A NOTE ON THE HANKINSON FORMULA

K. Y. Kim

Wood Products Research and Development
Weyerhaeuser Company, Tacoma, WA 98477

(Received June 1985)

ABSTRACT

Recent tests at the Weyerhaeuser Company Technology Center were aimed at determining the applicability of the Hankinson Formula for predicting the strength of wood loaded in tension at angles to the grain. Small specimens cut at varying grain angles were loaded in tension at a rate of 0.1 inches per minute. A stress versus grain angle plot of the data revealed that the Osgood Formula, a generalization of the Hankinson Formula, provided a better fit and that both could be applied in tension as well as compression loading.

Keywords: Hankinson Formula, Osgood Formula, grain angle, dog-boned.

REVIEW OF THE LITERATURE

The Hankinson Formula, developed by the U.S. Army in 1921 as a means of computing allowable stress in compression of spruce loaded at varying angles to the grain, is of the form:

$$n = \frac{p \times q}{p \sin^2\theta + q \cos^2\theta} \tag{1}$$

where:

n = the allowable stress at angle, θ, to the grain
p = the allowable stress parallel to the grain
q = the allowable stress perpendicular to the grain
θ = the angle between the direction of the load and the direction of the grain

Another less well-known formula, relating grain angle to the allowable stress at that angle, the Osgood Formula, a generalization of the Hankinson Formula, was promulgated in 1928. It had as a basis data from compression tests by Ayres at the University of Mississippi (1920), Martel at the California Institute of Technology (1920), and at Cornell University where Mr. Osgood was Assistant Professor of Structural Engineering. It was of the form:

$$n = \frac{p \times q}{q + (p - q)(\sin^2\theta + a \cos^2\theta)\sin^2\theta} \tag{2}$$

where: a = a coefficient that is species-dependent and for which Osgood gives the value 0.35 for southern yellow pine, and n, p, q, and θ are defined as in Hankinson. Note that when Osgood's coefficient, a, equals 1, the Osgood Formula collapses to the Hankinson Formula.

These studies were done on wood loaded in compression to failure. Often the point of failure was difficult to ascertain; hence a study by Kojis and Postweiler (1953) at the University of Wisconsin used the proportional limit stress instead of an "ultimate stress."

Wood and Fiber Science, 18(2), 1986, pp. 345–348

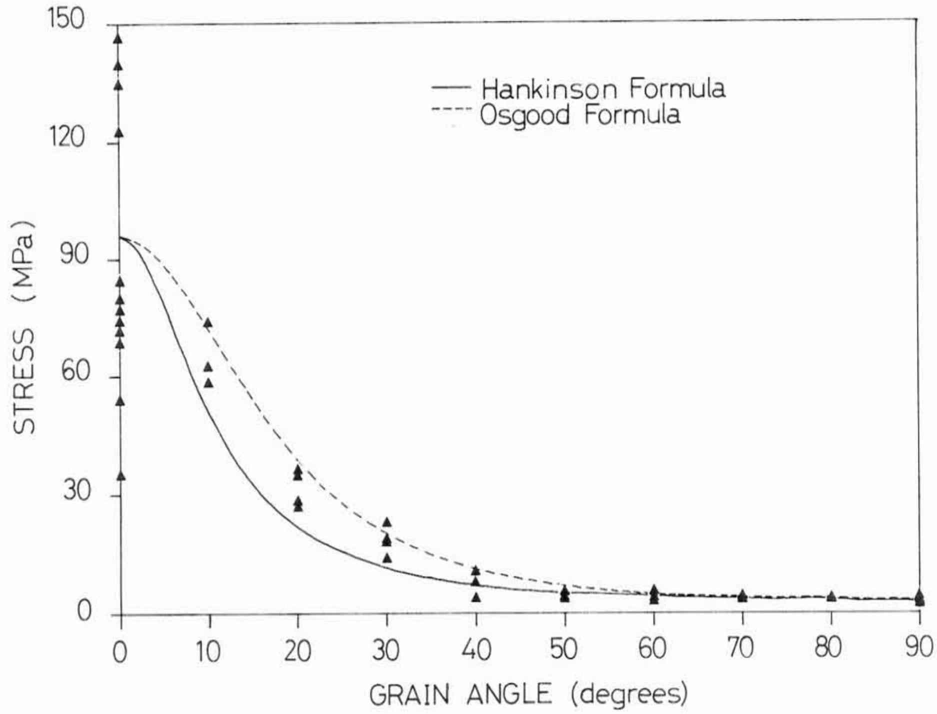

FIG. 1. Stress variance (Hankinson vs. Osgood), p = 95.83 MPa, q = 3.18 MPa, a = 0.35 MPa.

There is little indication in the literature that research of similar nature has been undertaken using tension loading.

PROCEDURE

Three southern yellow pineboards whose centers appeared to be relatively straight-grained were chosen. From one board, four rectangular-shaped, dog-boned specimens at each grain angle, varying in 10 degree increments between 0 and 30 degrees, were cut. The second board produced four specimens at each of the following angles: 40, 50, and 60 degrees. The third board was cut into four specimens at each 70, 80, and 90 degrees. Additional 0 degree angle specimens were cut from the third board. The final count was 12 0-degree specimens, 4 specimens at each 10 degree interval between 10 and 80 degrees, and 12 90-degree specimens. The specimens were roughly 1⅛ inch by ¼ inch through the cross section and varied in length. The smaller angles (0–30) were 8 inches in length, the larger angles (50–90) approximately 5½ inches long, and the 40 degree specimens about 7 inches in length. These specimens were carefully measured with a digital micrometer, and their dimensions were recorded. The specimens were weighed three times: first, after conditioning for two weeks; second, after dog-boning; and third, after drying. With these measurements, it was possible to determine the specific gravity and moisture content of each specimen. The specimens were conditioned for two weeks in a room with a humidity of 50% and a temperature of 70 F. After testing each specimen in tension until failure occurred, the specimens were dried at 221 F for 40 hours in order to determine the moisture content. Using ASTM test method D1037, the specimens were loaded in an Instron Universal Testing

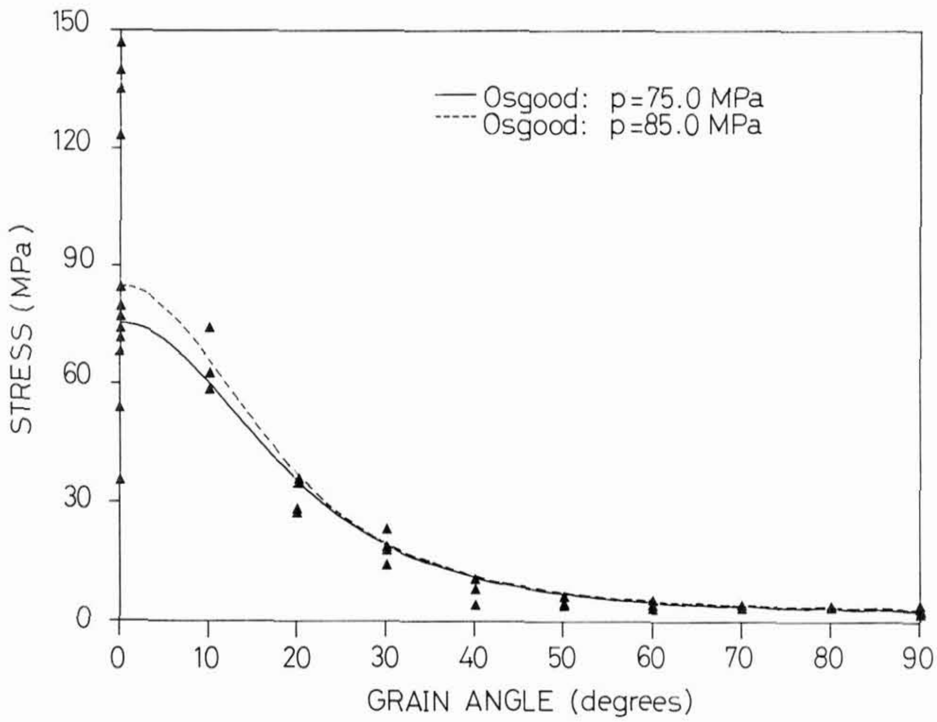

Fig. 2. Stress variance at angles to grain (Osgood), q = 3.18 MPa, a = 0.35 MPa.

Instrument with a load cell whose maximum capacity was 10,000 pounds. The specimens were pulled in tension until breakage occurred. The maximum load was recorded and the ultimate stress computed.

RESULTS

Figure 1 shows the fits of the Hankinson and Osgood Formulas to the actual data points from this experiment. The points tend to fall between the curves of the formulas when p and q are picked equal to the average of the experimental stresses parallel and perpendicular to the grain. This would seem to indicate that one formula overestimates and the other underestimates the actual stresses. Perhaps the formulas should be used as bounds to approximate the stresses.

The moisture content did not vary significantly from specimen to specimen. Hence, the moisture content could be considered a controlled variable. The specific gravity varied considerably among the 0-degree specimens. The spread of points in tensile stress parallel to the grain is due mainly to this variance in the specific gravities. The spread of data points would seem to indicate that specific gravity is a variable that should be more rigidly controlled in future tests.

Figure 2 shows Osgood curves with statistically calculated values of p that gave the best fit.

CONCLUSIONS

The two formulas are applicable both to compression loading and tension loading. Based on severely limited tests, the Osgood Formula appears to provide a better fit for tension loading than does Hankinson. This experiment was intended as a first glance at the appropriateness of the Osgood and Hankinson Formulas to loading in tension, as opposed to exclusively compressive uses. For more conclusive results, further study is recommended.

ACKNOWLEDGMENT

The assistance and guidance provided by Will Fohrell and Steve Means in this work is gratefully acknowledged.

REFERENCES

AYRES, Q. C. 1920. Crushing strength of southern pine at angles to grain. Eng. News-Record 85(14): 653–654.

KOJIS, D. D., AND R. H. POSTWEILER. 1953. Allowable loads for common bolts at various angles to the grain for southern yellow pine. For. Prod. J. 3(3):21–26.

MARTEL, R. R. 1920. Tests of bearing strength of redwood agree with Howe's Formula. Eng. News-Record 85(20):959.

OSGOOD, W. R. 1928. Compressive stress on wood surfaces inclined to the grain. Eng. News-Record 100(6):243–244.

U.S. ARMY, ENGINEERING DIVISION. 1921. Investigation of crushing strength of spruce at varying angles of grain. Air Service Information Circular 3(259):3–15.