RELATIONSHIP BETWEEN RADIAL COMPRESSIVE MODULUS OF ELASTICITY AND SHEAR MODULUS OF WOOD

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ABSTRACT

Wood properties in transverse compression are difficult to determine because of such factors as anatomical complexity, specimen geometry, and loading conditions. The mechanical properties of wood, considered as an anisotropic or orthotropic material, are related by certain tensor transformation rules when the reference coordinate system changes its orientation. In this paper, we used our verified shear modulus model to estimate compressive modulus of elasticity in the radial direction by means of certain established tensor transformation rules. The obtained basic engineering constants form a viable set that agrees with reliable test data and the anisotropic elasticity theory.

Keywords: Anisotropic material, elasticity, shear modulus, shear test, tensor transformation, wood.

INTRODUCTION

Wood is frequently assumed to be an orthotropic material with independent mechanical properties in three mutually perpendicular directions: longitudinal (L), radial (R), and tangential (T). In reality, wood is a cylindrically orthotropic material, and an orthogonal approximation may introduce bias. The mechanical properties of wood are known to be greatly influenced by its anatomical structure. In tangential compression, for example, Bodig (1965) suggested a spaced column theory assuming that springwood (earlywood) and summerwood (latewood) bands function as parallel columns that transmit the load from the loading surface to the supporting base. Most of the load is taken by the stronger latewood; the weaker earlywood functions mainly as a lateral support for the latewood. Bodig (1965) has successfully used this theory and the concept of the slenderness ratio of the column to explain failure mechanisms observed in experimental studies.

A weak band theory was proposed for radial compression (Bodig 1965; Kennedy 1967). In the radial direction, earlywood and latewood bands are arranged in series, perpendicular to the applied load. Both bands carry the same load, but the latewood deforms much less than does the earlywood. The first failure occurs in the weakest earlywood band, with subsequent failures occurring in other earlywood bands and then latewood bands as compression progresses.

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However, it is generally difficult to generalize the behavior of softwoods or hardwoods in transverse compression because of the complex interactions of various anatomical influences. For instance, the earlywood/latewood ratio and density differences between earlywood and latewood are common variables, even within the same species. For hardwoods, the wide variation of ray volume introduces yet another variable (Kennedy 1967; Reiterer et al. 2002).

In most types of loading, specimen size also influences test results. The contact stress distribution is very complex as a result of frictional forces at the contact areas between the specimen and the loading and support plates, due to Poisson's effect (Bodig 1966; Kobayashi 1962; Norris 1955). The influence of these forces diminishes with distance from the contact areas, making it necessary to increase the height of the specimen. Studies on the influence of height in radial compression indicate that various properties are affected to different degrees. For instance, Bodig (1966) reported that specimen height affected the following properties, in decreasing order: modulus of elasticity, strain at proportional limit, strain at ultimate stress, unit work at proportional limit, Poisson's ratio, proportional limit stress, and ultimate stress.

The present ASTM D143 standard test for transverse compression of wood (ASTM 1996) specifies a 50.8- by 50.8- by 152.8-mm clear specimen with the long dimension in the longitudinal direction resting on a support. Using a 50.8-mm-wide metal bearing plate, the specimen is loaded over the central third of the wood surface in the tangential direction; hence, this test is designated the partial-plate compression test (Bodig 1969; Pellicane et al. 1994). Design values for perpendicular-to-grain compression are based on the stress associated with 2.5 mm of deformation. The test provides no other information. The procedure was developed to evaluate the reaction force supporting capacity of solid wood joists.

The ASTM partial-plate compression test gives a higher strength value than does the full surface compression test because of the added "edge effect" (Bodig 1969). Following the ASTM standard (with the exception of specimen dimension and no special attention given to the direction of annual rings with respect to loading), Bodig (1969) found the edge effect contribution, based on bearing area, could be as much as 5 to 9 times that of full surface compression. The added load-carrying capacity is the result of the shear effect along the perimeter of the compression plate. These results were verified by the plane-stress finite element model of Pellicane et al. (1994), which also showed a complex stress state in members even when the load is distributed over the entire specimen surface. In particular, numerically determined stresses nearly 3.5 times the nominal stress were found for certain combinations of input parameter—specimen geometry, loading geometry, and material properties (Pellicane et al. 1994). All of these results serve to verify that there is no standard testing method for regulating the exploration of orthotropy in transverse compression (Lang et al. 2002).

The mechanical properties of anisotropic composites are known to be strongly dependent on the orientation of the reference coordinate system. These properties are related by certain tensor transformation rules when the reference coordinate system changes from one orientation to another (Wu et al. 1973).

In our study of shear modulus variation with grain slope of wood (Liu and Ross 1997), we derived a formula that shows that, in any principal material plane, if the values of shear modulus at two different orientations are known, the value at any other orientation in the plane is known. The formula was verified with high accuracy using the Arcan shear test on Sitka spruce specimens. In the process of deriving this formula, we identified an expression for elasticity modulus in the radial direction in terms of other more easily obtainable parameters. Just as off-axis tension tests (Tsai 1965) are used to supplement shear to determine anisotropic moduli, we will show how the Arcan shear test can be used to supplement the compression test in the radial direction to serve the same purpose.

METHODS

Derivation of shear modulus and modulus of elasticity in radial direction

Let the 1-2 coordinate system represent the principal material axes and the *x-y* coordinate system the geometrical axes with angle θ from the *x* axis to the 1 axis, as shown in Fig. 1. The transformed engineering constants can be expressed in terms of the four basic engineering constants: E_1 and E_2, elasticity moduli in the 1 and 2 axes; G_{12}, shear modulus in the 1–2 plane; and ν_{12}, Poisson's ratio, with 1 referring to direction of applied stress and 2 referring to direction of strain. In a two-dimensional analysis of Sitka spruce, we identify the 1 axis with the *L* axis and the 2 axis with the *R* axis.

The transformed shear modulus in the *x-y* plane is (Jones 1975)

$$\frac{1}{G_{xy}} = 2\left(\frac{2}{E_1} + \frac{2}{E_2} + \frac{4\nu_{12}}{E_1} - \frac{1}{G_{12}}\right) \sin^2\theta\cos^2\theta + \frac{1}{G_{12}}\left(\sin^4\theta + \cos^4\theta\right) \tag{1}$$

which can be reduced to the following form with G_{xy} replaced by $G(\theta)$:

$$\frac{1}{G(\theta)} = \frac{\theta}{E_2}\sin^2 2\theta + \frac{1}{G_{12}}\cos^2 2\theta \tag{2}$$

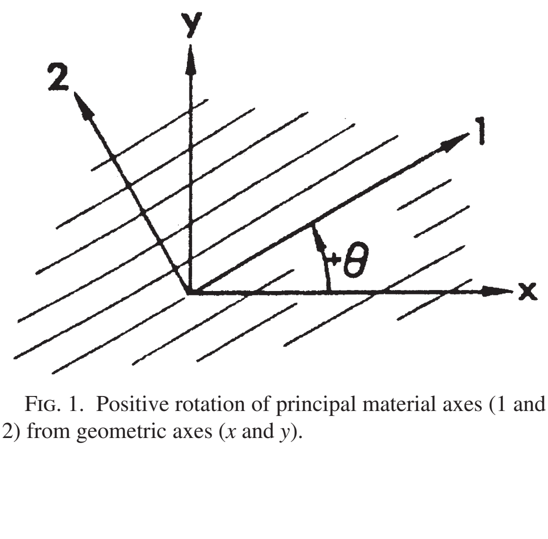

FIG. 1. Positive rotation of principal material axes (1 and 2) from geometric axes (*x* and *y*).

in which

$$\theta = 1 + \frac{E_2}{E_1}\left(1 + 2\nu_{12}\right) \tag{3}$$

At $\theta = 0°$ in Eq. (2),

$$1/G(0°) = 1/G_{12} \tag{4}$$

at $\theta = 45°$,

$$1/G(45°) = \theta/E_2 \tag{5}$$

Substituting Eqs. (4) and (5) into Eq. (2) yields

$$\frac{1}{G(\theta)} = \frac{1}{G(45°)}\sin^2 2\theta + \frac{1}{G(0°)}\cos^2 2\theta \tag{6}$$

or

$$G(\theta) = \frac{G(0°)G(45°)}{G(0°)\sin^2 2\theta + G(45°)\cos^2 2\theta} \tag{7}$$

Equation (7) indicates that in the range $0° \le \theta \le 90°$, the variation of $G(\theta)$ is symmetrical with respect to $\theta = 45°$. Knowing $G°(0)$ and $G(45)°$, we can calculate any $G(\theta)$. Conversely, when we know $G(\theta_1)$ and $G(\theta_2)$ with $\theta_1 \neq \theta_2$, we can also calculate $G(0°)$ and $G(45°)$ using the following relations:

$$G(0°) = \frac{G(\theta_1)G(\theta_2)}{G(\theta_2)\sin^2 2\theta_2 - G(\theta_1)\sin^2 2\theta_1} \left(\cos^2 2\theta_1 \sin^2 2\theta_2 - \cos^2 2\theta_2 \sin^2 2\theta_1\right) \tag{8}$$

and

$$G(45°) = \frac{G(\theta_1)G(\theta_2)}{G(\theta_2)\cos^2 2\theta_2 - G(\theta_1)\cos^2 2\mathrm{v}_1} \left(\sin^2 2\theta_1 \cos^2 2\mathrm{v}_2 - \cos^2 2\theta_1 \sin^2 2\theta_2\right) \tag{9}$$

Equations (8) and (9) permit arbitrary selection of θ_1 and θ_2 in a test program.

From Eq. (3) and (5), we obtain

$$\nu_{12} = \frac{1}{2}\left[\left(\frac{E_2}{G(45°)} - 1\right)\frac{E_1}{E_2} - 1\right] \tag{10}$$

which requires that $> G(45°)$ since $\nu_{12} > 0$. Eq. (10) can be put in the following form:

$$E_2 = \frac{E_1 G(45^\circ)}{E_1 - G(45°)(1+2\nu_{12})} \qquad (11)$$

in which E_1 and ν_{12} are known to be relatively stable (Bodig 1966; Bodig and Goodman 1969; Doyle et al. 1956) and, in addition, any variation of ν_{12} can only be small compared to 1. Therefore, the one parameter that is most sensitive to E_2 is $G(45°)$.

Modified Arcan shear test and specimen

The Arcan shear test (Arcan et al. 1978) and its modified versions (Liu and Ross 1997; Daniel and Ishai 1994) are based on the fact that a shear force transmitted through a section between two edge notches produces nearly uniform shear stress along the section. The original Arcan shear test fixture has two antisymmetrical portions forming a circular device, with a specimen located in the center and bonded to the fixture by adhesive. In a modified version reported by Daniel and Ishai (1994), the specimen is attached to the fixture by a bolted specimen holder. Liu and Ross (1997) adopted a six-sided configuration, which simplified the testing procedures without compromising the stress condition at the critical section of the specimen.

The geometrical dimensions of the specimen are shown in Fig. 2. The grain of the specimen is parallel to the surface, making an angle of θ with the critical section AB. The thickness of the specimen is parallel to the tangential direction, such that the specimen can be tightly clamped between the restraining plates.

A board of Sitka spruce (*Picea sitchensis*) of unknown history but of the desired grain and annual ring orientations was selected from storage at the Forest Products Laboratory. Fifteen specimens in three equal groups, each with a specified θ value 0°, 22.5°, and 45°, were cut from the board. The specimens were stored in a conditioning room at 20°C and 50% relative humidity for several weeks before testing. The average moisture content was 9.4%, and the average specific gravity was 0.33.

The shear modulus of each specimen was determined by using two side-by-side shear gauges (Ifju 1994) on both faces of a specimen. The strain gauge had a nominal length of 19.35 mm, which closely matched the critical section length of the specimen. Tensile loading was applied with a universal test machine, as shown in Fig. 2. Crosshead speed was 1.27 mm/min. Displacement and load data for shear modulus calculations were recorded electronically.

RESULTS AND DISCUSSION

The results of the Arcan shear test on Sitka spruce specimens are shown in Table 1. Note that the data dispersion decreased quickly as the grain slope increased and the highest coefficient of variation at the grain slope of 0° was 8.96%, well below the usual range of about 20% for wood mechanical properties (Schniewind 1979). Although the data look impressive, we cannot

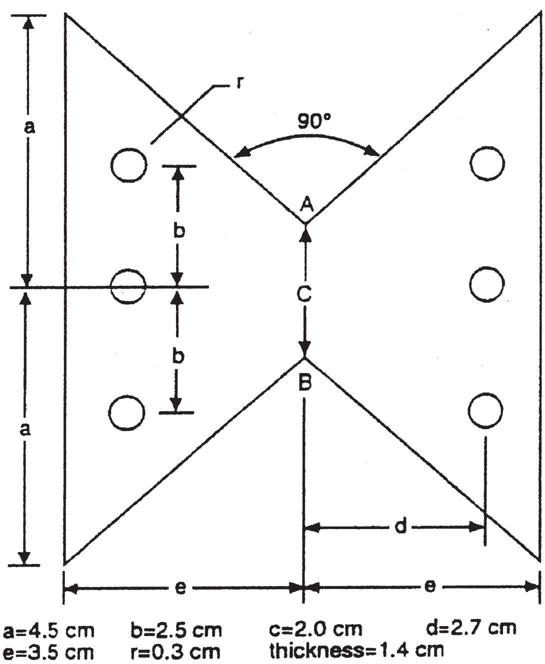

Fig. 2 Geometric dimensions of specimen.

ignore the fact that they were obtained from a relatively small sample.

Substituting the related data from Table 1 in Eq. (7) for different values of θ, we obtained the curve in Fig. 3. Note that the predicted shear modulus at θ = 22.5° is 1,179 MPa, very close to the measured average of 1,194 MPa, which demonstrates the reliability of the test data in Table 1.

Kubojima et al. (1996) performed torsion tests on Sitka spruce specimens and obtained an average value of 884 MPa for $G(0°)$. The $G(0°)$ value listed in Kollmann and Côté (1968) is 745 MPa, which was obtained using the method of March et al. (1942) or ASTM Standard D3044–76. This method, which is called the plate twist test, was designed for plywood plates and requires a square specimen with a ratio of length of edge to thickness to lie between 25:1 and 50:1. Considerable errors are introduced when the wood grain is markedly inclined to the specimen faces or edges in portions of the plate. Since the results in the literature are not consistent and our data in Table 1 are more complete than any known to us, we will use the data in Table 1 in the following discussion.

From Kubojima et al. (1996), we find E_1 = 11,800 MPa. Kollman and Côté (1968) gave E_1 = 11,600 MPa, E_2 = 902 MPa, and ν_{12} = 0.37. As stated earlier, these E_1 values are very stable, but since E_2 = 902 MPa is less than $G(45°)$ = 1,670 MPa, it cannot satisfy Eq. (10). The data for E_2 in Kollman and Côté (1968) were obtained using the compression test by Doyle at al. (1956), which specifies a 50.8- by 50.8- by 203.2-mm specimen with the long dimension in the radial direction. To obtain the required length and to avoid excessive annual ring curvature, four blocks were glued together to construct a specimen. The sources of error in Bodig (1966) and Pellicane et al. (1994)

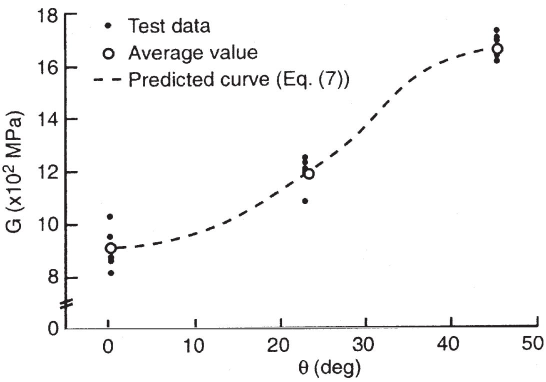

Fig. 3 Variation of shear modulus G with grain slope θ

are seen to converge in this test procedure. With $G(45°)$ = 1,670 MPa, E_1 = 11,800 MPa, and ν_{12} = 0.37, Equation (11) gives E_2 = 2,216 MPa, which is about 2.5 times the value of 902 MPa in Kollman and Côté (1968). Note that ν_{12} has little effect on E_2 in Eq. (11). When ν_{12} increases by 20% from 0.37 to 0.44, E_2 = 2,279 MPa, an increase of less than 3%; when ν_{12} decreases by 20% from 0.37 to 0.30, E_2 = 2,159 MPa, a decrease of less than 3%. Therefore, based on the anisotropic elasticity theory, our predicted value for E_2 = 2,216 MPa.

To lend additional support for the reliability of the four basic engineering constants thus obtained, i.e., E_1 = 11,800 MPa, E_2 = 2,216 MPa, G_{12} = 910 MPa, and ν_{12} = 0.37, all these constants satisfy the equations derived in the analysis of the off-axis tension test of wood specimens, where the principal stress components expressed in terms of the applied tensile stresses, mechanical properties, and grain orientation in anisotropic theory are the same as those in terms of only the applied tensile stresses and grain orientation (Liu 2002). This implies that the four basic engineering constants are correct as a set. If any one of them should change, the others should change accordingly (Wu et al. 1973).

TABLE 1. *Shear modulus test data for Sitka spruce*[a]

Slope of grain (degree)	Average shear modulus (MPa)	Coefficient of variation (%)
0	910	8.96
22.5	1,194	5.16
45	1,670	2.72

[a] Specimens stabilized in conditioning room at 20°C and 50% relative humidity; five tests for each group; 9.4% average moisture content; 0.33 average specific gravity.

CONCLUSIONS

Based on the tensor transformation rules, we derived a formula for shear modulus at any orientation in a principal material plane of wood in terms of the shear modulus values for two specified orientations at $\theta = 0°$ and 45° in the same plane. The formula was verified using a modified Arcan shear test on Sitka spruce specimens. In the process of deriving the formula, we also established a formula for elasticity modulus in the radial direction as a function of the shear modulus at $\theta = 45°$, the elasticity modulus in the longitudinal direction, and the Poisson's ratio associated with the principal material plane. The modulus of elasticity in the radial direction based on this formula can satisfy certain practical equations of anisotropic elasticity theory with which wood specimens are supposed to comply. The current testing methods for wood properties in radial compression cannot yield reliable results because of stress concentrations caused by anatomical structures, specimen geometry, and loading conditions. Since the derived formula needs to be satisfied, we find the determination of elasticity modulus in the radial direction by means of the proposed shear test a convenient and reliable approach to solve an otherwise complicated problem.

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