# A MODEL FOR PREDICTING WOOD FAILURE WITH RESPECT TO GRAIN ANGLE IN ORTHOGONAL CUTTING[1]

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(Received February 1982)

ABSTRACT

Hardwoods were orthogonally cut against the grain to determine a relation between slope of grain and wood failure. White ash, sugar maple, and basswood specimens at approximately 8% moisture content were cut with a 40° rake angle cutter, 0.010-inch depth of cut, and 6½ inches per minute feed rate. Certain types of wood failure were associated with the inflection points and maximum of the model $\mu = \Theta^a b^{(\Theta - c)} + d$, where $\mu$ was the cutting coefficient of friction determined from cutting forces and rake angle and $\Theta$ was the grain angle in radians.

*Keywords:* Surface quality, knife cutting, wood machining, orthogonal cutting, torn grain.

## INTRODUCTION

The most severe surface wood-machining defects, such as torn grain, generally occur during knife cutting against the grain. These defects often require substantial subsequent processing to produce a satisfactory surface. An earlier planing study (Stewart 1970) showed that chipped or torn grain was formed during peripheral milling against relatively low slopes of grain up to approximately 20°. Orthogonal wood cutting studies (Stewart 1969, 1971) have also shown that chipped grain occurs when cutting against the grain up to about 20° in white ash; different types of wood failure have been shown to occur when cutting against other grain angles (Stewart 1969, 1970). Further, similar types of wood failure have been found to occur against approximately the same slopes of grain for peripheral milling as for orthogonal cutting (Stewart 1969, 1970). Characteristic types of surfaces and wood failure appear to be commonly associated with knife cutting against particular slopes or a range of slopes of grain. The previous study (Stewart 1969) of orthogonal cutting of straight-grained white ash at various angles to the grain needed to be replicated with other species to show similarities of wood failure.

An orthogonal cutting study against the grain of three hardwoods—white ash, basswood, and sugar maple—was undertaken to determine the characteristic wood failures and to relate the occurrence of these failures to grain angle (slope of grain) with a curvilinear relation. If the formation of wood failure types can be predicted to occur within ranges or near characteristic points of a curvilinear relation, the study of the formation and reduction of wood-machining defects may be simplified by investigating the slopes of grain where the most severe defects occur. Such a study has not previously been undertaken.

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[1] This manuscript was written and prepared by U.S. Government employees on official time; it is therefore in the public domain.

*Wood and Fiber Science*, 15(4), 1983, pp. 317–325

The different types of wood failure resulting from orthogonal wood cutting indicate variations of tool forces. The tool force components parallel and perpendicular to the cutting edge direction reflect the resistance to the tool movement through the wood. The resistance is the combined interaction of the strength properties. In a previous study (Stewart 1969), the tool force components from orthogonal wood cutting against the grain of white ash did not appear to have a continuous curvilinear relation. However, by calculating the cutting coefficient of friction with the equation:

$$\text{CUTTING COEFFICIENT OF FRICTION, } \mu =$$
$$\text{TAN}\left(\text{ARCTAN } \frac{\text{NORMAL TOOL FORCE, } F_N}{\text{PARALLEL TOOL FORCE, } F_P} + \text{RAKE ANGLE, } \alpha\right), \quad (1)$$

a relatively continuous curvilinear relation between the cutting coefficient of friction and grain angle became apparent. The tool force components from orthogonal cutting at specified angles to the grain could be indirectly related to grain angle. Further, wood failure or chip types associated with changes of the cutting forces changed coincidentally at or near characteristic points, such as inflection points of the curve; for example, in white ash, chipped grain ceased to form and a relatively good surface was obtained during orthogonal cutting against slopes of grain greater than 20°. The range of grain angles where different types of wood failure occur may be described by a mathematical function. Although the cutting coefficient of friction is not the actual physical coefficient of friction, the cutting coefficient of friction provides a quantity that can be related to grain angle among the normal tool force, parallel tool force, and rake angle.

## PROCEDURE

Basswood and sugar maple specimens ¼-inch thick were prepared at specific angles to the grain and conditioned to approximately 8% moisture content prior to cutting. Grain angles were oriented from 90°–0° (cutting parallel to the grain) to 90°–90° (cutting end grain) at increments of 5°, i.e., 90°–0°, 90°–5°, 90°–10°, . . . 90°–90°. The increment is the angle between the direction of tool movement and the grain. The grain angle was varied in the radial-longitudinal plane to minimize the variation of a single growth ring.

A 40° rake angle knife with a 10° clearance angle was mounted in a dynamometer on a milling machine so that the normal and parallel tool force components could be measured. The specimens were trimmed with the grain so that damage from preparation could be minimized and so that a uniform 0.010-inch depth of cut could be maintained. The three cuts for each combination were 3 inches long at 6½ inches per minute.

The model applied to relating the cutting coefficient friction ($\mu$) to the grain angle ($\Theta$) in the radians was

$$\mu = \Theta^a b^{(\Theta - c)} + d, \quad (2)$$

where a, b, c, and d are constants. The constants were determined by nonlinear regression techniques. The cutting coefficient of friction was determined for each grain angle by substituting force data into Eq. (1).

Equation (2) was selected after observing the data for failure types with respect to grain in the previous study (Stewart 1969). Some failure types appeared to change abruptly and others gradually, depending on the grain angle. For instance, torn grain occurred when orthogonal cutting white ash between 0° and 20° slope of grain but stopped abruptly at between 15° and 20° slope of grain. Fiber deflection appeared to increase and decrease gradually between 35° and 60° slope of grain. The calculated coefficient of friction suggested that a skew distribution would fit the data. Since failure types appeared to change with grain angle and the changes seemed to be related to a change in relation between the cutting forces associated with the grain angle, the changes could also be related to the characteristics of a continuous function. Further, the cutting coefficient of friction started and returned to about the same value, which indicated that an intercept value was required. The model chosen appeared to fulfill the requirements.

## RESULTS AND DISCUSSION

The cut surfaces of basswood and sugar maple exhibited many of the same features previously described for white ash as the grain was oriented from 90°–0° (parallel to the grain) to 90°–90° (end grain). Chipped grain was formed at grain angles up to 20° in white ash (Stewart 1969) and was formed up to about 15° in basswood and sugar maple (Figs. 1 and 2). Whereas the fiber deflection was quite pronounced at 90°–50° for white ash, it was less pronounced at this slope for basswood and sugar maple; it was greatest at approximately 90°–35° and 90°–40° for the two species, respectively (Figs. 3 and 4). At intermediate slopes of grain (90°–25° for basswood and 90°–30° for maple) the surfaces were essentially intact (Figs. 5 and 6). As the cutting direction approached the longitudinal grain direction, a relatively good surface was produced. The more evenly textured, diffuse-porous basswood and sugar maple did not exhibit the extreme failure types shown by the unevenly textured, ring-porous white ash. Although the results are limited, coarse-textured, less uniform woods may be expected to show more severe failure. Further, the same types of failure occurred when cutting against slightly smaller grain angles for diffuse-porous species.

The relations between the cutting coefficient of friction and grain angle were similar for the three species (Fig. 7). The parallel and normal tool force component patterns were similar for basswood and sugar maple, and both were similar to previous results (Stewart 1969) for cutting white ash. The similarities of tool force component patterns accounted for the similarities among the relations between the cutting coefficient of friction and grain angle. Further, the relations between the cutting coefficient of friction and grain angle appeared continuous (Fig. 7), unlike the relations between either the normal or parallel tool force component and the grain (Stewart 1969).

The model resembles a skew distribution over the included range of data. The constants a, b, c, and d, which were determined by nonlinear regression techniques for each species, are given in Table 1. The skew distribution has inflection points and a maximum.

The model selected for the relation between the cutting coefficient of friction and grain angle has some characteristics that are coincidental with surface features. The inflection points were determined from $\Theta = -a \pm \sqrt{a/\ln b}$. The

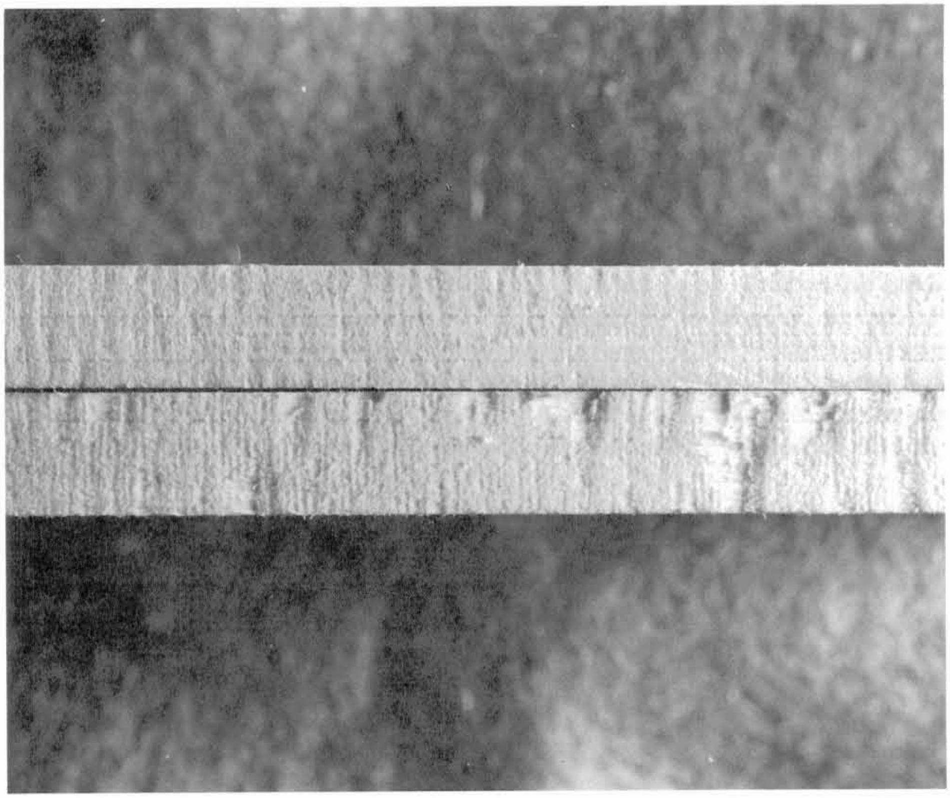

FIG. 1. The reduction of chipped grain as the grain angle increases from 10° (bottom) to 15° (top) in basswood.

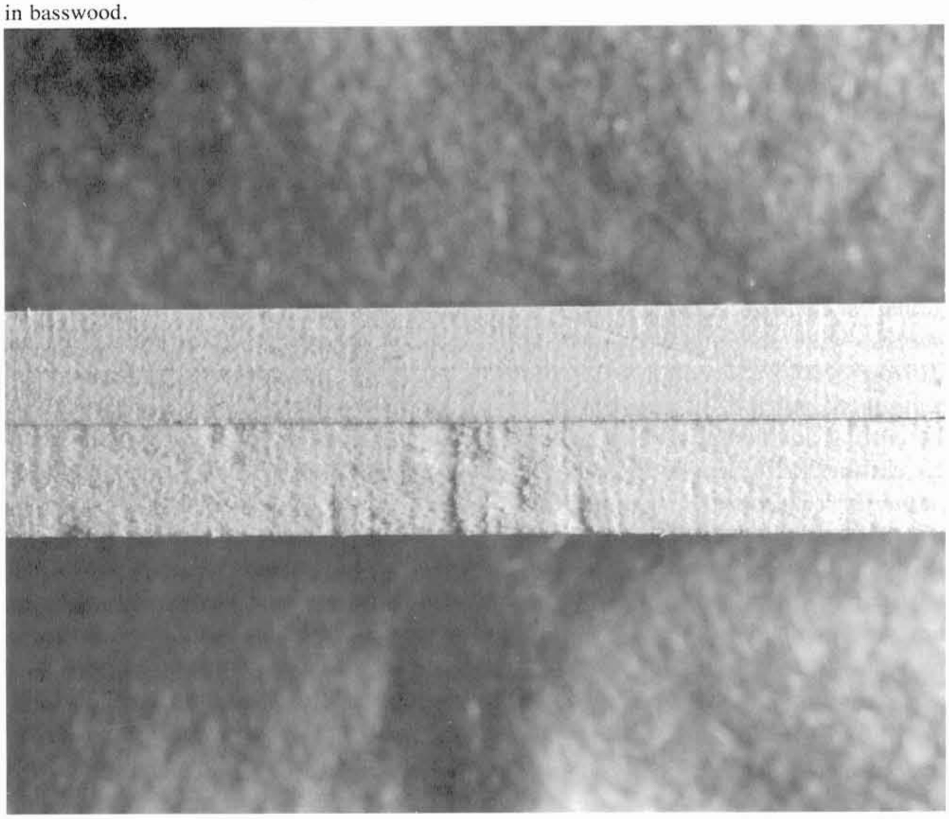

FIG. 2. The reduction of chipped grain as the grain angle increases from 15° (bottom) to 20° (top) in sugar maple.

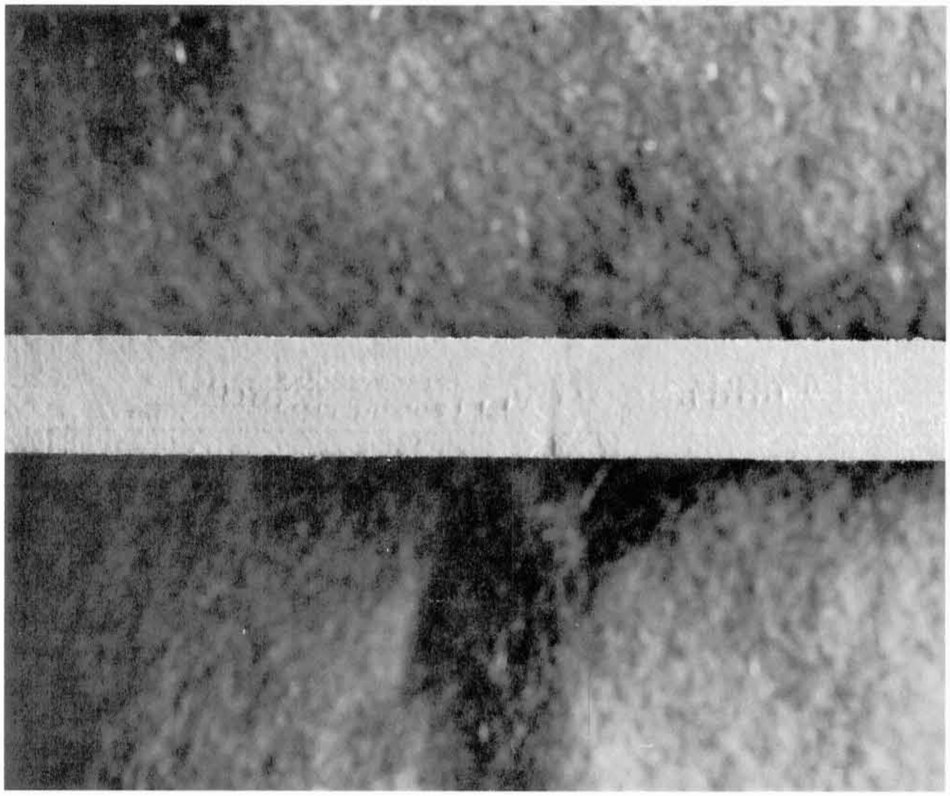

FIG. 3. The fiber deflection in basswood at a 35° slope of grain.

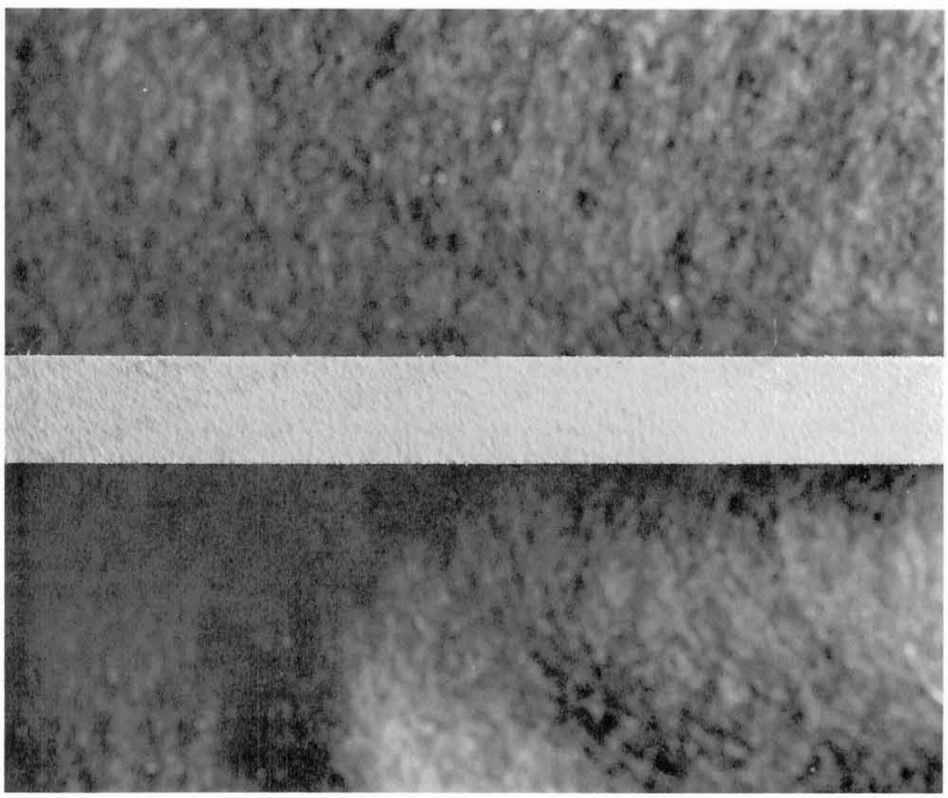

FIG. 4. The fiber deflection in sugar maple at a 40° slope of grain.

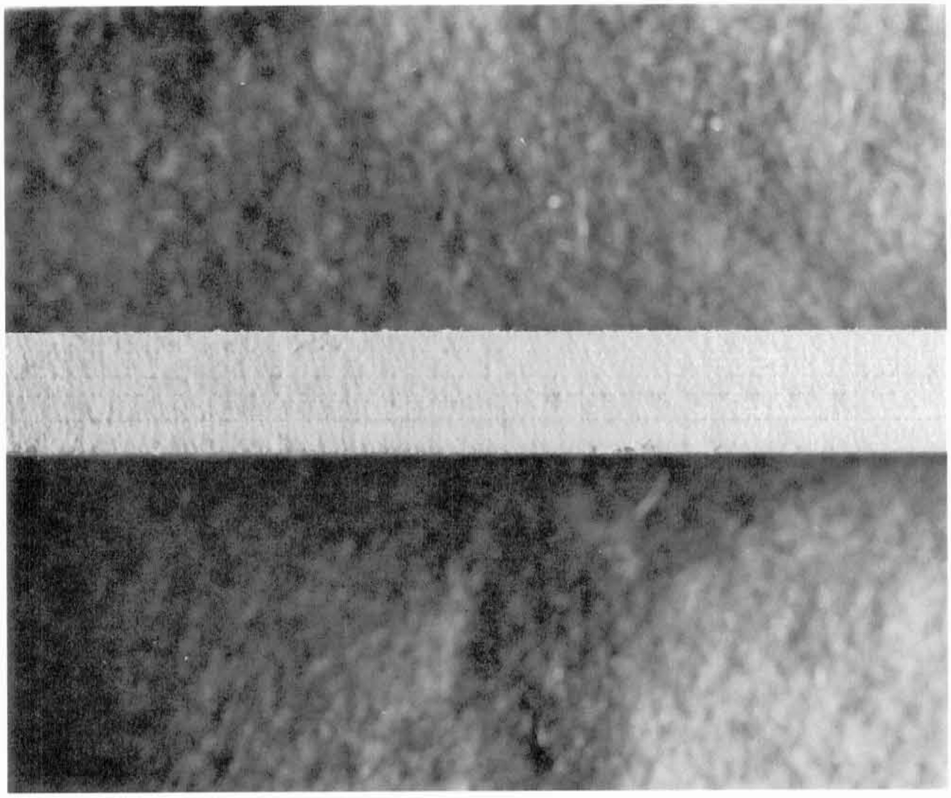

FIG. 5. A relatively good surface at a 25° slope of grain in basswood.

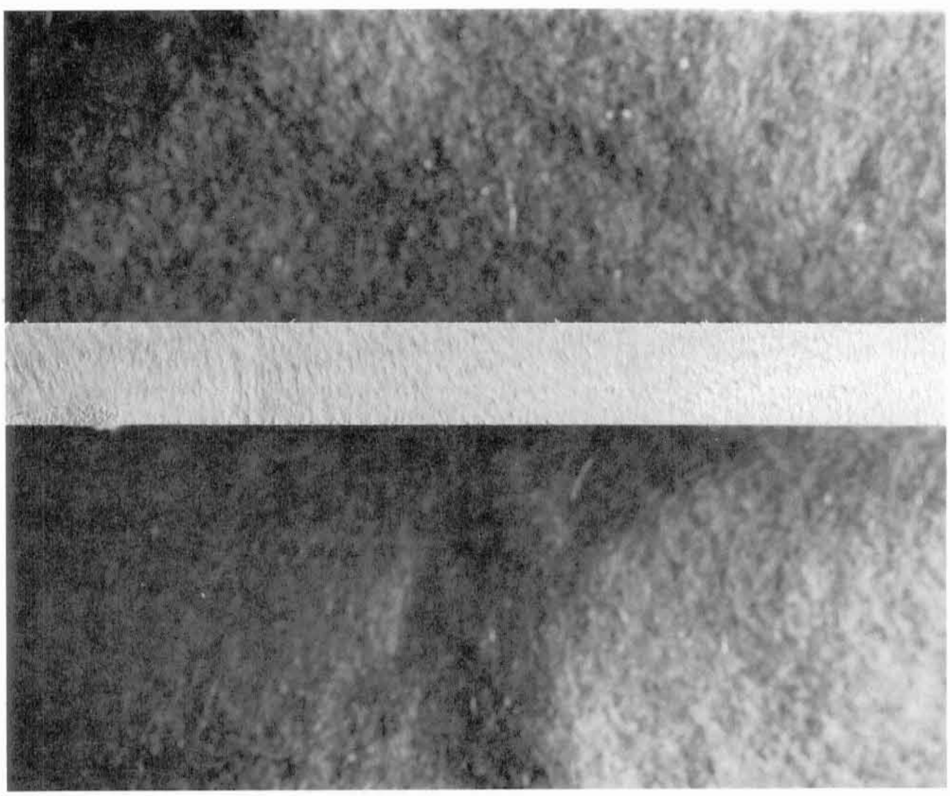

FIG. 6. A relatively good surface at a 30° slope of grain in sugar maple.

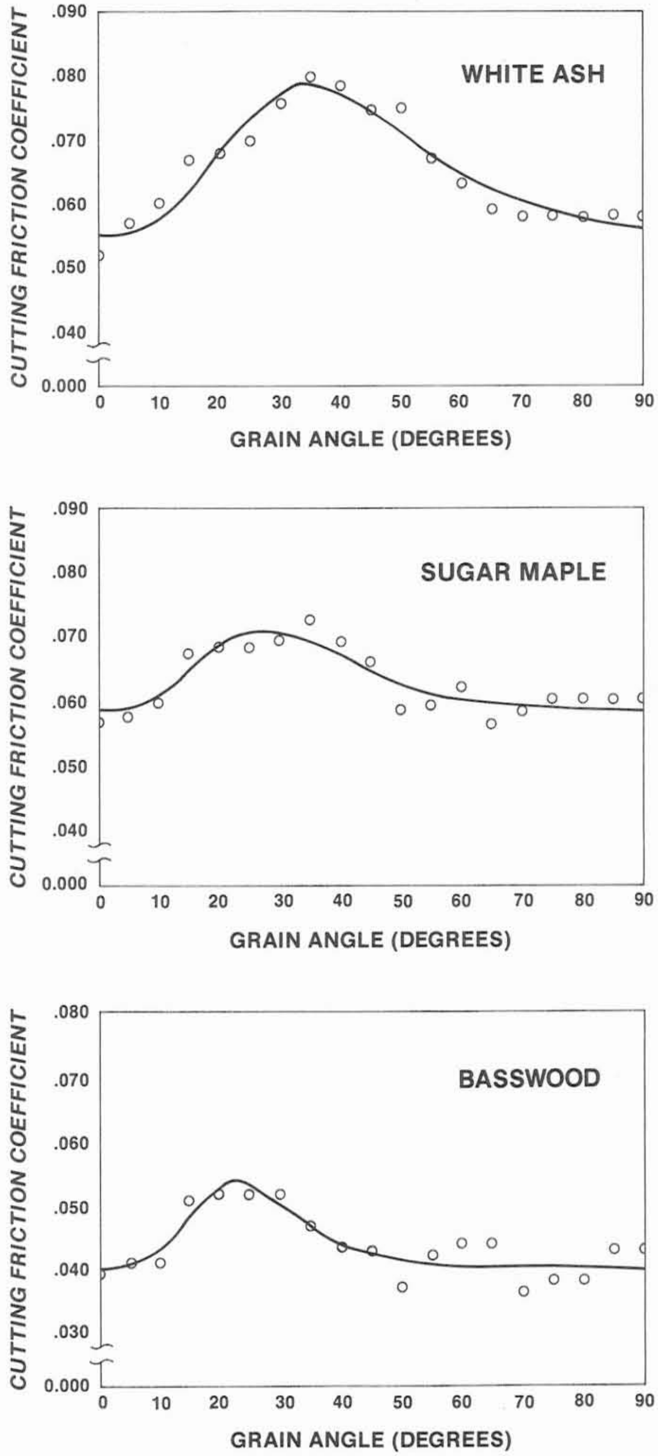


FIG. 7. The cutting friction coefficient as a function of grain angle using a rake angle of 40° and depth of cut of 0.010 inch for white ash, sugar maple, and basswood conditioned at 8% moisture content. The circles are data and the solid lines are estimated values from the model $\mu = \Theta^a b^{(\Theta - c)} + d$.

TABLE 1. *The constants a, b, c, and d; maximum (M); inflection points (IF); and standard error of the estimate SE for the regression model $\mu = \Theta^a b^{(\Theta-c)} + d$, where $\mu$ is the cutting coefficient of friction and $\theta$ is the grain angle in radians, for three hardwood species conditioned at approximately 8% moisture content and orthogonally cut with a 40° rake angle and 0.010-inch depth of cut.*

| Species | (a) | (b) | (c) | (d) | (M) | $(IF)_1$ | $(IF)_2$ | (SE) |
|---|---|---|---|---|---|---|---|---|
| | | | | | Degrees | | | |
| White ash | 4.84 | $2.99 \times 10^{-4}$ | 0.724 | 0.547 | 34.2 | 18.6 | 49.7 | ±0.027 |
| Sugar maple | 4.87 | $0.395 \times 10^{-4}$ | 0.629 | 0.580 | 27.5 | 15.0 | 40.0 | ±0.025 |
| Basswood | 6.41 | $0.781 \times 10^{-7}$ | 0.636 | 0.393 | 22.4 | 13.4 | 31.3 | ±0.027 |

first inflection point occurred at approximately the greatest slope of grain, where torn grain occurred in all three species. The calculated inflection points were 18.6, 15.0, and 13.4 degrees for white ash, sugar maple, and basswood, respectively. Torn grain did not occur at angles greater than 20, 20, and 15 degrees for white ash, sugar maple, and basswood, respectively. Considering that the specimens were prepared at 5° increments in slope of grain, and that the grain can vary naturally within a specimen, the calculated inflection points coincided remarkably well with the actual observations.

The second inflection point occurred at the slope of grain where the fibers were severely deflected before being severed. These inflection points were at 49.7, 40.0, and 31.3 degrees for white ash, sugar maple, and basswood, respectively. The apparent maximum fiber deflection in the specimens tested occurred at 50, 40, and 35 degrees for white ash, sugar maple, and basswood. Again, the calculated inflection points coincided remarkably well with actual observations.

The maximum of the model for the cutting coefficient of friction was determined from $\Theta = -a/\ln b$. As indicated previously, the cutting coefficient of friction was not the actual physical coefficient of friction. The cutting coefficient calculated from the cutting forces reflects to some extent the interference from the workpiece as a result of recovery acting along the clearance face as well as the effect of the edge radius. The variation in the cutting coefficient of friction illustrated in Fig. 7 is also partially the result of deformation recovery forces acting against the tool clearance face. However, the recovery forces acting on the clearance face do not explain the increase of the parallel tool force from 0 to approximately 35, 30, and 25 degrees for white ash, sugar maple, and basswood, respectively. The increase of the cutting coefficient of friction is apparently due to the increased parallel force accompanied by a decreased negative normal force, in turn due to the effects of the increasingly stiff chip, which can be imagined as a cantilever beam. The maximum resistance to bending appears to occur at grain angles beyond those where chipped grain or a cleavage failure occur and where the cross-section of the imaginary beam (Stewart 1971) ahead of the knife becomes sufficiently large to prevent bending as a cantilever beam (Table 1).

The maximum cutting coefficient of friction occurs at approximately the same slope of grain as the inflection point for Hankinson's formula (USDA 1974), which relates strength properties parallel and perpendicular to the grain when the tensile

TABLE 2. *Inflection point of Hankinson's formula (IPH) determined from the maximum fiber stress (MFS) at the proportional limit in static bending and the tensile strength perpendicular (TSP) to the grain.*

| Species | MFS (psi) | TSP (psi) | IPH |
|---|---|---|---|
| White ash | 10,990 | 1,320 | 30.5 |
| Sugar maple | 30,140 | 1,680 | 25.9 |
| Basswood | 7,505 | 375 | 25.3 |

strength perpendicular to the grain and a value (USDA 1974) for the tensile strength parallel to the grain at the proportional limit are applied. When cutting against lower slopes of grain, a cleavage failure along the grain precedes the cantilever beam-type failure as a result of the bending stresses caused by the chip movement up the tool face. The sequence of failures is characteristic of a Type I chip (Franz 1958). The cleavage or tension failure perpendicular to the grain occurs substantially before the rupture of the chip as a beam and probably before the maximum fiber stress at the proportional limit (which is considered a conservative estimate of tension strength parallel to the grain). The values substituted for white ash were from previous data (Stewart 1971), the values for hard maple were from the *Wood Handbook* (USDA 1974), and the values for hard maple were estimated from *Canadian Woods* (FDL 1951). Some common characteristics among models for strength properties and models derived from cutting forces may be expected, since the cutting forces are a measure of the combined resistance to the wedge-shaped tool.

Relations among the tool force components, rake angle, cutting coefficient of friction, grain angle, and a possible connection to selected strength properties and wood failures were shown empirically. Knowledge of such relations may have future value for improving knife cutting processes.

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