I, J), and 1 from Wengert and Lamb (cutting bill E), were used to verify the original findings. Table 2 summarizes these cutting bills and indicates their respective estimated level of difficulty in terms of obtaining the parts required. To allow comparisons, the Buehlmann cutting bill was also included in Table 2.

Experimental design

Only two- and three-grade combinations were tested for this study, because higher-grade combinations (e.g., four- and five-grade combinations) are unlikely to be used in industrial settings. Also, tests were always made between combinations of high quality grades (FAS, SELECTS, SEL&BETR) and lower quality grades (1 Common, 2A Common, 3A Common). For

TABLE 1. *Number of parts of each size required by the Buehlmann cutting bill.*

Part width (in.)	Part length (in.) 10	17.5	27.5	47.5	72.5
1.50	136	297	433	243	103
2.50	152	298	480	262	98
3.50	46	102	146	88	57
4.25	49	99	158	85	40

TABLE 2. *Eleven cutting bills used in the study including 10 that were used to compare findings from Buehlmann's cutting bill.*

Cutting bill	Rank [a]	# of parts	# of width	# of length
A	1	5	3	4
B	2	10	4	9
C	3	25	7	16
D	4	5	3	5
E	5	4	4	4
F	6	12	4	6
Buehlmann	7	20	4	5
G	8	20	7	12
H	9	8	2	8
I	10	16	4	11
J	11	9	5	4

[a]The cutting bills were ranked from easiest to hardest as defined in Thomas's study (1996). The ranking for Wengert and Lamb's (1994, cutting bill E), and the Buehlmann (1998) cutting bills were done using the same criteria as employed in Thomas's 1996 study.

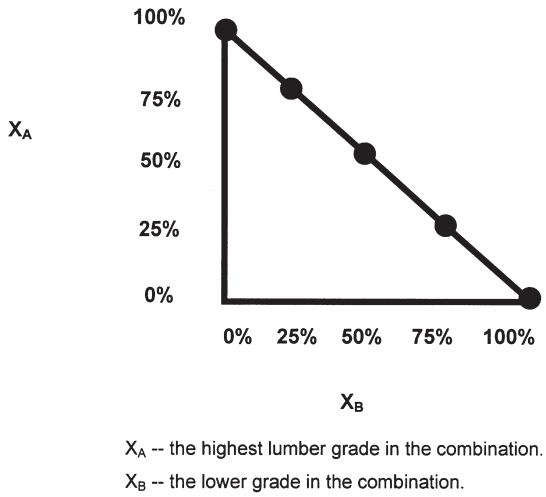

FIG. 1. Experimental points used for combination tests employing two lumber grades.

all experiments conducted, each grade was considered a factor, and its weight had to be between zero and one. In addition, the sum of all grade proportions had to equal one. A mixture design, a special response surface design (Kuehl 2000), was applied to satisfy these requirements and allow for statistical analysis of the results. Three replicates were made of all tests performed.

Two-grade combinations

The two-grade combination experiment was designed to test if there is a simple linear relationship between yield and a mix of two different lumber grades. The grade mixes used were combinations of two grades in 25% increments. This study set-up is shown in Fig. 1. For each grade combination, X_A always represents the better of the two grades according to the NHLA grading rules (NHLA 1998), and X_B represents the lower grade. For example, when testing the FAS-1 Common lumber grade mix, FAS is denoted by X_A and 1 Common is denoted by X_B.

Preliminary testing showed that long and/or wide parts, such as dimension parts 72.5 in. long and 4 in. wide, could not be obtained in sufficient numbers from 3A Common lumber. Therefore, no tests were conducted using 100% 3A Common lumber. Thus, all grade combinations containing 3A Common lumber only have four test points (100%–0%, 75%–25%, 50%–50%, and 25%–75% for X_A and X_B grades, respectively), instead of five (i.e., 0%–100% is missing).

Three-grade combinations

A similar approach as described above was used for the three-grade combinations tested. A Simplex-Lattice {3,2} design (Kuehl 2000) was applied for all grade combinations that did not contain 3A Common lumber. For combinations containing 3A Common lumber, an 80% upper bound constraint was imposed for the same reasons discussed previously for the two-grade combinations. Preliminary tests showed that cutting bills using grade combinations containing up to 80% 3A Common lumber could produce all the part sizes requested in sufficient numbers. To obtain accurate analysis results for the three-grade combination tests, two different designs were employed, one for all grade combinations not containing 3A Common lumber and one for tests employing 3A Common lumber. Figure 2 shows the design points for grade combinations without 3A Common lumber, whereas Fig. 3 shows the design points for the grade combinations with 3A Common lumber. As in the two-grade combination, X_A represents the highest lumber grade in the combination, X_B is the next lower grade, and X_C the lowest grade involved in any given test.

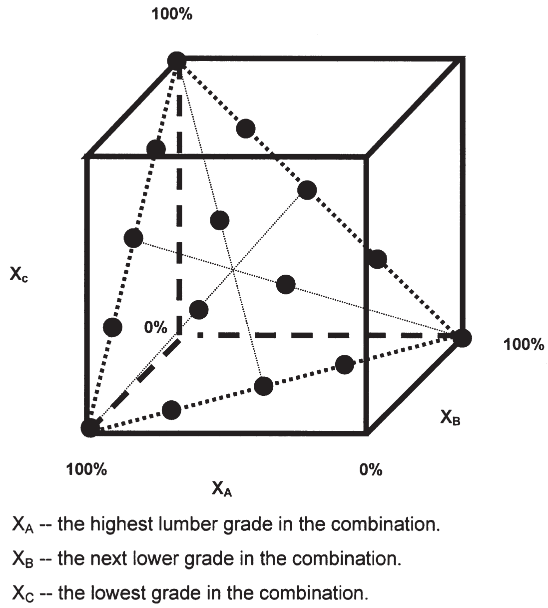

FIG. 2. Design points for Simplex–Lattice {3,2} design.

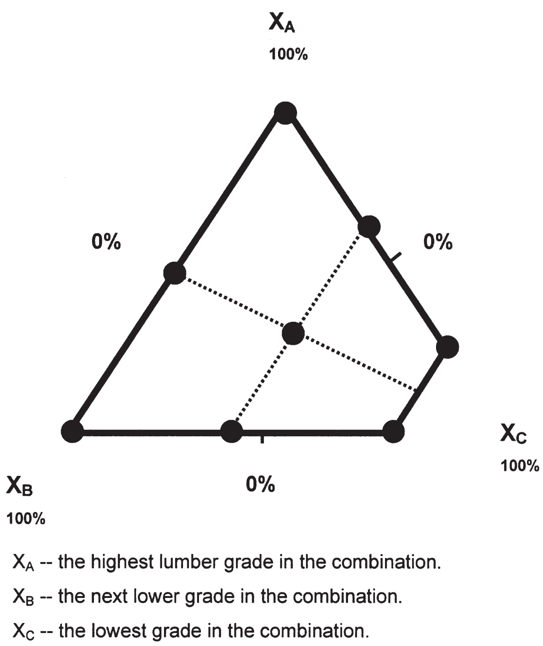

FIG. 3. Mixture design points with 80 percent upper bound restraints on 3ACom lumber.

Statistical analysis

The general second-order polynomial model for a response surface is

$$\mu_y = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n} \beta_{ii} x_i^2 + \sum_{i<} \sum_{j} \beta_{ij} x_i x_j \quad (1)$$

where μ_y is the yield of a given cutting bill, x_i are the proportions of each lumber grade, n is 2 for two-grade combinations and 3 for three-grade combinations, β_0 is the intercept, β_i are the coefficients of linear terms, β_{ii} are the coefficients of quadratic terms, and β_{ij} are the coefficients of the interaction terms. Because the constraint $\sum_{i=1}^{n} x_i$ = 1 applied in the mixture design, Eq. (1) can be reduced to

$$\mu_y = \sum_{i=1}^{n} \beta_i^* x_i + \sum_{i<} \sum_{j} \beta_{ij}^* x_i x_j \quad (2)$$

by transforming

$$\beta_i^* = \beta_0 + \beta_i + \beta_{ii}, \text{ and } \beta_{ij}^* = \beta_{ij} - \beta_{ii} - \beta_{jj} \ ,$$

(Kuehl 2000).

If simple linearity holds between yield and grade combinations, then the higher order coefficients β_{ij}^* are non-significant. Thus, the hypothesis of this study was:

$$H_0 : \beta_{ij}^* = 0; \text{ vs. } H_a : \beta_{ij}^* \neq 0 \quad (3)$$

All the conclusions made were based on a 0.05 level of significance.

Verification of findings

To verify the findings made using the Buehlmann cutting bill, the 10 industry cutting bills (Thomas 1996b; Wengert and Lamb 1994) described previously were subjected to the same, yet less detailed, statistical analyses as the Buehlmann cutting bill (Buehlmann 1998) described above. Since the product terms in Eq. (2) for three-grade combinations include the relationship of the two-grade combination, the verifications were done only for three-grade combination.

To verify the applicability of the findings made using the advanced rough-mill lumber cut-up techniques employed (e.g., all-blades movable, complex dynamic exponential part prioritization, among others), a test using a scenario considered similar to actual rough mills in use today also was performed. The set-up and cutting bill from an earlier study by Thomas and Buehlmann (2002), where RR2 (Thomas 1999) was validated as a true simulator of an actual rough mill were used for this test. Only the FAS-3A Common lumber grade mix was tested at 100%–0%, 75%–25%, 50%–50%, 25%–75%, and 0%–100% for FAS and 3A Common grade, respectively. Three replicates of each test were performed.

RESULTS AND DISCUSSION

The discussion focuses first on the more thoroughly tested cutting bill by Buehlmann (1998). The observations from these tests are then verified using the industry cutting bills from Thomas (1996b) and Wengert and Lamb (1994).

Two-grade combinations

For 6 out of the 12 grade mixes tested using the Buehlmann cutting bill (1998), the null hypothesis was rejected, e.g., yield did not linearly and proportionally increase/decrease with a change in the lumber grade mix composition. Lack-of-fit tests were conducted for each grade combination, and the corresponding *P*-values are shown in Table 3. Table 3 also shows the yield levels for the different two grade combinations tested using Buehlmann's cutting bill. As indicated in the column "P-value for lack of fit test," six grade combinations were found not to have a simple linear relationship between grade mix and yield at the 0.05 level of significance. As was explained previously, the higher lumber grade employed in each test was always assigned the notation X_A, whereas the lower grade was assigned the notation X_B.

As shown in Table 3, the FAS-2A Common, FAS-3A Common, SEL-2A Common, SEL-3A Common, SEL&BETR-2A Common, SEL&BETR-3A Common grade combinations require higher order polynomial terms to describe the yield–grade mix relationship, thus invalidating the linearity assumption made by other researchers (Hanover et al. 1973; Martens and Nevel 1985; Timson and Martens 1990; Harding 1991; Fortney 1994; Lawson et al. 1996). It is interesting to note that all the grade mixes found to cause nonlinear yield behavior do involve one higher quality (e.g., FAS, SEL, or SEL&BETR) and one lower quality (e.g., 2A Common or 3A

TABLE 3. *Yield and statistical results from testing the simple linearity assumption of two-grade lumber combinations.*

		Average yield of each grade combination (%)				
Combinations	P-value for lack of fit test	X_A (100%) X_B (0%)	X_A (75%) X_B (25%)	X_A (50%) X_B (50%)	X_A (25%) X_B (75%)	X_A (0%) X_B (100%)
FAS-1 Common[ns]	0.4900	77.02	74.07	70.57	66.73	64.03
FAS-2A Common	0.0012	77.02	71.96	65.51	57.49	47.5
FAS-3A Common	0.0002	77.02	68.11	58.13	44.75	a
SEL-1 Common[ns]	0.0729	65.90	66.07	65.99	65.65	64.03
SEL-2A Common	0.0263	65.90	62.90	58.98	56.65	47.50
SEL-3A Common	0.0057	65.90	58.49	51.04	41.33	a
SEL&BETR-1 Common[ns]	0.3695	72.22	70.08	67.97	66.92	64.03
SEL&BETR-2A Common	0.0393	72.22	66.89	62.05	55.29	47.50
SEL&BETR-3A Common	0.0029	72.22	63.71	54.47	42.67	a
1Common-2A Common[ns]	0.2868	64.03	60.26	57.35	52.68	47.50
1Common-3A Common[ns]	0.0510	64.03	58.00	48.15	40.00	a
2ACommon-3A Common[ns]	0.2945	47.50	36.78	30.93	16.14	a

ns—non-significant at 0.05 level.
a—grade combination was not tested.

Common) grade. Grade mixes consisting of similar grades (FAS-1 Common, SEL&BETR-1 Common, SEL-1 Common, 1 Common-2A Common, 1 Common-3A Common, and 2A Common–3A Common) do exhibit linear behavior and thus can be described by a simple linear function. Linearity between yield and the 1 Common-3A Common lumber combination was barely proven with a *P*-value for the lack of fit test only slightly above 0.05 (*P*-value 0.051).

The phenomenon that lumber grade mixes consisting of similar grades behaving linearly whereas non-alike mixes do not may be due to the increasing differences in lumber quality among grades. When the percentage-composition of alike grades (e.g., FAS-1 Common) is changed, yield increases or decreases proportionally over the entire span of the solution space. However, when the percentage composition of not-alike grades (e.g., FAS-3 A Common) is changed, quality gaps between not-alike grades lead to over proportional yield changes resulting in nonlinear behavior of the yield curve. For example, in a FAS-3A Common grade mix, when less FAS is used, larger parts previously obtained from the high-quality FAS boards now are much harder to obtain in the 3A Common grade, and yield suffers disproportionally. This leads to a nonlinear relationship between grade mixes and yield for grade combinations made up of dissimilar lumber grades.

Three-grade combinations

Observations for the three-grade lumber combinations using the Buehlmann cutting bill (Buehlmann 1998) showed that only the SEL&BETR-1 Common-2A Common combination behaves linearly over its entire grade yield response surface. Even for this case, the 1 Common-2A Common interaction is weak (*P*-value 0.059). Table 4 shows the level of significance for the estimated model parameters for the three-grade combination cases investigated. The terms X_A, X_B, X_C always designate a particular lumber grade, as shown in the three top rows of Table 4.

The results in Table 4 show that a simple linear model does not accurately characterize the lumber grade mix–yield relationship. Only 16 out of a total of 30 interaction terms were found to be non-significant at the 0.05 level. Each three-grade combination tested had at least one significant interaction term except the SEL&BETR-1 Common-2A Common combination. Thus, 9 of the 10 three-grade combinations tested behaved nonlinearly. In 5 out of 10 cases, the model required two interaction terms to be included. Dissimilar grades, as was observed for the two-grades model, lead to more nonlinear behavior of the yield-grade mix relationship. The interaction term for the lowest and highest grade (e.g., X_A*X_C) of any given grade mix combination was found to be significant in all cases except the SEL&BETR-2A Common grades at the SEL&BETR-1 Common-2A Common grade mix.

These tests show that the linearity assumption for the grade mix–yield relationship assumed and used in several least-cost lumber cost grade mix models (Hanover et al. 1973; Martens and Nevel 1985; Timson and Martens 1990; Harding

TABLE 4. *Significance of model parameters when using the Buehlmann cutting bill.*

X_A X_B X_C	FAS- 1Com- 2ACom	FAS- 1Com- 3ACom	FAS- 2ACom- 3ACom	SEL& BETR- 1Com- 2ACom	SEL& BETR- 1Com- 3ACom	SEL& BETR- 2ACom- 3ACom	SEL- 1Com- 2ACom	SEL- 1Com- 3ACom	SEL- 2ACom- 3ACom	1Com- 2ACom- 3ACom
X_A	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
X_B	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
X_C	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
X_A*X_B	ns	ns	ns	ns	ns	ns	0.0035	ns	ns	ns
X_A*X_C	0.0001	0.0038	0.0001	ns	0.0107	0.0001	0.0001	0.0175	0.0001	0.0001
X_B*X_C	0.0016	0.0029	ns	ns	0.0026	ns	ns	0.0026	ns	ns

ns—non-significant terms in the model at the 0.05 level of significance

1991; Fortney 1994; Lawson et al. 1996) does not reflect the true relationship between grade mix and yield for the Buehlmann cutting bill given the settings employed in this study. To verify these findings on a broader scale, 10 cutting bills from industry (Thomas 1996; Wengert and Lamb 1994) were tested using the three-grade mix set-up.

Verification of findings

All 10 cutting bills used to verify the findings made with the Buehlmann cutting bill were found to require higher order polynomial term(s) to describe the yield–grade mix response surface for three lumber grade combinations. Thus, the observations made using the Buehlmann cutting bill were confirmed. Table 5 shows the 10 cutting bills used for verification purposes and the Buehlmann cutting bill for comparison purposes (cutting bills are listed on the left and lumber grade mixes on top). Each cutting bill–grade mix combination that required at least one higher order term in the model to describe the response surface is marked with a dash in the matrix. The cutting bills are ranked in decreasing order based on the frequency a higher order model was needed to describe the relationship between yield and grade mix.

Table 5 demonstrates that cutting bill requirements, in addition to lumber grades, do have an impact on the relationship between lumber grade mix and yield. There is a tendency for cutting bills that are viewed as more difficult to be processed and satisfied to require more complex models (e.g., more higher order terms) to describe the yield–grade mix response surface. Thomas (1996b) ranked his cutting bills in order of difficulty from 1 to 10, with 1 denoting the "easiest" cutting bill. The rank of individual cutting bills is shown in Table 2 in the second column. The Wengert and Lamb (1994) and Buehlmann (1998) cutting bills were ranked later in the same way as Thomas's original bills. This column shows that cutting bills that are viewed as more difficult by experts also tend to require more complex models to describe the yield–grade mix response surface. For example, cutting bills I and J, the most difficult cutting bills in the study, ranked 9th and 11th (e.g., third last and last) in terms of complexity of the models required that describe their response. However, this relationship is not as simple as stated, since there are cutting bills that, although ranked more difficult than others, require less complex models for the description of the grade mix–yield relationship. For example, cutting bill F is classified as being of medium difficulty by Thomas (1996b) (6th out of 10), but requires a more complex model than do more difficult cutting bills such as G (8th), H (9th), or I (10th).

The findings of this study also clearly illustrate the interconnected relationship between difficulty of cutting bills (e.g., how difficult it is

TABLE 5. *Cutting bill – three grade lumber combinations with and without linear relationships.*

Cutting bill	FAS-2ACom-3ACom	SEL&BETR-2ACom-3ACom	SEL&BETR-1Com-3ACom	1Com-2ACom-3ACom	FAS-1Com-3ACom	SEL-1Com-3ACom	SEL-2ACom-3ACom	SEL-1Com-2ACom	BETR-1Com-2ACom	FAS-1Com-2ACom
A		—								
D	—									
C	—	—	—	—		—		—		
B	—	—	—	—	—	—				
H	—	—	—	—	—	—	—			
G	—	—	—	—	—	—	—			
E	—	—	—	—	—		—		—	—
"Buehlmann"	—	—	—	—	—	—	—	—	—	—
I	—	—	—	—	—	—	—	—	—	—
F	—	—	—	—	—	—	—	—	—	—
J	—	—	—	—	—	—	—	—	—	—

Note: Dashes denote cutting bills where a higher order polynomial model is needed at the 0.05 level of significance to obtain required parts given a specific lumber grade) and lumber grade. The linear relationship between lumber grade mix and yield is less likely to be violated or partially violated when either an easy cutting bill or a high lumber grade or both are tested. Difficulty of cutting bill and quality of lumber grade are in fact correlated. A difficult cutting bill may require less nonlinear terms when higher quality lumber is used as cutting material while lower grade material can be used for an easy cutting bill and may still not require a large amount of nonlinear terms. However, the correlation between difficulty of cutting bill and lumber grade is not 1.0 and may differ based on small changes in cutting bill or lumber grade composition.

Also, proving that the grade mix–yield relationship is nonlinear does not necessarily say to what extent a linear model produces nonoptimal results. Nonetheless, the findings presented here serve as a red flag to be critical of the results produced by the traditional linear programming based least-cost lumber grade mix models. Further research will have to show by how much costs can decrease when a more appropriate (e.g. a statistical model is used).

To better assess reasons for this inconsistent behavior, Table 6 shows the basic characteristics of the 11 cutting bills used in this study. The cutting bills are listed in the same order as in Table 5. As pointed out above, the tests conducted show a strong, although not perfect, relationship between the difficulty of a cutting bill according to Thomas (1996b) and the complexity of the model required to describe the grade mix–yield response surface. Part length distribution turns out to be a crucial factor affecting the linearity of the grade mix–yield relationship. The model for the response surface tends to require more complex models when there is a more pronounced requirement for longer parts (Table 6). Also, uneven length distribution of the cutting bill requirements tends to require more interaction terms in the model. For example, cutting bill J, which was ranked as the most difficult cutting bill by Thomas (1996b), requires 75% of its parts to be shorter than 41 in. and 25% to be longer than 70 in. However, no parts are required with lengths between 41 and 70 in. This cutting bill requires a second order polynomial model to describe the relationship between yield and lumber grade combinations for all grade mixes tested. Similarly, cutting bills F, I, and Buehlmann require higher order polynomial models to describe all grade mix combinations tested. These four cutting bills each require at least 25% of their parts to be longer than 41 inches and a minimum of 50% to be wider than 3 inches.

As the quantities of long-length parts (>41 in.) and/or wide parts (>3 in.) decreases, the complexity of the model to describe the yield response surface for the different grade mix combinations decreases. Simple linearity does hold for a very easy cutting bill (A was ranked as the

TABLE 6. *Basic characteristics of the 11 cutting bills used in this study.*

Cutting bill	Total number of widths	Percentage of narrow-width ($W \leq 3.0$ in.) parts	Percentage of wide-width ($W > 3.0$ in.) parts	Total number of lengths	Percentage of short-length parts ($L \leq 41$ in.)	Percentage of long-length parts ($41 < L \leq 70$ in.)	Percentage of longer-length parts ($L > 70$ in.)
A	3	100	0	4	100	0	0
D	3	100	0	5	100	0	0
C	7	43	57	16	100	0	0
B	4	100	0	9	78	22	0
H	2	50	50	8	63	25	12
G	7	43	57	12	58	25	17
E	4	75	25	4	50	50	0
"Buehlmann"	4	50	50	5	60	20	20
I	4	50	50	11	64	27	9
F	4	50	50	6	67	33	0
J	5	40	60	4	75	0	25

"easiest" cutting bill by Thomas 1996). Cutting bill A requires only narrow and short parts with just five different part sizes to be cut. However, cutting bill B, which was ranked second easiest by Thomas and has very similar requirements to cutting bill A, requires higher order terms for four out of seven lumber grade mix combinations. It appears there is no single, easy to measure indicator as to which cutting bill characteristics are responsible for simple linear versus non-simple linear behavior with respect to the grade mix–yield relationship. As with other phenomena observed when it comes to lumber cut-up, the issue is highly complex and depends on many interrelated characteristics of the cutting bill and the lumber.

As was observed previously for the tests involving the Buehlmann cutting bill, the yield–grade mix relationship becomes more complex when the different lumber qualities combined are more varied. For example, 10 of the 11 cutting bills require a non-simple linear model to describe the yield–lumber grade mix relationship when involving the FAS-2A Common-3A Common grade mix. The same applies for the SEL&BETR-2A Common-3A Common grade mixes. Both these grade mixes consist of dissimilar lumber qualities. Cutting bills satisfied with similar lumber qualities mixed together, as in the case of SEL-1 Common-2A Common, SEL&BETR-1 Common-2A Common, or FAS-1 Common-2A Common, however, require fewer higher order models to describe the yield-grade mix relationship (5 out of 11).

It also appears that the simple linearity is affected by the overall quality of a given lumber grade combination. As Table 5 shows, 9 out of 11 cutting bills require a nonlinear model to describe the relationship between yield and lumber grade mix when the lowest lumber quality combination (1 Common-2A Common-3A Common) is used. Conversely, only five cutting bills require a nonlinear model to describe the yield-lumber grade mix relationship when higher quality lumber grade mixes, such as FAS-1 Common-2A Common, SEL&BETR-1 Common-2A Common, and SEL-1 Common-2A Common are involved. Thus, the complexity of the relationship between lumber grade mix and yield also is dependent on individual lumber grade quality and the overall quality of the lumber involved. Since these tests used a simulation scenario that represents rough mill technology and practices widely used today (Thomas and Buehlmann 2002), the linearity assumption between yield and lumber grade mix for industry cutting bills does not always hold, either. The lack-of-fit test of simple linearity for the 10 industry bills was found to be highly significant ($P<0.0001$). Thus, the rough mill technology used does not prevent non-simple linear results for the yield–lumber grade mix relationship. The findings of this study therefore do apply to current rip-first rough mill set-ups used in mills. Further research will have to reveal if the findings from this study also apply to crosscut-first rough mills.

Based on today's understanding of the yield–lumber grade mix relationship, it is impossible to predict if a particular cutting bill–grade mix combination will result in a simple linear or a non-simple linear relationship between grade mix and yield. More than half of the cutting bills tested using two lumber grade combinations and all cutting bills tested using three lumber grade combinations were found not to have a simple linear relationship between lumber grade mix and yield. This high percentage of non-simple linear behavior combined with the inability to predict which cutting bill–grade mix combination will result in simple linear or non-simple linear relationships raises questions about the validity of the linearity assumption made by earlier developers of least-cost lumber grade mix search algorithms (Hanover et al. 1973; Martens and Nevel 1985; Timson and Martens 1990; Harding 1991; Fortney 1994; Lawson et al. 1996). Therefore, efforts should be undertaken to create a new least-cost lumber grade mix model that does not rely on the assumed linear behavior of the relationship between lumber grade mix and yield.

SUMMARY AND CONCLUSIONS

Solving the least-cost lumber grade mix problem is, due to its large economical implications, a pressing problem. In the past, efforts were mainly undertaken using linear programming models, which were all based on the assumption that the relationship between lumber grade mix and yield is a simple linear relationship. This crucial assumption has never been verified or rejected scientifically, so far.

Findings from this study indicate that the simple linearity assumption does not apply for many cutting bills. Tests with a cutting bill created by Buehlmann (which is based on industry-relevant requirements) showed that the simple linear yield–grade mix relationships exist only in certain cases, but not in general. For example, linearity exists for some two-grade lumber mix combinations that contain two similar grades, and for only one three-grade lumber mix combination, SEL&BETR-1 Common-2ACommon. These findings were substantiated when testing 10 additional cutting bills used by industry and research. In addition, it was observed that cutting bill characteristics, especially the length requirements, have effects on the simple linear or nonlinear relationship between yield and grade mix. The number of different lumber grades combined is another factor affecting the shape of the response surface of the yield and grade–mix interaction. Generally, it can be observed that the more dissimilar grade qualities are used for one grade mix, the more likely a nonlinear response will occur.

Predicting the relationship between yield and grade mix appears to be highly complex. However, the high percentage of non-simple linear relationships observed here raises questions about the validity of the linearity assumption made by previous developers of least-cost lumber grade mix. Further efforts are needed to construct a new least-cost lumber grade mix model that will not rely on the assumption of a simple linear relationship between lumber grade mix and yield.

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