LASER INCISING TO INCREASE DRYING RATE OF WOOD[1]

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ABSTRACT

The objective of this study was to determine the extent to which laser incising can decrease drying time of wood. Specifically, the effect of hole spacing and diameter were investigated. Hard maple heartwood was incised with holes of 0.012-, 0.018-, and 0.027-inch diameter with edge-to-edge hole spacings of 0.03, 0.06, 0.09, 0.12, 0.15, and 0.20 inch at each diameter. The time required for 90% of total drying to occur was decreased by as much as 70% for close hole spacings. As spacing increased, the reduction in drying time decreased until at a spacing of 0.20 inch it was only 5% or less. The effect of hole diameter was less well defined. The decreases in drying time for a hole diameter of 0.012 inch were less than for the two larger hole diameters. Speculation is that at this small diameter water vapor cannot escape from the holes at a rate fast enough to keep pace with the rate of delivery of water from the wood to the internal surfaces of the holes. The relationship between the time required for 90% of drying to occur and edge-to-edge hole spacing was modeled using a combination of moisture diffusion analysis and an empirical correction to the diffusion analysis that takes into account the effect of drying from the normal center-to-surface movement in addition to the movement to hole surfaces.

Keywords: Incising, drying, laser.

INTRODUCTION

Drying has always been somewhat of a bottleneck in lumber processing. While it might only take minutes to reduce a log to lumber, it might take days or weeks to dry the lumber. If drying time could ever approach sawing time, rapid new systems of log to finished product processing might be possible. In order to accomplish this goal, two interdependent limitations to such rapid drying must be overcome. A significant increase is necessary in the speed with which water can move through and out of wood, and the stresses that cause drying defects must be controlled. Various pretreatments have been devised in attempts to increase drying rate. The objective of this study is to explore the effectiveness of one particular pretreatment, laser incising, to open up the structure of wood and make it more permeable and thus faster drying. More specifically, the objective is to determine the effect of hole diameter and spacing on the expected increase in drying rate of incised wood.

Previous research on increasing drying rate

Several different pretreatments have been applied to wood to try to increase drying rate. Erickson et al. (1966) showed that by prefreezing green redwood, madrone, tanoak, and black walnut at temperatures ranging from zero to −75 C before drying, the subsequent drying time was reduced significantly for some species and less so for others. Most of the increase in drying rate occurred early

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in drying when the mass flow of free water, rather than the diffusion of water vapor, was the dominant mechanism in drying. Further work by Cooper et al. (1970) and Chen and Cooper (1974) confirmed that prefreezing was somehow increasing the permeability of the wood to water during drying. No conclusive evidence was apparent to explain the mechanism of the increase, but since prefreezing also reduced the occurrence of collapse, the authors speculated that prefreezing was somehow lowering the hydrostatic tension of water in the wood cells.

Cech (1971) developed a precompression process whereby wood is momentarily compressed from 2.5 to 20% of its thickness before drying. The result is creation of many microscopic splits in the impermeable pit membranes of hardwoods and an increased drying rate. The increased permeability reduces susceptibility to drying defects and allows acceleration of drying.

Presteaming green lumber before drying has also been shown to increase permeability and drying rates. Ellwood and Ecklund (1961), Comstock (1965), and Benvenuti (1963), all found increases in longitudinal permeability of wood that had been presteamed at 212 F for several hours under saturation conditions. Mackay (1971) showed that presteaming increased the rate of moisture diffusion in wood. Simpson (1975) found that presteaming increased the drying rates of northern red oak, cherrybark oak, sweetgum heartwood, and white fir wetwood. None of these authors determined the mechanism responsible for the observed increases.

Several authors have explored the concept of mechanically drilling holes in wood in order to create new surfaces, especially end grain surfaces, from which water can evaporate. Drilling reduces the distance water must travel within the wood structure before it reaches an evaporating surface, and the result is an increase in drying rate. Chudnoff (1972) proposed the idea of drilling patterns of holes in wood primarily as a way to equalize the wide ranging specific gravity of mixtures of tropical species. He hypothesized that this equalization would narrow the range of processing and mechanical properties to such an extent that these mixtures of species could be processed and utilized as if they were only one species. His experimental work was limited to the effects of holes on mechanical properties, but he recognized that his concept would reduce both drying and preservative treating times. Erickson and Demarel (1972) drilled patterns of ¼- or ⅜-inch-diameter holes on 4- or 6-inch centers perpendicular to the grain and parallel to the wide face of aspen lumber 1 or 2 inches thick. Drying time reductions ranged from one-third to one-half.

THEORY

Analysis of hole patterns and effect on drying rate

The rationale for drilling holes to increase drying rate is that moisture will only have to move to the nearest hole to escape, rather than across the entire half-thickness of a board. Closely spaced holes could reduce flow path length by a factor of 100 or more in common lumber sizes. Thus edge-to-edge hole spacing is the most important parameter in these considerations.

The ultimate practicality of the concept of drilling holes in wood to increase drying rate (and also ease preservative treating) is unknown; and if practical, the exact nature of applicability is also unknown. Would it be preferable to drill small numbers of relatively large and noticeable holes, or larger numbers of smaller, less conspicuous holes? Will partial penetration of thickness, as practiced in commercial incising for preservative treatment, be as effective as full penetration? Will appearance and noticeability be critical considerations? On what types of products might predrilling be applicable? Because of these and other unknowns this initial study focuses on exploring fundamentals and establishing the basic effect of predrilling holes through the full thickness of specimens on drying rate.

Geometry of hole patterns

To conduct a systematic study of the effect of holes on drying rate, it is useful to relate number, size, and spacing mathematically. Specific gravity is also an important variable that depends on pattern and size, and it too should be included in the mathematical relationships.

The mathematics are developed in Appendix A. The variables in the analysis (Fig. 1) are number of holes (n^2), hole diameter (d), center-to-center hole spacing (c), edge-to-edge hole spacing (e), and the fraction of specific gravity retained after drilling (a). The relationships of interest developed in Appendix A are:

$$e = c - d$$

and

$$c = \left(\frac{\pi r^2}{1 - a}\right)^{1/2} = \left(\frac{1}{n^2}\right)^{1/2} \tag{1}$$

and are shown graphically in Fig. 2.

The edge-to-edge spacing is important for two reasons: (1) if it becomes too great, holes overlap and structural integrity is lost; (2) if holes are effective in increasing moisture flow, the path length of movement will be approximately e/2 instead of x/2, and if e/2 is smaller than x/2, the distance moisture has to flow to reach an evaporating surface and escape from the wood is reduced.

Moisture movement in drilled wood

Moisture movement is a complex combination of several mechanisms occurring at the same time, i.e., diffusion in response to moisture content gradients, mass flow of moisture due to total pressure gradients, and heat transfer into the wood due to temperature gradients. Also, the geometry of flow patterns of moisture from wood to drilled holes is not easily detailed analytically, particularly when coupled with moisture movement that occurs irrespective of holes. Despite these complexities it is possible to make an analysis that can at least offer a first approximation to the effect of hole drilling on drying rate and provide a model to correlate the hole pattern to drying rate.

The mathematics of diffusion makes a quantitative approximation possible, even though the approximation suffers from the qualifications mentioned above. Mathematically we will consider moisture flowing radially from the walls of a hollow cylinder (Fig. 1) into the hole at the center of the cylinder. To keep the mathematics manageable, the areas between the outside boundaries of the cylinder walls will be ignored, and all drying will be assumed to occur via holes. The general mathematics of radial diffusion in a hollow cylinder have been presented by Crank (1975). By writing the diffusion equation in cylindrical coordinates and using a finite difference technique to solve the equation, it is possible to develop a theoretical curve relating moisture movement out of the cylinder wall into a

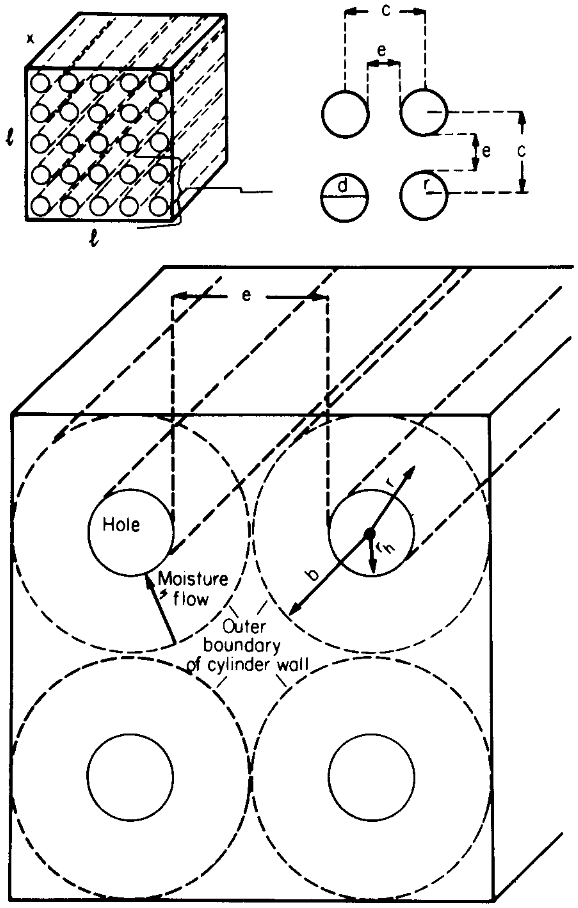

FIG. 1. Model of geometry of hole patterns and how they affect drying.

hole to time. In terms of dimensionless variables, and with a constant diffusion coefficient, the diffusion equation in cylindrical coordinates is:

$$\frac{\partial E}{\partial T} = \frac{\partial^2 E}{\partial R^2} + \frac{1}{R}\frac{\partial E}{\partial R} \tag{2}$$

where

E = fractional drying; i.e.,

$$E = \frac{m_i - m}{m_i - m_f} \tag{3}$$

and

m = moisture content
m_i = initial moisture content
m_f = final moisture content

where

$$T = \frac{\tau D}{(b - r_h)^2} \tag{4}$$

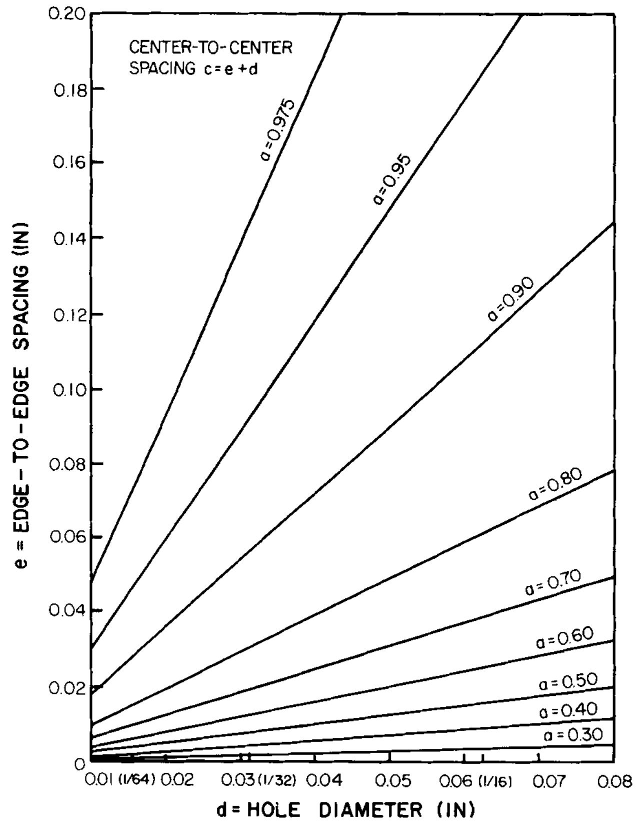

FIG. 2. Edge-to-edge (e) and center-to-center (c) hole spacing as functions of hole diameter (d) and specific gravity retention (a).

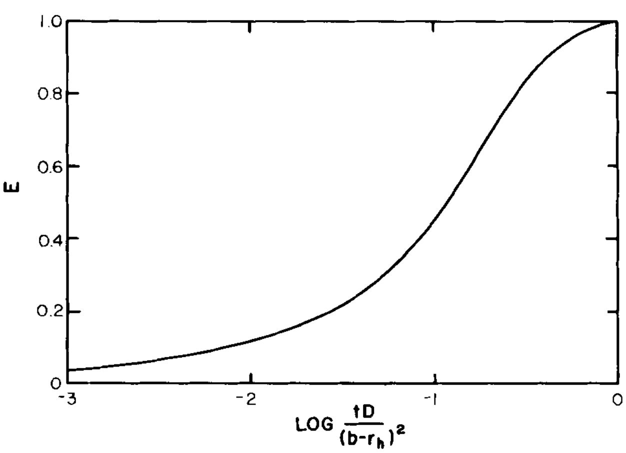

FIG. 3. Theoretical rate of diffusion radially inward in a hollow cylinder.

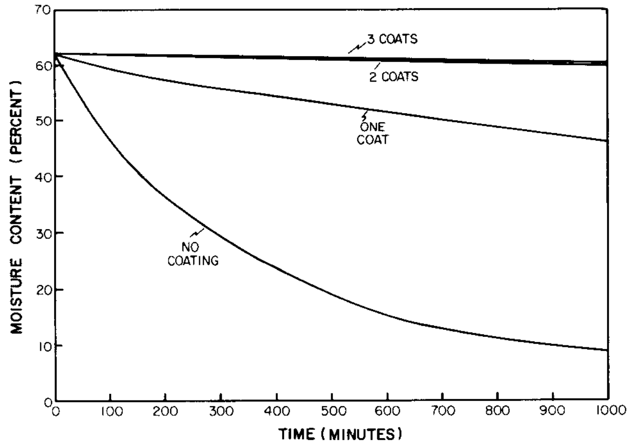

FIG. 4. The effect of aluminum paint coatings in retarding moisture loss from wood.

and

τ = time (sec)
D = diffusion coefficient (cm^2/sec)
b = radius of cylinder (cm)
r_h = radius of hole (cm)

where

$$R = \frac{(b - r)}{(b - r_h)} \tag{5}$$

and

r = variable cylinder radius (cm).

Details of the solution are given in Appendix B.

After determining the distribution of E as a function of position and time by the method outlined in Appendix B, the results are numerically integrated from

TABLE 1. *Experimental hole pattern.*

Hole diameter d		Edge-to-edge spacing e (in.)					
		0.03	0.06	0.9	0.12	0.15	0.20
in.		*in.*					
0.012	c	0.042	0.072	0.102	0.132	0.162	0.212
	n^2	567	193	96	57	38	22
	n	23.8	13.9	9.8	7.6	6.2	4.7
	a	0.935	0.978	0.989	0.994	0.996	0.997
0.018	c	0.048	0.078	0.108	0.138	0.168	0.218
	n^2	434	164	86	53	35	21
	n	20.8	12.8	9.3	7.2	6.0	4.6
	a	0.890	0.958	0.978	0.987	0.991	0.995
0.027	c	0.057	0.087	0.117	0.147	0.177	0.277
	n^2	308	132	73	46	32	19
	n	17.5	11.5	8.5	6.8	5.6	4.4
	a	0.824	0.924	0.958	0.974	0.982	0.989

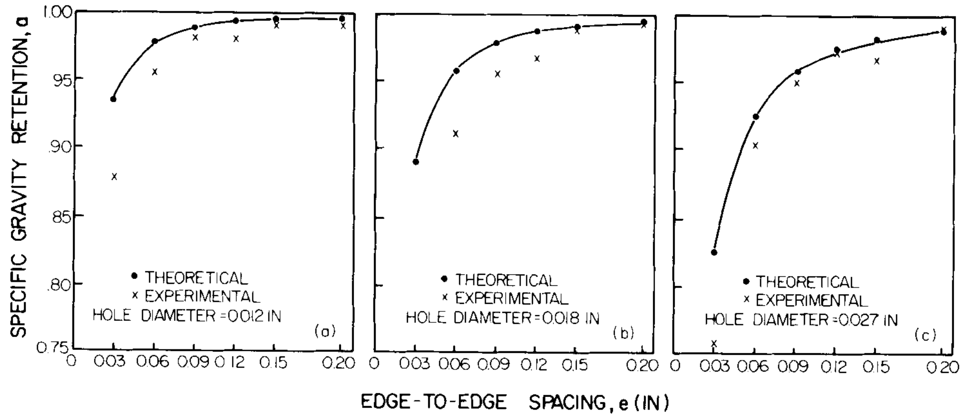

FIG. 5. Comparison of theoretical and actual specific gravity of incised hard maple: a) 0.012-inch hole diameter; b) 0.018-inch hole diameter; c) 0.027-inch hole diameter.

R = 0 to 1 to determine the average value of E as a function of T. The theoretical curve of E versus T is shown in Fig. 3.

The relationship shown in Fig. 3 allows estimates of time from estimates of the diffusion coefficient and hole patterns. From Fig. 1,

$$b = \frac{e}{2} + r_h \tag{6}$$

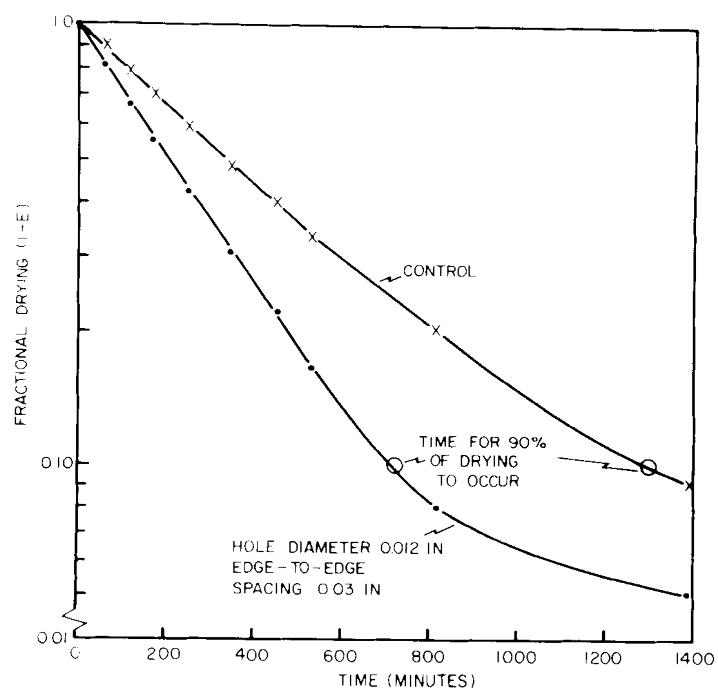

FIG. 6. Typical drying curves of incised hard maple compared to matched, unincised control.

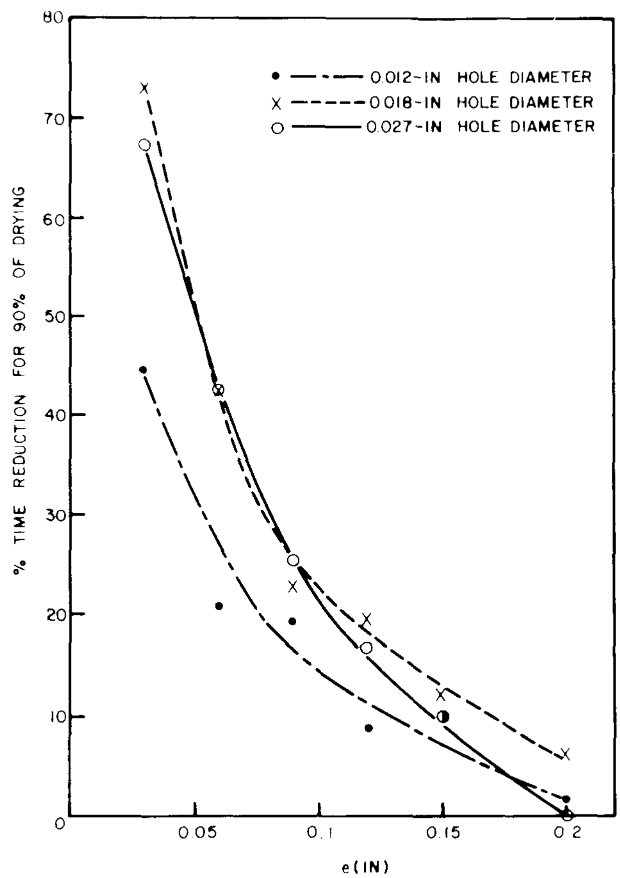

FIG. 7. Relationship between time required for 90% of drying to occur, hole diameter, and hole spacing.

Substituting Eq. (6) into Eq. (4), we can then solve Eq. (4) for the time required to reach a certain value of E:

$$\tau = \frac{Te^2}{4D} \tag{7}$$

Equation (7) predicts that drying time depends only on diffusion coefficient and edge-to-edge hole spacing. Dependence on the square of edge-to-edge hole spacing is expected from a diffusion analysis and emphasizes the importance of hole spacing to drying rate.

Method of testing the moisture movement model

Equation (7) can be tested with experimental data to determine if the concept of hole effect on moisture movement is valid. If we plot the time required for E to reach a certain value, for example 0.9 (when 90% of total drying has occurred), versus edge-to-edge hole spacing e, we can determine the exponent of e and compare it to the theoretical value of 2. Specifically, we will test the equation

TABLE 2. *Time required to 90% of total drying to occur for incised and unincised hard maple.*

Hole diameter	Edge-to-edge spacing e (in.) 0.03		0.06		0.09		0.12		0.15		0.20	
	I[1]	C[2]	I	C	I	C	I	C	I	C	I	C
in.	*min.*											
0.012	743	1,358	1,059	1,334	1,065	1,325	1,123	1,233	1,104	1,224	1,196	1,220
0.018	351	1,339	732	1,299	1,005	1,313	1,086	1,355	1,185	1,355	1,267	1,355
0.027	444	1,371	768	1,339	958	1,306	1,084	1,299	1,101	1,221	1,075	1,062

[1] I = incised.
[2] C = unincised control.

TABLE 3. *Analysis of variance results for 6 by 3 factorial design: 6 levels of edge-to-edge hole spacing and 3 levels of hole diameter.*

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Treatments				
A (spacing)	5	27,539.94	5,570.99	248.7[1]
B (diameter)	2	1,885.21	942.60	42.6[1]
AB	10	1,728.11	172.81	7.8[1]
Error	54	1,195.45	22.14	

[1] Indicates significance at greater than the 99.5% level.

$$\tau = \frac{T}{4D} e^{z} \tag{8}$$

or

$$\ln \tau = \ln \frac{T}{4D} + z \ln e \tag{9}$$

A plot of $\ln \tau$ versus $\ln e$ should be linear with

$$\text{intercept} = \ln \frac{T}{4D} \tag{10}$$

$$\text{slope} = z. \tag{11}$$

The intercept and slope can be determined by linear regression. T can be calculated so that D can be determined from Eq. (10).

EXPERIMENTAL

The study was conducted on hard maple (*Acer saccharum* Marsh.) heartwood, which is quite impermeable and should benefit by incising. Because the experiment includes a variety of hole patterns with replication, a large number of holes had

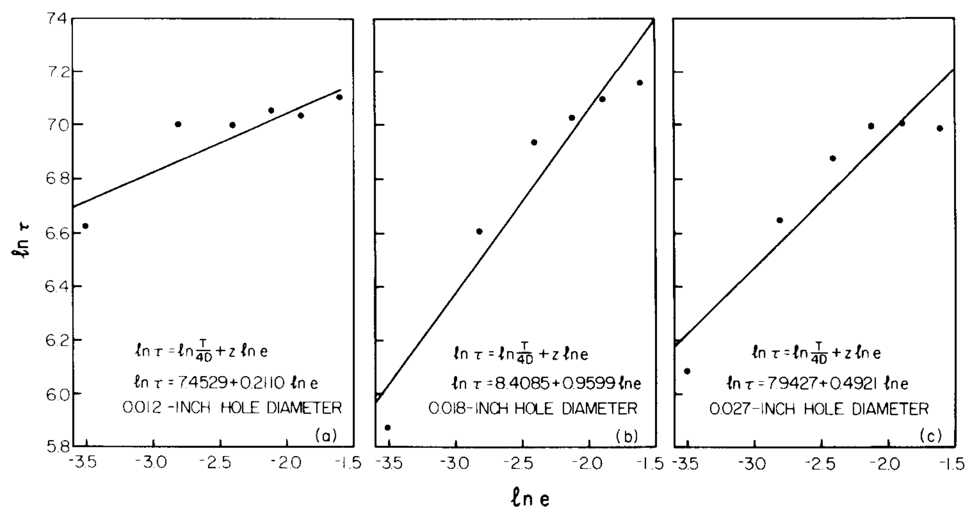

FIG. 8. Results of attempt to fit the experimental relationship of logarithm time (t) required for 90% of drying to occur and logarithm of edge-to-edge spacing (e) to the linear equation ln t = ln T/4D + z ln e: a) 0.012-inch hole diameter; b) 0.018-inch hole diameter; c) 0.027-inch hole diameter.

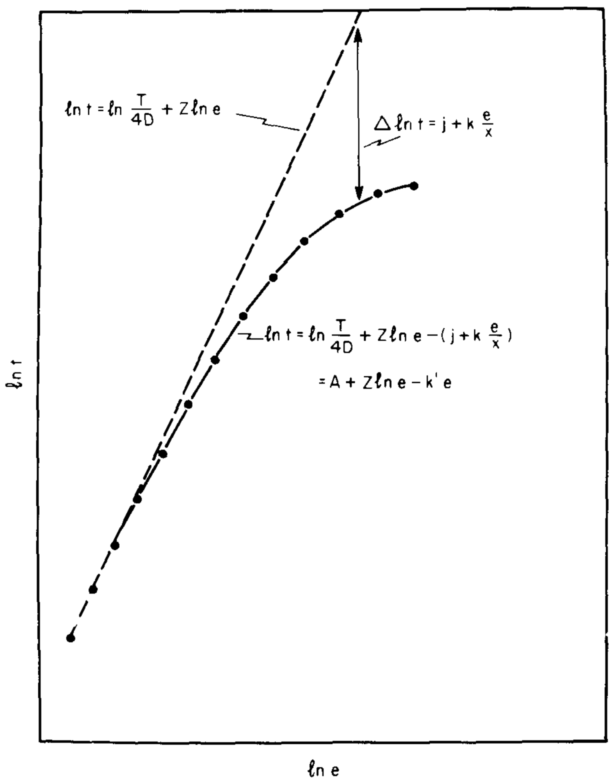

FIG. 9. Schematic diagram illustrating method of empirically modifying the model to predict time required for 90% of drying to occur as a function of edge-to-edge hole spacing.

to be drilled. Thus fairly small specimens were used to keep the amount of laser drilling within practical limits. They were 1 inch square ($\ell \times \ell$) by ½ inch thick (x) with the holes drilled in the x direction. The edges were covered with two coats of aluminum paint to force most water to leave through the holes and the 1-inch-square face. Figure 4 shows the effectiveness of aluminum paint in retarding moisture loss. Drilling was done with an 800-watt laser. Hole spacing was computer controlled so that the laser head moved automatically by the correct amount between each hole. Drying was in a temperature- and humidity-controlled cabinet at approximately 120 F, 60% relative humidity, and 200 feet per minute air velocity.

The experimental hole patterns are listed in Table 1, and are arranged in a 6 by 3 factorial; 6 levels of edge-to-edge spacing e (from 0.03 to 0.20 in.) and 3 levels of hole diameter d (from 0.012 to 0.027 in.). Each of the 18 experimental conditions included four replicates each of drilled and end-matched controls. Weights were recorded periodically; often early in drying, and then at increasing time intervals. Drying was continued for about 6 weeks to ensure that the specimens were close to equilibrium, even though 90% of the total drying took place within the first 24 hours. After reaching equilibrium, the specimens were then oven-dried. The dimensions of each specimen were also measured for specific gravity determinations.

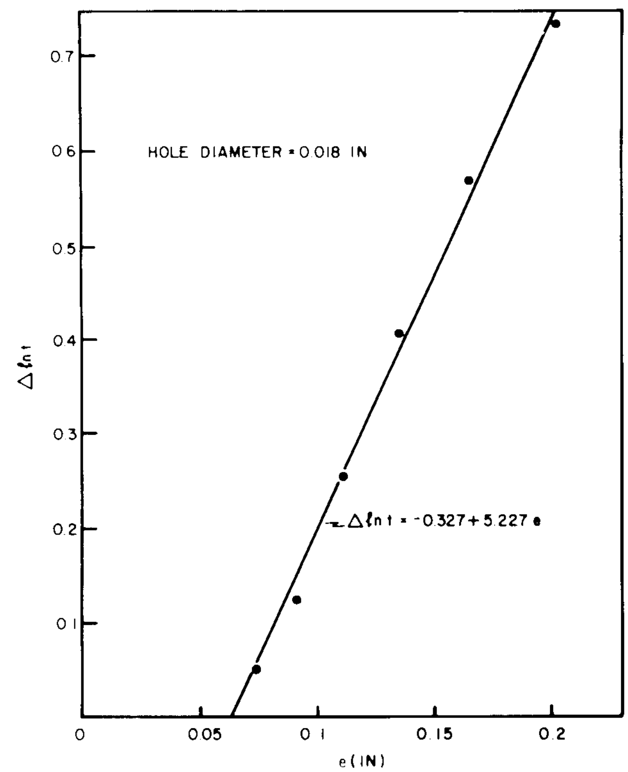

FIG. 10. Example of relationship between the divergence of the logarithm of time required for 90% of drying to occur from the linear model of Eq. (9).

RESULTS

Drying time reduction

One indication of the accuracy of the hole patterns is to compare theoretical specific gravity retentions, a, calculated from Eq. (30) using the values of hole spacing and diameter in Table 1, to actual specific gravity measurements taken on the specimens. This comparison is shown in Fig. 5. In most cases the actual specific gravity retention is less than the theoretical value. This would suggest that either the holes were a little oversize or the edge-to-edge spacing was less than expected. Since some evidence of charring was noted, a more likely explanation is that some of the wood adjacent to the holes was charred during incising and thus reduced in specific gravity.

Typical drying rate curves for an incised and control pair of specimens is shown in Fig. 6, where hole diameter is 0.012 inch and edge-to-edge spacing is 0.03 inch. The times for 90% of drying to occur are shown for all experimental conditions in Table 2, and as expected the incised specimens consistently dried faster than the unincised controls. The percent time reduction in drying due to incising is shown graphically for all experimental conditions in Fig. 7. Analysis of variance (Table 3) shows that both edge-to-edge hole spacing and, contrary to the simple model developed, hole diameter affect drying time. From Fig. 7 it appears that the decrease in drying time at hole diameter of 0.012 inch is not as great as at

TABLE 4. *Constants of the equation relating time, t, required for 90% of drying to occur and edge-to-edge hole spacing, e: ln t = A + z ln e − k′e, where*

Hole diameter	A	z	k′
in.			
0.012	8.420	0.474	2.890
0.018	10.642	1.289	7.218
0.027	10.067	1.079	6.697

the two larger hole diameters, 0.018 and 0.027 inch. This observation suggests that with small diameter holes the water vapor may not be removed from the holes as fast, relative to the rate of delivery of moisture from the wood to the internal surface of the holes, as with larger diameter holes.

Model evaluation

The relationship between the logarithm of time for 90% of drying to occur and the logarithm of edge-to-edge spacing for each hole diameter are shown in Fig. 8. If the simple model resulting in Eq. (9) was valid the plots would be linear. For the sake of completing the proposed method of evaluating the model, the results of the linear regression are shown in Fig. 8. However, the plotted points are obviously not arranged linearly, and fall off from linear as ln e increases. The oversimplifications of the model were stated earlier, and the shape of the relationships suggests that the most serious oversimplification is the assumption that all drying occurs via the holes. When edge-to-edge hole spacing is small, this may be a good assumption; but as edge-to-edge spacing increases, the time required for moisture to reach a hole increases and the normal moisture movement from the center of the specimen to the surface becomes an increasingly important second mechanism of drying. Thus, as the ratio of edge-to-edge hole spacing, e, to specimen thickness, x, increases, the divergence from the linear relationship of Eq. (9) increases. We can use this reasoning to make an empirical modification to the model that significantly improves the relationship between the logarithm of time for 90% of drying to occur and the logarithm of edge-to-edge hole spacing.

The modified model is shown schematically in Fig. 9. We assume that when e is small compared to x, i.e., the initial part of the ln t versus ln e relationship, Eq. (9) holds and the relationship is linear. At some point as ln e increases, the relationship begins to diverge from Eq. (9) in a way we will assume is a linear function of the ratio e/x. In fact, if we graphically estimate this divergence of ln t from Eq. (29) and plot it versus e, the result is close to linear (Fig. 10), and can be well represented by the function,

$$\Delta \ln t = j + ke/x \tag{12}$$

where k and j are constants that can be estimated by linear regression. In the modified model we simply subtract the divergence represented by Eq. (12) from Eq. (9):

$$\ln t = \ln T/4D + z \ln e - (j + ke/x) \tag{13}$$

or

$$\ln t = A + z \ln e - ke/x \tag{14}$$

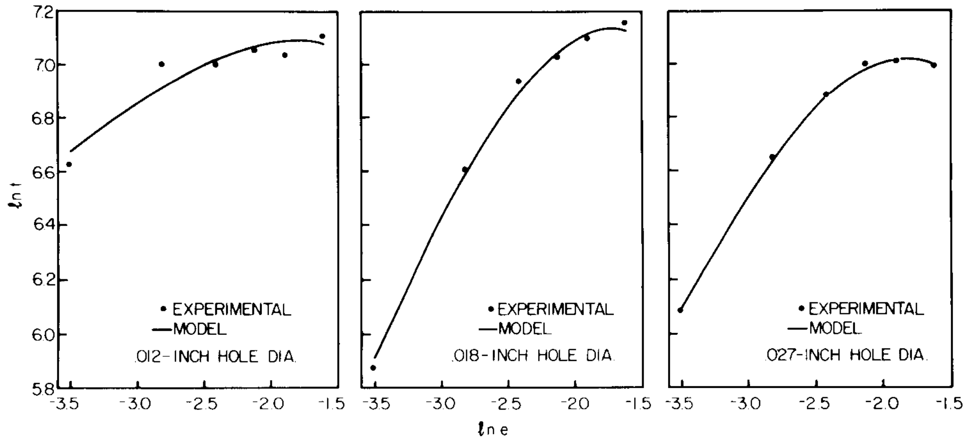

FIG. 11. Comparison of experimental and model relationships between the logarithm of time (t) required for 90% of drying to occur and logarithm of edge-to-edge hole spacing (e); ln t = A + z ln e − k′e: a) 0.012-inch hole diameter; b) 0.018-inch hole diameter; c) 0.027-inch hole diameter.

where

$$A = \ln T/4D - j \quad (15)$$

In this study specimen thickness was constant, so Eq. (14) can be written

$$\ln t = A + z \ln e - k'e \quad (16)$$

where

$$k' = k/x \quad (17)$$

The constants of Eq. (16) can be estimated by the graphical and linear regression techniques outlined above and illustrated in Fig. 9, or estimated directly from Eq. (16) by nonlinear regression. In this study we used the graphical/linear regression technique to obtain initial estimates of A, z, and k′ that could be used to start the iterative nonlinear regression analysis. The constants are listed in Table 4, and plots of ln t versus ln e are shown in Fig. 11. The agreement between model interpolation and experimental results is good, and Eq. (16) is a good representation of the experimental data of this study.

CONCLUSIONS

The results of this study show that the drying time of hard maple heartwood can be reduced by up to 70% by incising with various patterns of holes. Edge-to-edge hole spacing is the most important variable affecting the decrease in drying time, and as spacing decreases the reduction in drying time increases. Hole diameter has a smaller and less defined effect. The smallest hole diameter of this study, 0.012 inch, was not as effective as the two larger hole diameters. Speculation is that because the holes were so small, water vapor was not removed from them fast enough to keep pace with the rate of moisture movement from the wood to the internal surface of the holes. The relationship between drying time of incised wood and edge-to-edge hole spacing can be modeled semi-empirically, resulting in an expression that well represents the experimental data of this study.

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APPENDIX A

Assume we have a piece of wood of unit area ℓ^2 and thickness x. The specific gravity of this piece is:

$$G_1 = \frac{W_{01}}{V_m} \tag{18}$$

where

G_1 = original specific gravity

W_{01} = original oven-dry weight

V_m = original volume at moisture content m.

A pattern of n^2 holes of radius r (and diameter d) is now drilled, with center-to-center spacing c and edge-to-edge spacing e (Fig. 1). The new specific gravity at moisture content m is:

$$G_2 = \frac{W_{02}}{V_m} \tag{19}$$

where

G_2 = specific gravity after drilling

W_{02} = oven-dry weight after drilling.

Since the gross volume is unchanged by drilling, Eq. (18) and (19) can be equated through V_m:

$$\frac{W_{01}}{G_1} = \frac{W_{02}}{G_2} \tag{20}$$

Drilling reduces the specific gravity to a fraction, a, of the original specific gravity, i.e.,

$$a = \frac{G_2}{G_1} \tag{21}$$

Substituting Eq. (21) into Eq. (20) and rearranging,

$$\frac{W_{01} - W_{02}}{W_{02}} = \frac{1}{a} - 1 \tag{22}$$

Substituting Eq. (19) into Eq. (22),

$$\frac{W_{01} - W_{02}}{G_2 V_m} = \frac{1}{a} - 1 \tag{23}$$

and then Eq. (21) into Eq. (23),

$$V_{mr} = \frac{W_{01} - W_{02}}{G_1} = V_m(1 - a) \tag{24}$$

where V_{mr} is the volume of wood removed by drilling a pattern of n^2 holes that reduces specific gravity by $1 - a$.

For practical use we should now express the volume of wood removed of Eq. (24) in terms of the volume of holes. The volume of each hole, drilled through the entire thickness, is $\pi r^2 x$, so

$$V_{mr} = \pi r^2 x n^2 \tag{25}$$

or, the number of holes required to remove V_{mr} of wood is

$$n^2 = \frac{V_{mr}}{\pi r^2 x} \tag{26}$$

Substituting Eq. (24) into Eq. (26),

$$n^2 = \frac{V_m(1 - a)}{\pi r^2 x} \tag{27}$$

The center-to-center hole spacing c (Fig. 1) must be known to produce any hole pattern. From Fig. 1, we can write side dimension ℓ in terms of the number of holes n and spacing c:

$$\ell = c(n - 1) + \frac{c}{2} + \frac{c}{2} \tag{28}$$

or

$$c = \frac{\ell}{n} \tag{29}$$

Remembering that $V_m = \ell^2 x$, and substituting Eq. (27) into Eq. (29),

$$c = \left(\frac{\pi r^2}{1 - a}\right)^{1/2} \tag{30}$$

and, from Eq. (27) and (30), for unit area

$$c^2 = \frac{1}{n^2} \tag{31}$$

Center-to-center spacing (c) and edge-to-edge spacing ($e = c - d$) are shown as functions of hole diameter d and specific gravity retention (a) in Fig. 2.

The limiting edge-to-edge spacing before overlap occurs can be expressed by realizing that when $e = 0$, $c = 2r$ (Fig. 1). Substituting this relationship into Eq. (30), the edge-to-edge limitation can be expressed as

$$a = \frac{4 - \pi}{4} = 0.215 \tag{32}$$

That is, holes will overlap if center-to-center spacing is equal to or less than hole diameter, or if one attempts to reduce specific gravity as low as 0.215 of its original value.

APPENDIX B

The boundary conditions for solution are:

(1) The initial moisture content is uniform:

$$E(R,T) = E(R,O) = 0, \qquad T = 0, \qquad 0 \le R \le 1$$

(2) The inner surface of the cylinder immediately comes to the new moisture equilibrium:

$$E(R,T) = E(1,T) = 1, \qquad T > 0, \qquad R = 1$$

The Schmidt method (Crank 1975), with modifications to apply to a hollow cylinder, was used to solve Eq. (2). The interval R is divided into i equal increments ΔR, and T is divided into j equal increments ΔT. The terms E_{i-1}, E_i, and E_{i+1} are the concentrations at $(i - 1)\Delta R$, $i\Delta R$, and $(i + 1)\Delta R$ at $T = j\Delta T$. The terms E_i^+ and E_i^- are the values of $R = i\Delta R$ at $T = (j + 1)\Delta T$ and $(j - 1)\Delta T$. Using the first term of Taylor's expansion, the finite difference approximations are

$$\frac{\partial^2 E}{\partial R^2}i = \frac{E_{i+1} - 2E_i + E_{i-1}}{(\Delta R)^2} \tag{33}$$

$$\frac{\partial E}{\partial R}i = \frac{E_{i+1} - E_{i-1}}{2(\Delta R)} \tag{34}$$

$$\frac{\partial E}{\partial T}i = \frac{E_i^+ - E_i}{\Delta T}\,. \tag{35}$$

Substituting these approximations into the diffusion equation

$$E_i^+ = E_i + \frac{\Delta T}{2i(\Delta R)^2}(2i + 1)E_{i+1} - 4iE_i + (2i - 1)E_{i-1} \tag{36}$$

For i = 0,

$$E_i^+ = E_i + 4\frac{\Delta T}{(\Delta R)^2}(E_i - E_o) \tag{37}$$

In this analysis, $\Delta T = 0.001$ and $\Delta R = 0.05$. To start the solution at some small time ($T = 0.001$), the following formal solution is used (Crank 1975):

$$E = 1 - \operatorname{erf}\frac{1 - R}{2T^{1/2}} \tag{38}$$