THE MODULUS OF ELASTICITY OF HYBRID LARCH PREDICTED BY DENSITY, RINGS PER CENTIMETER, AND AGE

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ABSTRACT

A total of 492 specimens from eighteen trees representing fast-growth larch were studied. The mean values of the modulus of elasticity (MOE), density, age, and rings per centimeter were determined for each. Mathematical models were developed to predict the MOE from the other parameters. Linear models are first presented showing the dependence of MOE on each of the three parameters, density, age, and rings per centimeter. A multivariate linear regression model is then developed for MOE as a function of all three parameters. The correlation coefficient of this model is 0.66, an improvement over each of the models for which each parameter is taken individually. A prediction of MOE by visual means alone, i.e., using only age and rings per centimeter in a linear model, is also presented. The correlation coefficient of this model is 0.58, which is superior to any of the parameters taken individually. In order to develop a model useful for extrapolation beyond the range of test data, a nonlinear model is presented. The parameters of this nonlinear model can easily be interpreted in terms of (1) the maximum attainable stiffness, (2) the ring age for maximum MOE growth rate vs. age, and (3) the shape parameter of the model. We found that the nonlinear model matched the data well and incorporated the realistic conditions of zero MOE at zero ring age and a limit on the maximum attainable MOE.

Keywords: Modulus of elasticity, wood density, age, rings per centimeter, larch, *Larix* × *eurolepis*.

INTRODUCTION

Mechanical properties of wood have often been correlated with specific gravity. There is some degree of uncertainty regarding the appropriate method of modeling the relationship between the two. Bodig and Goodman (1973) tested and analyzed a large number of softwoods and hardwoods and applied regression analysis to the relationships between the moduli of elasticity and specific gravity. They applied both linear and exponential relationships and found little difference in accuracy between the two methods. Bodig and Jayne (1982) opted to use only the exponential relationship. In their study of 37 specimens of softwoods, Guitard and El Amri (1987) found a higher correlation for the linear relationship than the exponential relationship. They also presented data showing consistently strong linear behavior above a specific gravity value of 0.4 for all elastic constants, and any departure from linearity occurred at levels below 0.4.

Zhang (1994) analyzed the relationships of mechanical properties, including MOE, to specific gravity for 342 Chinese woods. He found the linear relationship to predict the static bending properties better than the exponential

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relationship. More recently Zhang (1997) reviewed the history of differing points of view and reported preferences on both sides. He also reexamined the mechanical properties and specific gravity relationships at the species level. His results for larch show a slight bias in favor of the exponential relationship for tests on at least 30 specimens.

These results, particularly those of Guitard and El Amri, show that these are empirical relationships for which there exists no physical law that would prove any one relationship to be valid.

MATERIALS AND METHODS

A total of 492 samples from eighteen trees of fast-growth hybrid larch grown in Brittany, France, representing a fast-growth hybrid larch, were tested for modulus of elasticity (MOE) by means of the resonance flexure method. This method has been employed previously for samples of spruce and fir (Haines et al. 1996; Haines and Leban 1997). The larch specimens of our study all had values for specific gravity well above 0.4.

The wood samples were sawn from eighteen randomly chosen trees of a hybrid (*Larix × eurolepis*) between *Larix decidua* and *Larix kaempferi*. This hybrid is an open-pollinated progeny from the Danish hybridization seed orchard FP201DX, which is used as a standard in all French trials. This progeny was planted in 1959 in a replicate of the 2nd International IUFRO Provenances trials located in Coat-An-Noz, Brittany, France. Details about plantation and experimental design can be found in Pâques (1996). The eighteen trees yielded a total of 492 specimens, at least 16 specimens from each tree. When harvested in February 1994, the trees were 35 years from the seed. The attained tree height was close to 20 meters, and the average of the DBH for the sampled trees was 24 cm. The specimens were selected from butt logs.

We report on the individual relationships of the modulus of elasticity to wood density (WD), rings per centimeter (RPC), and mean age (AGE) and then present linear models for each. (Note: Wood density = mass(kg)/volume(m^3) = 1,000*(specific gravity).) RPC is determined by dividing 20 mm, the sample section width, by ring width (in mm) and by 2. This variable was selected in place of ring width because it provides a better correlation with MOE than ring width.

Resonance flexure tests

The resonance flexure method utilizes specimens in the form of bars, nominally 380 mm in length with a 20-mm-square cross section. These dimensions conform to those specified in the French standard NF B 51 016 for tests of wood specimens in four-point static bending. The specimens are all oriented with the length parallel to the longitudinal axis of the tree and with the sides coinciding with the tangential (T) and radial (R) directions. The resonance test consists of supporting the bar on two stretched threads with the tangential direction vertical. The supports are located at the nodal points of free-free flexural vibration, 22% of the length from each end. The bar is struck perpendicularly in the center. This process excites the bar in flexural vibration. The resulting sound is received by a microphone placed near the surface, and the signal is analyzed by Fourier analysis to identify the principal resonance. A full description of this test method is presented in Haines et al. (1996). With the principal resonance frequency so determined, the modulus of elasticity is calculated in SI units from the equation

$$MOE = 0.946 \rho f^2 L^4 / T^2, \qquad (1)$$

where

- MOE is the modulus of elasticity in the longitudinal direction (N/m^2),
- ρ is the density of the wood at 12% MC (kg/m^3),
- f is the frequency of the principal resonance (Hz),
- L is the length of the bar (m),
- T is the bar thickness, direction T (m).

TABLE 1. *Descriptive statistics for all the measured samples. Number of samples = 492.*

	MOE (Mpa)	Ring width (mm)	AGE (Years)	DENSITY (Kg/M^3)
Minimum	3643	1.34	7	444
Maximum	18559	9.90	35	723
Mean	11789	3.34	21.33	584
Standard Dev	3618	1.33	7.59	51.21

We present the modulus of elasticity result in units of megapascals (MPa) which is the value of the modulus of elasticity calculated from Eq. (1) divided by 10^6.

RESULTS AND DISCUSSION

Repeatability of test results

The frequencies obtained with wood specimens of the selected size fall in the range of 300 to 1,000 Hz. With modern instrumentation, it is possible to readily determine the frequency with accuracy to 1.0 Hz, or better. To determine the repeatability of results, five researchers tested the same 23 specimens and the results compared. For all but two of the 115 comparisons, the frequencies differed by no more than 4 Hz, and the difference for the two exceptions was only 6 Hz. Thus, with sufficiently accurate measurements of dimensions, it is possible to calculate the MOE to an accuracy of better than two significant figures with confidence.

Basic tree properties

The basic tree properties of importance in this study are the ring width, age, wood density at 12%, moisture content, and modulus of elasticity. For the entire set of 492 specimens, the maximum, minimum, mean, and standard deviation of these properties are presented in Table 1.

The MOE to density ratio is a useful material performance index (Ashby 1992). It is therefore meaningful to compare this ratio for larch (about 20) to other softwoods available in our data base (Leban and Mothe 1996). The best (i.e., the highest) value in the data base for this ratio is 26 for Norway spruce (503 specimens) and the worst is 17 for Corsican pine (387 specimens).

Table 2 presents a breakdown of these data for each tree in order to illustrate the individual tree variability. One can see that for close values of average ring width and ring age, the

TABLE 2. *Breakdown of the data for each tree.*

		MOE MPa				RW (mm)				AGE years				DENS (Kg/M^3)			
TREE	NB	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD	Min	Max	Mean	SD
2464	25	4546	15378	9448	3498	1.48	8.00	3.61	1.79	8	30	19	8	488	611	536	36
2478	42	4681	14450	10255	2550	2.34	8.00	4.08	1.13	10	33	21	7	502	607	559	30
2468	32	4332	14526	10310	3010	2.08	6.63	3.57	1.15	7	30	19	7	452	584	521	32
2463	27	3765	15812	10555	2999	2.01	5.72	3.11	0.88	9	33	22	7	472	618	557	39
2466	24	3643	16800	10570	4200	1.47	9.90	3.45	2.12	8	35	22	8	444	648	576	56
2469	35	4168	16643	10946	4094	1.89	6.67	3.75	1.63	8	33	20	8	467	649	576	53
2474	31	4700	15860	11210	3046	2.10	6.68	3.40	1.02	9	34	22	8	509	641	594	35
2470	26	6131	17268	11716	3599	1.74	6.70	3.64	1.61	9	32	19	8	487	681	599	56
2479	26	4428	17140	11983	3684	1.91	5.72	3.63	1.31	10	32	21	7	503	652	574	40
2471	30	5800	16591	11993	3291	2.00	5.73	3.27	1.23	10	32	21	8	465	649	584	49
2476	22	4511	17375	12353	4198	1.66	5.68	2.87	1.18	10	33	23	8	495	668	605	43
2475	17	6677	18223	12661	3585	1.54	4.46	2.63	0.92	12	33	23	7	562	723	649	41
2462	16	4824	17264	12766	4100	1.34	5.70	2.57	1.26	11	31	22	7	527	708	612	49
2477	26	6270	17872	12987	3017	1.71	5.73	2.87	0.98	11	33	23	7	526	659	584	33
2467	28	7009	18095	13162	3071	1.53	5.70	3.06	1.13	8	32	20	7	515	667	615	40
2465	25	5755	18076	13565	3461	1.82	5.74	2.79	0.96	14	33	24	7	541	675	623	29
2472	22	4912	17835	13582	3948	1.43	6.69	2.78	1.34	10	35	24	8	499	720	628	55
2473	38	7712	18559	13764	3102	2.35	5.01	3.57	0.82	10	33	22	8	517	658	588	38

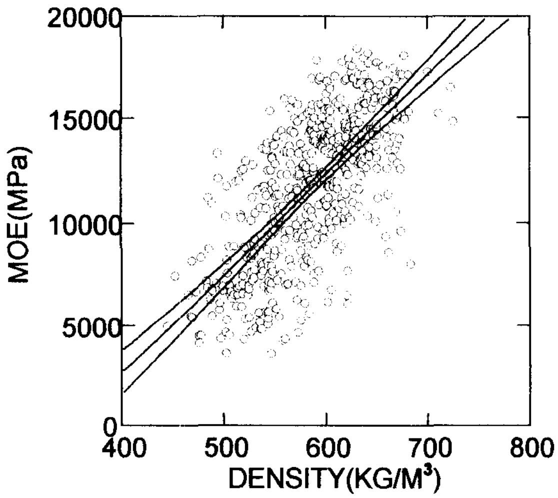

FIG. 1. MOE vs. mean density with confidence intervals on regression line.

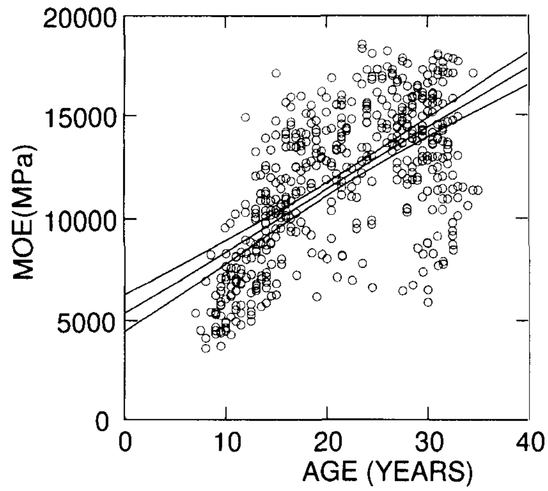

FIG. 2. MOE vs. mean age with confidence intervals on regression line.

average individual tree MOE can vary from 9,448 to 13,764 MPa (trees No. 2464 and 2473, respectively). In terms of the MOE to density ratio, the values for these same two trees are respectively 17 (=9,448/536) and 23 (=13,774/588). These values cover about the same range of variation as that between species of the data base (17 to 26). In the following, we present the steps for analyzing the data for the MOE mode construction.

The modeling approach

Considering that there is no physical law that establishes validity of linear versus nonlinear models and because of the simplicity of a linear model, we first employ linear relationships. A linear model relates MOE to each of wood density, age, and rings per centimeter, a total of three. In order to develop an improved model, a single multivariate regression analysis was performed using all three parameters. The correlation coefficient obtained is better than any of the three linear models taken individually, as expected.

The prediction of MOE based on data that can be obtained by visual means alone has the appeal of simplicity. With this goal in mind, a multivariate linear regression analysis was performed using only the parameters of rings per centimeter and age. The correlation coefficient for this case is also better than any of the three taken individually.

It is readily observed that one deficiency of the linear models is the uncertainty of extrapolating beyond the range of the test data that produced the linear models. To overcome this problem and to make the model more physically realistic and biologically interpretable, a nonlinear exponential growth model is proposed that imposes zero MOE for zero ring age and a maximum attainable MOE.

Linear regression

The first presentation of data shows the measured MOE versus each of the three parameters: mean wood density, mean age, and rings per centimeter. The linear model was selected for each, and a linear equation was fitted to the data of each case. Figure 1 displays the plot of MOE versus the mean wood density for all specimens tested. The regression analysis of the data for a straight line fit is shown as well as the equation and correlation (R^2). The value of R^2 is 0.47.

Figure 2 shows the plot of MOE versus the mean mean age of the specimen. In this case R^2 is 0.39, not as high as for the density.

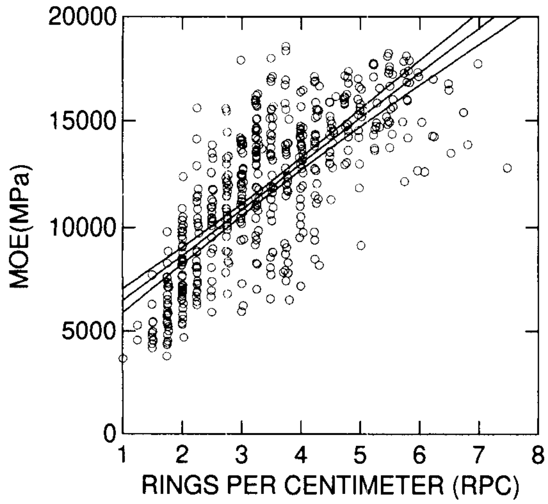

FIG. 3. MOE vs. mean rings per centimeter with confidence intervals on regression line.

Figure 3 displays the plot of MOE versus the average rings per centimeter for each specimen. The linear regression analysis yields a value of $R^2 = 0.56$, the highest of the three parameters.

Multivariate linear regression

Given that the dependence of MOE on each parameter has similar characteristics, viz. increasing MOE with increasing values of each parameter, we investigated the outcome of combining the trends by expressing the modulus of elasticity as a function of several parameters. The first choice selected in this approach is all three parameters: wood density (WD), age (AGE), rings per centimeter (RPC). This is expressed as:

$$MOE = \alpha_1(WD) + \beta_1(AGE) + \gamma_1(RPC) + \mu_1. \quad (2)$$

A multivariate linear regression analysis performed with the software SYSTAT (1992) by using the REG procedure to obtain the values of α_1, β_1, γ_1 and μ_1. This process yielded the following values:

$$\alpha_1 = 25595 \qquad \beta_1 = 87.019,$$

$$\gamma_1 = 1148.4 \quad \text{and} \quad \mu_1 = -8979.3$$

with a calculated correlation coefficient R^2 of 0.66 (see results in Table 3). This value can be considered as an estimate of the maximum possible R^2 for this data set, because all the available variables are taken into account in the model.

With an objective to develop a model that is based on parameters determined by visual means alone, a second analysis was performed on the two parameters AGE and RPC. The equation that expresses this relation is:

$$MOE = \beta_2(AGE) + \gamma_2(RPC) + \mu_2. \quad (3)$$

TABLE 3. *Statistical ouput for model Eq (2).*

Dependant	Var:MOE		N:492			
Multiple R:			0.81			
Squared	multiple R:		0.66			
Adjusted	squared multiple R:		0.66			
Standard	error of estimates		2109.35			
Effect	Coefficient	Std error	Std coef	Tolerance	t	P (2 tail)
CONSTANT	−8979.31	1204.17	0	.	−7.457	0
AGE	87.02	18.02	0.182	0.484	4.83	0
RPC	1148.49	122.75	0.400	0.377	9.36	0
WD	25.595	2.34	0.362	0.633	10.9	0

Analysis of Variance

Source	Sum-of-squares	df	Mean-square	F-ratio	P
Regression	4.26E + 09	3	1.42E + 09	319.08	0
Residual	2.17E + 09	488	4.45E + 06		

TABLE 4. *Statistical output for model Eq. (3).*

Dependant	Var:MOE		N:492				
Multiple R:			0.76				
Squared	multiple R:		0.58				
Adjusted	squared multiple R:		0.58				
Standard	error of estimates		2352.2				
Effect	Coefficient	Std error	Std coef	Tolerance	t	P (2 tail)	
CONSTANT	3799.9	335.19	0	.	11.33	0	
AGE	86.3	20.09	0.18	0.484	4.29	0.00002	
RPC	1780.5	120.8	0.62	0.484	14.73	0	

Analysis of variance

Source	Sum-of-squares	df	Mean-square	F-ratio	P
Regression	3.72E + 09	2	1.86E + 09	336.59	0
Residual	2.71E + 09	489	5.53E + 06		

The model was fitted to the data by use of the REG procedure of SYSTAT. The multivariate linear regression yielded the following values for this case:

$$\beta_2 = 86.341, \quad \gamma_2 = 1{,}780.5 \quad \text{and}$$

$$\mu_2 = 3{,}799.9$$

with a calculated correlation coefficient R^2 of 0.58 (see results in Table 4). This model is therefore superior to taking either age or rings per centimeter alone as the variable.

Nonlinear regression

A valid physical model will show that MOE = 0 when either WD = 0, AGE = 0 or RPC = 0, i.e., these conditions define points where the wood can have no stiffness. None of the linear models presented are physical models since they do not exhibit this property. This requirement may be of little practical interest since no existing tree will exhibit these conditions. However, if one were to use the linear models outside the range of the data used to create the model, e.g., very young trees, extrapolation of the linear model will lead to inaccurate predictions. Another difficulty with a linear model is that it predicts ever-increasing MOE with age. It is more realistic to incorporate the observation that the MOE approaches a maximum limit as the tree ages.

These observations can be gained by using a nonlinear model that incorporates the conditions. As an example we reexamine the dependence of MOE on tree age. This behavior can be modeled with an exponential equation with three constants of the following form:

$$\text{MOE} = \beta_1[1 - \exp(\beta_2(\text{AGE}))]^{\beta_3}. \quad (4)$$

In the growth and yield sciences, it is very common to use Eq. (4), which is derived from Richards' growth curve (1959). The parameters of this model can be interpreted as follows:

β_1 is the asymptote, i.e., an estimate of the ultimate MOE value which can be reached.

β_2 (note $\beta_2 < 0$) and β_3 are both related to the growth rate and shape of the MOE versus AGE model.

The growth rate of MOE is expressed by the first derivative with respect to age. From Eq. (4) we have

$$\frac{\partial \text{MOE(AGE)}}{\partial \text{AGE}} = -\beta_1 * \beta_2 * \beta_3 * (1 - \exp(\beta_2 * \text{AGE}))^{\beta_3 - 1} * \exp(\beta_2 * \text{AGE}). \quad (5)$$

The maximum growth rate is reached when the second derivative with respect to age of Eq. (4) is set equal to zero. The second derivative is displayed in Eq. (5.1).

TABLE 5. *Statistical output for the nonlinear model established by Eq (6).*

Source	Sum-of-squares	df	Mean-square
Regression	7.17571E + 10	3	2.39190E + 10
Residual	3.05953E + 09	489	6.25672E + 06
Total	7.48166E + 10	492	
Mean corrected	6.43051E + 09	491	
Raw R-square (1-Residual/Total)		= 0.959	
Mean corrected R-square (1-Residual/Corrected)		= 0.524	
R (observed vs. predicted) square		= 0.524	

	Parameter estimate	A.S.E.	Param/ASE	Wald confidence interval Lower	< 95% >	Upper
β_1	14009	215	65.13	13587		14432
β_3	6.68	1.84	3.62	3.06		10.3
β_2	8.78	0.38	22.82	8.02		9.53

$$\frac{\partial^2 MOE(AGE)}{\partial AGE^2} = [\beta_1*(1 - \exp(\beta_2*AGE))^{\beta_3}*\beta_2^2 *\beta_3\exp(\beta_2*AGE)*(\beta_3*\exp(\beta_2*AGE) - 1)] \div [(1 - \exp(\beta_2*AGE))^2] \quad (5.1)$$

Equation (5.1) equals zero when $\beta_3*\exp(\beta_2*AGE) - 1 = 0$, and the ring age for which the maximum growth rate is reached is therefore $\ln(1/\beta_3)/\beta_2$. This value is proposed as a useful definition of the limit between juvenile and mature wood at the stand level for this wood property, i.e., MOE. By using such a definition it is easier to define and interpret the juvenile mature limit, at least easier than the age at which the asymptote is reached.

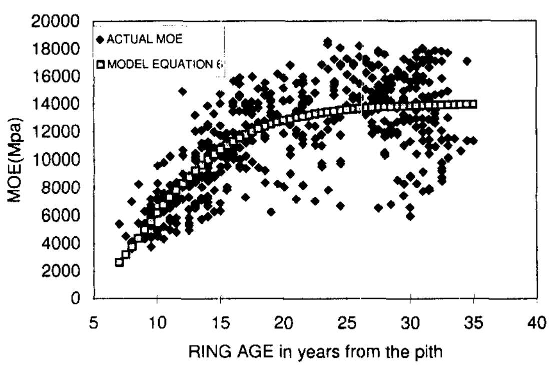

FIG. 4. Actual and simulated MOE vs. AGE. Model defined by Eq. (6).

In order to make all the parameters as easy as possible to interpret, we have transformed Eq. (4) as follows (Hervé 1998):

$$MOE(AGE) = \beta_1[1 - \exp(\beta_{21}*AGE)]^{\beta_3} \quad (6)$$

where

$$\beta_{21} = \ln(1/\beta_3)/\beta_2.$$

With this formulation β_1 remains the asymptote (i.e., an estimate of the maximum value for MOE), β_2 is now directly the ring age of maximum growth rate for MOE and β_3 is a parameter defining the shape of the growth curve of MOE vs. AGE. The model was fitted to the data by use of the NLIN procedure of SYSTAT (1992) with the Gauss-Newton method that computes the exact derivative. The estimates of the parameter converged after 19 iterations and the results are presented in Table 5. With this set of parameters, the predicted maximum MOE (β_1) is 14,009 Mpa, the age of maximum MOE growth rate (β_2) is 8.78, and the shape parameter is β_3 is 6.68. Figure 4 shows the comparison of the data points for MOE vs. AGE with the curve of Eq. (6).

The results obtained from applying Eq. (6) to the data appear satisfactory. The parameters are well estimated and their confidence intervals are low. The mean corrected R^2 is 0.524 with only one variable, i.e., the age. This value is comparable to the R^2 calculated with (1) the

TABLE 6. *Statistical output for the nonlinear model established by Eq. 7.*

Source	Sum-of-squares	df	Mean-square		
Regression	7.24431E + 10	4	1.81108E + 10		
Residual	2.37352E + 09	488	4.86378E + 06		
Total	7.48166E + 10	492			
Mean corrected	6.43051E + 09	491			
Raw R-square (1-Residual/Total)			=	0.968	
Mean corrected R-square (1-Residual/Corrected)			=	0.63	
R (observed vs. predicted) square			=	0.63	
	Parameter estimate	A.S.E.	Param/ASE	Wald confidence interval < 95% > Lower	Upper
a	1467	120	12.22	1231	1702
b	7541	538	14	6483	8599
β_3	12.1	6.6	1.8	0.83	25.1
β_2	7.55	0.49	15.22	6.57	8.52

linear model between MOE and AGE (R^2 = 0.56) and (2) the multivariate linear model between the MOE and both RPC and AGE (R^2 = 0.58).

Modified nonlinear model

Despite the relatively correct R^2, foresters would prefer a model for MOE variations based also on the growth rate in addition to the ring age. We therefore have introduced in the previous model (Eq. 6) the growth rate, expressed here in terms of rings per centimeter. We define β_{11} = a*RPC + b such that

$$\text{MOE(AGE)} = \beta_{11}*[1 - \exp(\beta_{21}*\text{AGE})]^{\beta_3}. \quad (7)$$

In this model the asymptote β_{11} varies now as a linear function of RPC. It is necessary to

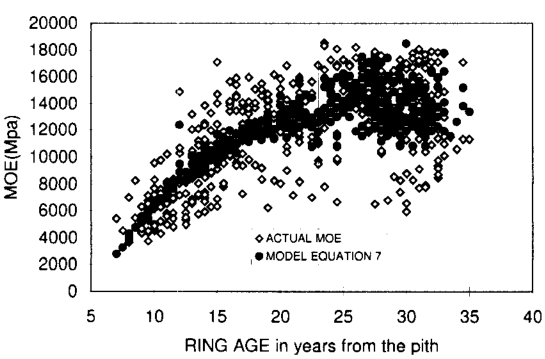

FIG. 5. Actual and simulated MOE vs. AGE. Model defined by Eq. (7).

estimate four parameters with this formulation. The fitting of the model to the data, by use of the same statistical procedure as before, gives the estimates of the parameters as shown in Table 6. Both a and b parameters determine the value of the asymptote β_{11} according to the RPC value of the samples. The parameter β_2, as before, defines the ring age value for which the MOE growth rate versus age is maximum, and β_3, as always, is the parameter that determines the shape of the curve. In comparison with the estimates performed with Eq. (6), the value for β_3 has changed because of the new formulation of the asymptote. The value for β_2 is 7.55 years, not greatly changed from the value estimated previously, i.e., 8.78 years. The mean corrected R^2 calculated for this model is now 0.63, almost as high as the R^2 reached for the linear multivariate model with density, age and rings per centimeter as variables. The modified nonlinear model can now be written as follows

$$\text{MOE(AGE)} = (1467*\text{RPC} + 7541) *[1 - \exp(-0.330*\text{AGE})]^{12.1}. \quad (8)$$

The simulated values are plotted with the measured values in Fig. 5. The improvement gained in the model behavior appears clearly by comparing the simulated values in Fig. 4 to the simulated values in Fig. 5.