# ANALYTICAL STUDIES ON THE NONLINEAR BENDING BEHAVIOR OF NAILED LAYERED COMPONENTS: PART I. NAILED LAYERED BEAMS

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ABSTRACT

The objective of this study is to develop a basic theory for the analysis of the nonlinear bending behavior of nailed layered components. For the analytical procedure, a numerical repetitive calculation using an electronic computer is needed. A simple procedure to predict the approximate behavior without the computer is also presented. The validity of the procedures is examined by conducting experiments. Both procedures give excellent agreements with the experimental results.

*Keywords:* Beams, fasteners, deformation, loads, nonlinear analysis.

## INTRODUCTION

The nailed layered system is widely used in wood construction. The method for analytical determination of the nonlinear structural performance, however, is very complicated. In this situation, full-scale laboratory tests are usually conducted to estimate the stiffness and strength. However, this method is costly and time-consuming, and furthermore, it is hardly possible to predict the performance of nailed laminated construction under many loading conditions and environments. Actually, many conditions and factors affect performance properties of materials, nailing schedules, and type and duration of load.

In order to solve this problem, it is useful to develop an analytical method to predict the stiffness and strength of the nailed laminates from load-slip curves of a single nail and the properties of constituent members and to use this analytical method with the empirical method. The load-slip characteristics of nailed joints under various conditions can be easily obtained by an empirical method. Joint tests are far simpler and easier to do than full-size tests of the components. If we input the results of the nailed joint tests under a certain condition to the analytical procedure, we might obtain the structural performances of the components under the same condition.

An analytical procedure for calculating the racking stiffness and strength of the sheathed walls has been already developed (Kamiya 1981). The bending stiffness of the nailed layered components was chosen as the subject of this study (Part I). The nailed stressed-skin components are reported in Part II.

In previous works, analyses of the nailed components used the concept of the constant slip modulus (Hishida and Mano 1959; Amana and Booth 1967a, b; Sawada 1976). Using this concept, the differential equation can be derived and solved. However, as the deflection of the beams, which is calculated by those theories, is proportional to the load, the application is constrained to the range in which the nail slip is small.

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The viscoelastic curve of the nail becomes fairly linear after several one-directional cycles. Therefore, those analyses indicate a range in which there is substantial nail slip. However, as the nails are subjected to two-directional load in the majority of cases, the nonlinear analysis seems to have rather a wider application.

If a stepwise load-slip modulus relationship, secant modulus, or tangent modulus is taken, the nonlinear load-deflection curve of the component can be obtained. Goodman (1969) has developed such a nonlinear analysis. In this study, the nail force used in each step was taken to be the average value over the left end of the beam.

The nail force varies not only by the level of the external load but also along the length of the beam. Thus, to analyze more accurately, the secant modulus or the tangent modulus should be varied according to the nail position. Tremblay et al. (1976) have applied FEM (Finite Element Method) to solve this problem. In their study, the energy method was used, and the deflection and axial displacements of the layered beam were approximated with the finite-element form of the Rayleigh-Ritz procedure.

This calculation, however, needs a complicated computer program and a powerful computer. In the study presented here the nonlinear analysis used is much simpler to find the relationship between the forces and slips of adjacent nails. The calculation does not need the FEM or the Rayleigh-Ritz procedure. The memory capacity required was only about five kilobytes.

Itani and Hiremath (1980) and Itani et al. (1981) have presented a theoretical method of analysis for multilayered diaphragms. The multilayered diaphragm tested in their works was similar to the nailed multilayered beam because the layers were glued with an elastomeric adhesive, and the interlayer slip occurred. The analysis for the multilayered beam is more complex because the matrix of $n - 1$ order (n is the number of layers) should be solved.

The method presented here can be applied to two- or three-layered beams but not to multilayered beams. The objective of this paper, however, is to develop a simple basic theory for the nailed stressed-skin components that are composed of two or three layers. Therefore, the application described addresses the problem sufficiently.

## THEORY

### *Two-layered beam*

Denote the total direct force at any section perpendicular to the axis of the beam as F (see Fig. 1). F is caused by the resistance of nails and acts at the interface of the beam. The equilibrium of the force is

$$M = M_1 + M_2 + FZ \tag{1}$$

where

$$Z = \frac{h_1}{2} + \frac{h_2}{2}$$

$h_1$ and $h_2$ are the depth of the layers, M is the bending moment due to the applied

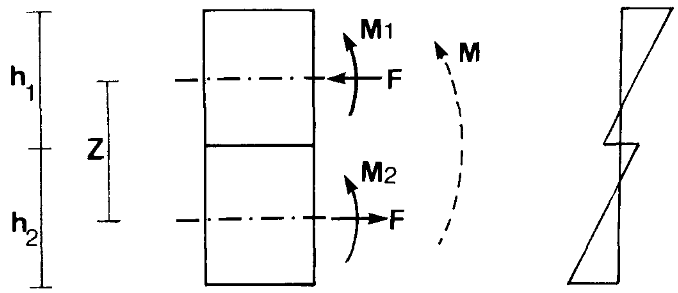


FIG. 1. Two-layer beam forces and strains.

load, and $M_1$ and $M_2$ are the moments of resistance of the layers about their centroidal axes. Subscripts 1 and 2 show, respectively, layer 1 and layer 2.

For equal curvature of layers

$$\frac{M_1}{E_1I_1} = \frac{M_2}{E_2I_2} = \frac{M - FZ}{D} \tag{2}$$

where

$$D = E_1I_1 + E_2I_2 \tag{3}$$

$E_1$ and $E_2$ are the Young's moduli for the layers, and $I_1$ and $I_2$ are the second moments of area of the layers.

In deriving the equation of the nailed beam, the following assumptions and limitations are imposed:

1. The stress of the axial force distributes uniformly at any section of the beam.
2. The applied load and the nailing pattern are symmetrical with the midspan.

*Slip and resistance of nail.*—If $\epsilon_1$ and $\epsilon_2$ are, respectively, the strains of layer 1 and layer 2 at the contact surfaces, we have

$$\epsilon_1 = \frac{M_1}{E_1I_1}\frac{h_1}{2} - \frac{F}{A_1E_1} \tag{4}$$

$$\epsilon_2 = -\frac{M_2}{E_2I_2}\frac{h_2}{2} + \frac{F}{A_2E_2} \tag{5}$$

where $A_1$ and $A_2$ are the cross-sectional areas of layers. The slip at X from the support, $\gamma$ is

$$\gamma = \int_X^{L/2} \epsilon_1\, dX - \int_X^{L/2} \epsilon_2\, dX \tag{6}$$

Substitution of Eqs. (4) and (5) in Eq. (6) gives

$$\gamma = \frac{Z}{D}\int_X^{L/2} M\, dX - \left(\frac{Z^2}{D} + C\right)\int_X^{L/2} F\, dX \tag{7}$$

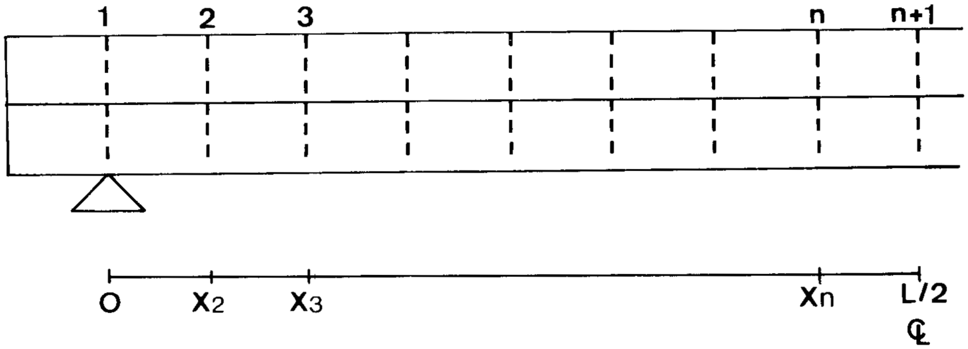


FIG. 2. Numbering of nails.

where

$$C = \frac{1}{A_1E_1} + \frac{1}{A_2E_2} \tag{8}$$

The axial force at $X_i$ from the support, F is (see Fig. 2)

$$F = \sum_{j=1}^{i} q_j \tag{9}$$

where $q_i$ is the resistance of $i^{th}$ nail.

Substituting for F in Eq. (7), slip of each nail is expressed as

$$\gamma_1 = \frac{Z}{D}\int_{X_1}^{L/2} M\,dX - \left(\frac{Z^2}{D} + C\right)\left[\left(\frac{L}{2} - X_1\right)q_1 + \left(\frac{L}{2} - X_2\right)q_2 + \ldots + \left(\frac{L}{2} - X_n\right)q_n\right] \tag{10}$$

$$\gamma_2 = \frac{Z}{D}\int_{X_2}^{L/2} M\,dX - \left(\frac{Z^2}{D} + C\right)\left[\left(\frac{L}{2} - X_2\right)q_1 + \left(\frac{L}{2} - X_2\right)q_2 + \ldots + \left(\frac{L}{2} - X_n\right)q_n\right]$$

$$\vdots \qquad\qquad \vdots$$

$$\gamma_i = \frac{Z}{D}\int_{X_i}^{L/2} M\,dX - \left(\frac{Z^2}{D} + C\right)\left[\left(\frac{L}{2} - X_i\right)\sum_{j=1}^{i} q_j + \sum_{j=i+1}^{n}\left\{\left(\frac{L}{2} - X_j\right)q_j\right\}\right] \tag{11}$$

$$\vdots \qquad\qquad \vdots$$

$$\gamma_n = \frac{Z}{D}\int_{X_n}^{L/2} M\,dX - \left(\frac{Z^2}{D} + C\right)\left(\frac{L}{2} - X_n\right)\sum_{j=1}^{n} q_j$$

where 2n + 1 is the total number of nails. From these equations, we get

$$\gamma_{i+1} = \gamma_i - \frac{Z}{D}\int_{X_i}^{X_{i+1}} M\,dX + \left(\frac{Z^2}{D} + C\right)(X_{i+1} - X_i)\sum_{j=1}^{i} q_j \tag{12}$$

If we have the $q - \gamma$ relation for a single nail, we can obtain all slips and resistances of the nails for an arbitrary bending moment by Eq. (12). The procedure is the repetitive numerical solution:

1. Assume the value, $\gamma_1$, and call this assumptive value $\gamma_a$. $q_1$ can be obtained by the $q - \gamma$ relation.
2. For $\gamma_2$, set $i = 1$ in Eq. (12), and we have

$$\gamma_2 = \gamma_1 - \frac{Z}{D}\int_{X_1}^{X_2} M\,dX + \left(\frac{Z^2}{D} + C\right)(X_2 - X_1)q_1$$

where $\gamma_1$ and $q_1$ are already obtained. Then $q_2$ can be obtained by the $q - \gamma$ relation.
3. Change the value, i, stepwise in Eq. (12), and all the values of $\gamma$ and q can be obtained.
4. Substitute $\gamma_2 \sim \gamma_n$ and $q_1 \sim q_n$ in Eq. (10), and we get $\gamma_1$, which is different from the assumptive $\gamma_a$.
5. Examine whether $\gamma_a$ can be considered to be equal to $\gamma_1$. The standard of judgment seems to be sufficient if $\gamma_a$ is nearly equal to $\gamma_1$ with 1% accuracy. In this case all the values of $\gamma$ and q can be considered as the solutions.

If $\gamma_a < \gamma_1$, increase $\gamma_a$ and repeat the procedure from 2. If $\gamma_a > \gamma_1$, decrease $\gamma_a$ and repeat the procedure from 2.

*Deflection of the beam.*—As all resistances of the nails for an arbitrary bending moment are obtained by the procedure mentioned above, the axial force F, at any section of the beam can be obtained by Eq. (9). The deflection of the nailed beam is considered to be the same as that of the no connections beam which is acted by the moment, $M - FZ$. The superposition principle can be used:

$$W = Wo(M) - Wo(FZ) \tag{13}$$

where W is the deflection of the nailed beam, Wo(M) is the deflection of the no connections beam which is acted by the bending moment, M, and Wo(FZ) is the deflection of the no connections beam which is acted by the couple moment, FZ.

Wo(M) can be easily obtained by the beam theory. Wo(FZ) can be obtained by the principle of virtual work without regard to the loading method;

at the location of $m^{th}$ nail (at X)

$$Wo(FZ) = \frac{Z}{2D}\left[\sum_{i=1}^{m-1} \{(XL - X^2 - X_i^2)q_i\} + \sum_{i=m}^{n}\left\{2X\left(\frac{L}{2} - X_i\right)q_i\right\}\right] \tag{14}$$

at the midspan

$$Wo(FZ) = \frac{Z}{2D}\sum_{i=1}^{n}\left\{\left(\frac{L^2}{4} - X_i^2\right)q_i\right\} \tag{15}$$

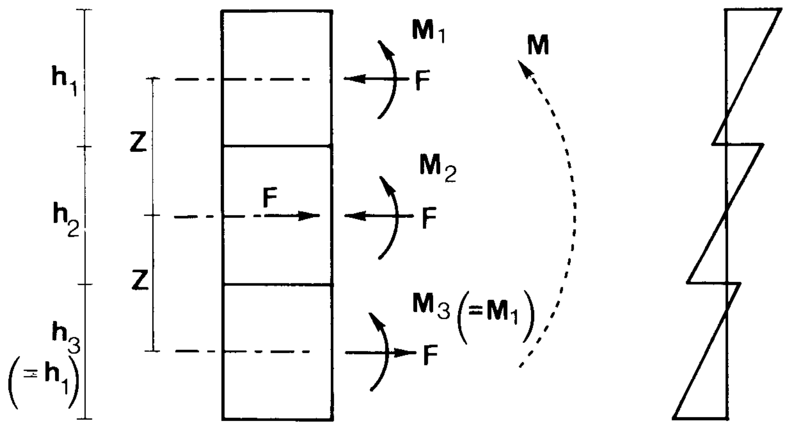


FIG. 3. Three-layer beam forces and strains.

In addition, if the nails are evenly spaced

$$Wo(FZ) = \frac{ZL^2}{8D}\frac{1}{n^2}\sum_{i=1}^{n}\{[n^2 - (i - 1)^2]q_i\} \tag{16}$$

*Three-layered beam*

In case of a three-layered beam, there are two contact surfaces; therefore the numerical solution is hard to use, because two values of slips should be assumed. But, if the dimensions and the mechanical properties of the outer layers are both the same, i.e., $I_1 = I_3$, $A_1 = A_3$, $h_1 = h_3$ and $E_1 = E_3$, slips and resistances are equal at both contact surfaces and can be obtained by the same way as the two-layered beam.

In this case, all equations for the two-layered beam are available for the three-layered beam except Eqs. (1)–(5), (8) and (13). Instead of these equations, the following equations should be used (see Fig. 3).

$$M = 2M_1 + M_2 + 2FZ \tag{1'}$$

$$\frac{M_1}{E_1I_1} = \frac{M_2}{E_2I_2} = \frac{M - FZ}{D} \tag{2'}$$

$$D = 2E_1I_1 + E_2I_2 \tag{3'}$$

$$\left.\begin{aligned} \epsilon_1 &= \frac{M}{E_1I_1}\frac{h_1}{2} - \frac{F}{A_1E_1} \\ \underset{(\text{upper})}{\epsilon_2} &= -\frac{M}{E_2I_2}\frac{h_2}{2} \end{aligned}\right\} \tag{4'}$$

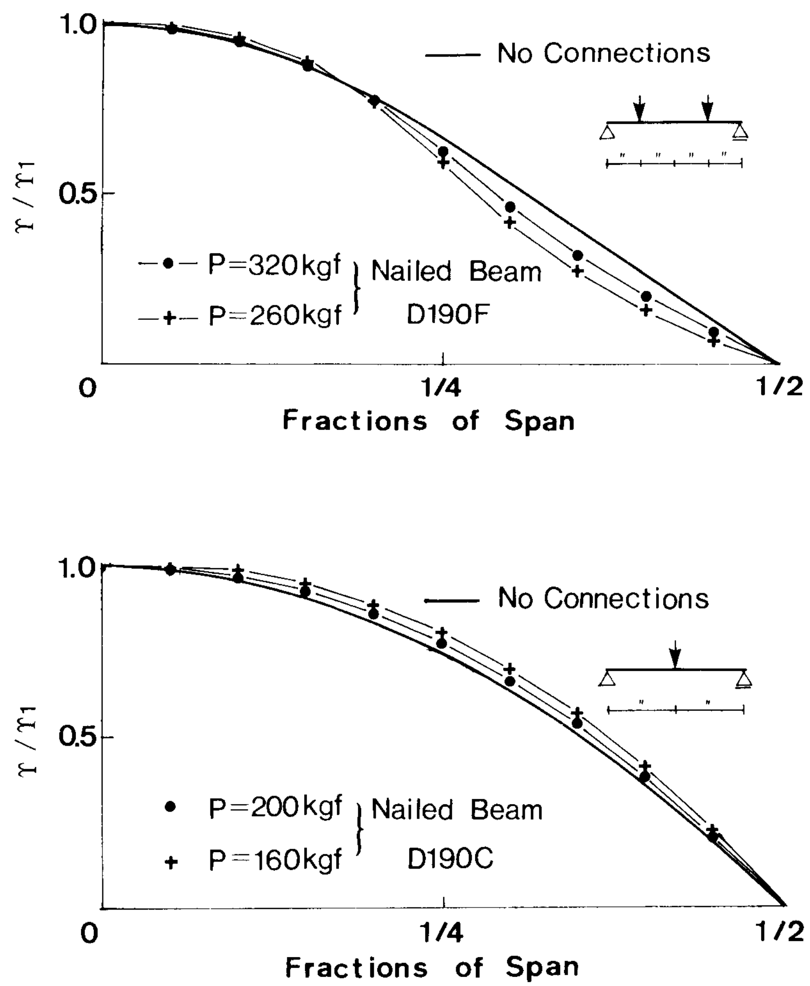


FIG. 4. Comparison of slip distribution of nailed beam with that of no connections beam.

$$\left.\begin{aligned} \underset{(\text{lower})}{\epsilon_2} &= \frac{M_2}{E_2 I_2}\frac{h_2}{2} \\ \epsilon_3 &= -\frac{M_1}{E_1 I_1}\frac{h_1}{2} + \frac{F}{A_1 E_1} \end{aligned}\right\} \tag{5'}$$

$$C = \frac{1}{A_1 E_1} \tag{8'}$$

$$W = Wo(M) - 2Wo(FZ) \tag{13'}$$

## APPROXIMATE SOLUTION

The numerical solution for calculating the slips and resistances of the nails needs the electronic computer. This is somewhat complicated. In this item, a

TABLE 1. *Test specimens.*

| Specimen | Ply | Layers* MOE (kgf/cm²) | | Nailing*** row | Loading |
|---|---|---|---|---|---|
| D190F | 2 | 122,600 | 117,400 | 1 | Two point |
| D290F | 2 | 85,800 | 92,600 | 2 | Two point |
| T290F | 3 | 145,400** | 148,600 | 2 | Two point |
| D190C | 2 | 135,900 | 104,800 | 1 | Central point |
| D290C | 2 | 127,400 | 140,000 | 2 | Central point |
| T290C | 3 | 135,900** | 123,700 | 2 | Central point |

* The dimension is 84.2 mm (W) × 36.8 mm (D).
** Average MOE of outer layers.
*** Nail spacing of a row is 90 mm.
1 mm = 0.0394 in.
1 kgf/cm² = 98.1 kPa = 14.2 psi.

simple procedure for predicting the approximate bending behavior of the nailed layered beams without the electronic computer was presented.

Figure 4 shows the nail slip distributions along the span of the nailed beam and no connections beam. The subjects for these calculations are the test specimens, D190F and D190C, which are shown in Table 1. This figure shows that, as the

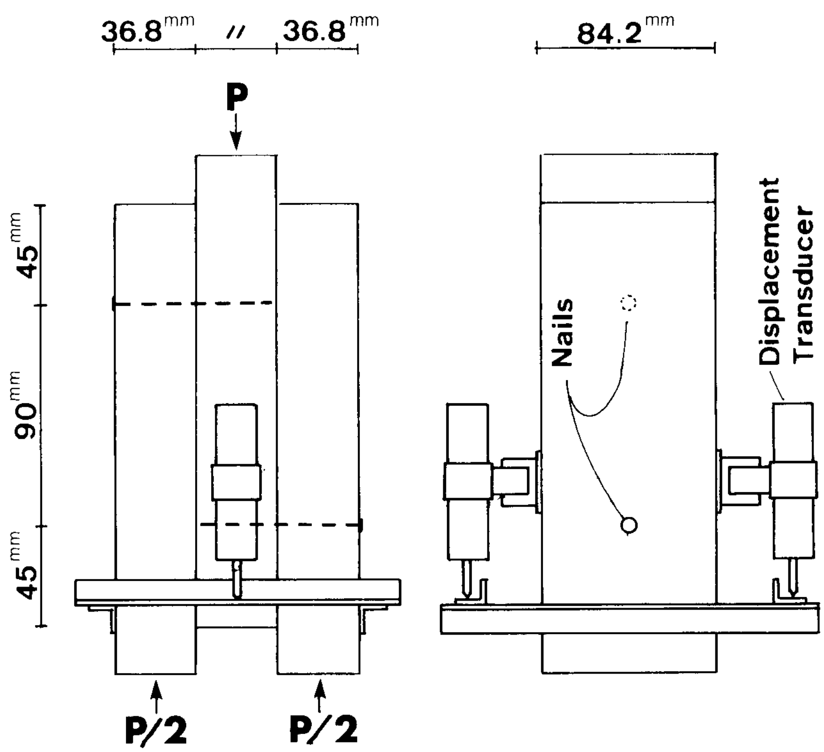


FIG. 5. Nailed joint test.

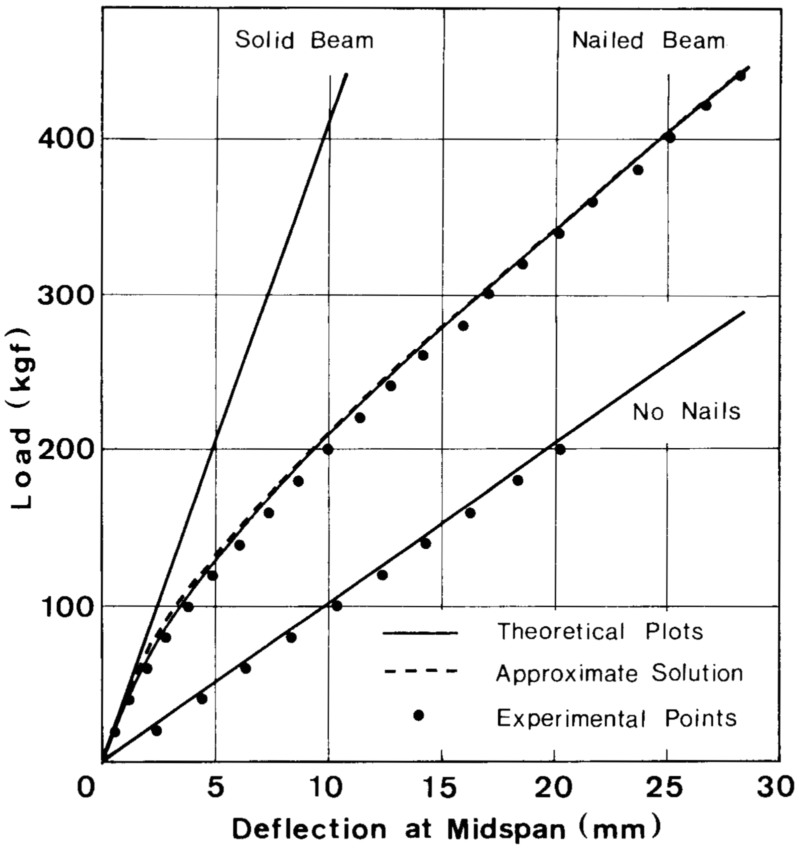


FIG. 6. Typical deflection versus load curves, for D190F.

slip becomes large, the slip distribution of the nailed beam approximates to that of the no connections beam. Therefore the assumption "the slip distribution of the nailed beam is equal to that of the no connections beam" is set.

The slip of the no connections beam at X is

$$\gamma_X = \frac{Z}{D}\int_X^{L/2} M\,dX$$

From this, the ratio of slip at X to slip at the support (i = 1) is

$$\frac{\gamma_X}{\gamma_1} = \left(\int_X^{L/2} M\,dX\right) \Big/ \left(\int_0^{L/2} M\,dX\right) \tag{17}$$

The approximate procedure is as follows:

1. Set the value of $\gamma_1$, and we can get all slips and resistances of nails by Eq. (17) and the $q - \gamma$ relation for a single nail.

TABLE 2. *Comparison of calculated load with experimental load.*

| | At W = 6 mm | | At W = 18 mm | |
|---|---|---|---|---|
| Specimen | Exp. load / Theo. load | Exp. load / Approx. load | Exp. load / Theo. load | Exp. load / Approx. load |
| D190F | 0.94 | 0.92 | 0.99 | 0.98 |
| D290F | 0.99 | 0.91 | 0.99 | 0.97 |
| T290F | 1.08 | 1.03 | 1.02 | 1.01 |
| D190C | 0.93 | 0.98 | 0.94 | 0.94 |
| D290C | 0.90 | 0.93 | 0.93 | 0.93 |
| T290C | 0.91 | 0.93 | 0.92 | 0.92 |
| (Ave.) | 0.96 | 0.95 | 0.97 | 0.96 |

2. Substitute all q and $\gamma$ values in Eq. (10) to obtain the bending moment (applied load).
3. Substitute all q values in Eq. (15) to obtain Wo(FZ), then, the deflection of the beam from Eq. (13).

Thus, the applied load and the deflection of the beam can be obtained for an arbitrary slip at the support without the electronic computer. The validity of this solution is examined in the experiments.

## EXPERIMENTS

### *Test specimens and testing method*

Two-layered and three-layered nailed beams shown in Table 1 were tested to examine the validity of the theory. The lumber was selected from kiln-dried 2 by 4 western hemlock, and was planed to reduce the effect of the friction at contact surface. The nails used were N65 with 3.05-mm diameter and 65-mm length, which is defined in JIS A5508.

The central point load or the two point loads were applied to the beam with 1,800-mm span. The deflections at each point divided the span into eight equal lengths and the slips of nails at both ends were measured by the displacement transducers. A special instrument and technique were used to measure the slips of each nail. The principle is as follows: a pair of razor marks was made on the edge planes of the layers at each nailing position. When slip occurs, the distance between a pair of razor marks increases, and it is equivalent to the slip displace-

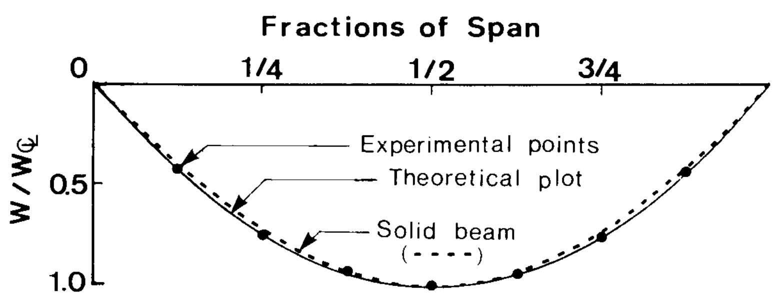


FIG. 7. Typical deflection curve, for D190F at P = 200 kgf.

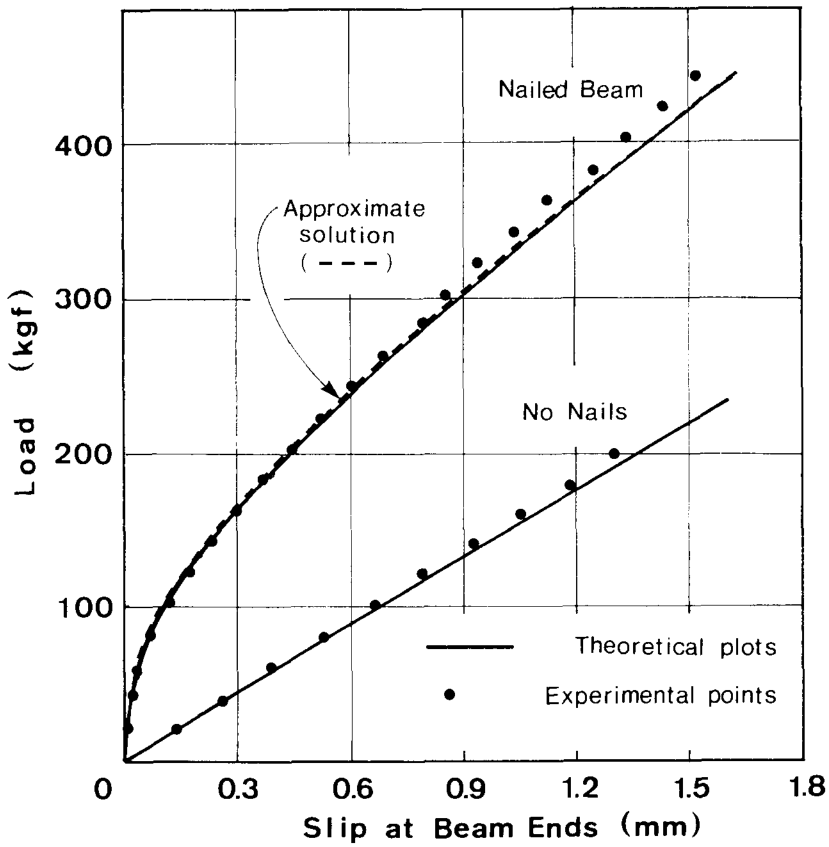


FIG. 8. Typical load versus slip curve at beam ends, for D190F.

ment. A screw comparator was used to measure the distance. The moved distance of the telescope was measured by the electronic dial gauge. The accuracy of this measuring method mainly depends on the thickness of the marks. It was estimated to be ±0.05 mm from the results of the preliminary tests repeated ten times.

In the case of the test specimens, D290F and D290C, the strain distribution across the depth at near midspan and a quarter span were measured by the electrical resistance strain gauges.

The $q - \gamma$ relation for a single nail was obtained by the nailed joint test as shown in Fig. 5. It was derived from a single loading. The measuring data were automatically recorded by the electronic data logger.

### *Comparisons of theoretical and experimental results*

*Deflection.*—Figure 6 shows typical load-deflection curves. Comparison shows that the theoretical result agrees closely with the experimental result; furthermore, the approximate solution gives an almost equal result to the theoretical result.

This can be seen for the other test specimens. For all test specimens, the ratio of experimental load to calculated load when the deflection at midspan is 6 mm (L/300) or 18 mm (L/100) as shown in Table 2. The theory gives a slightly higher stiffness, but the difference is very small. The approximate solution gives higher

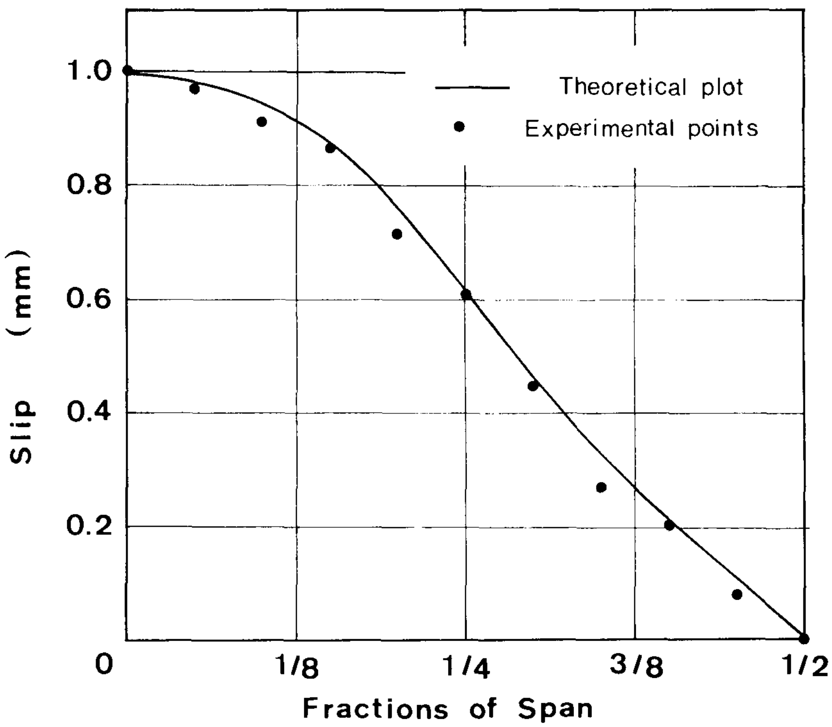


FIG. 9. Typical slip distribution, for D190F at P = 320 kgf.

stiffness than the theoretical one for the two point loads, and lower stiffness for the central point load.

Figure 7 shows the deflection curve along the span. The deflection curve of the nailed layered beam is almost equal to that of the solid beam. The calculated curve agrees fairly well with the observed one.

*Slip of nail.*—The relation between the load and the slip at beam ends is shown in Fig. 8. The good agreement of the theoretical and approximate solutions with the experimental result can be seen in this figure. An example of the slip distribution along the span is shown in Fig. 9. The calculated results agree well with the observed results.

*Strain of the layers.*—Strain distribution across the depth of layers is shown in Fig. 10. The good agreement of the theoretical results with the experimental results shows that the nailed layered beams satisfy the assumption 1 concerning the stress distribution of the axial force.

## NUMERICAL EXPERIMENTS

The solution was used to study the effects of some parameters on the behavior of the nailed layered beams. A two-layered beam was chosen as the subject for

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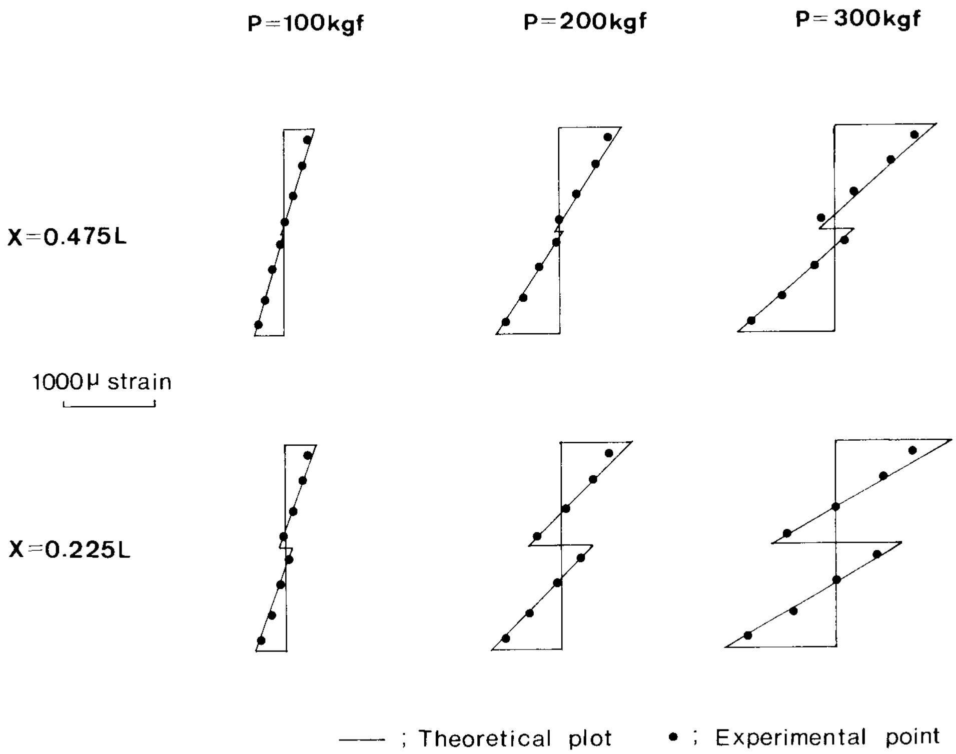
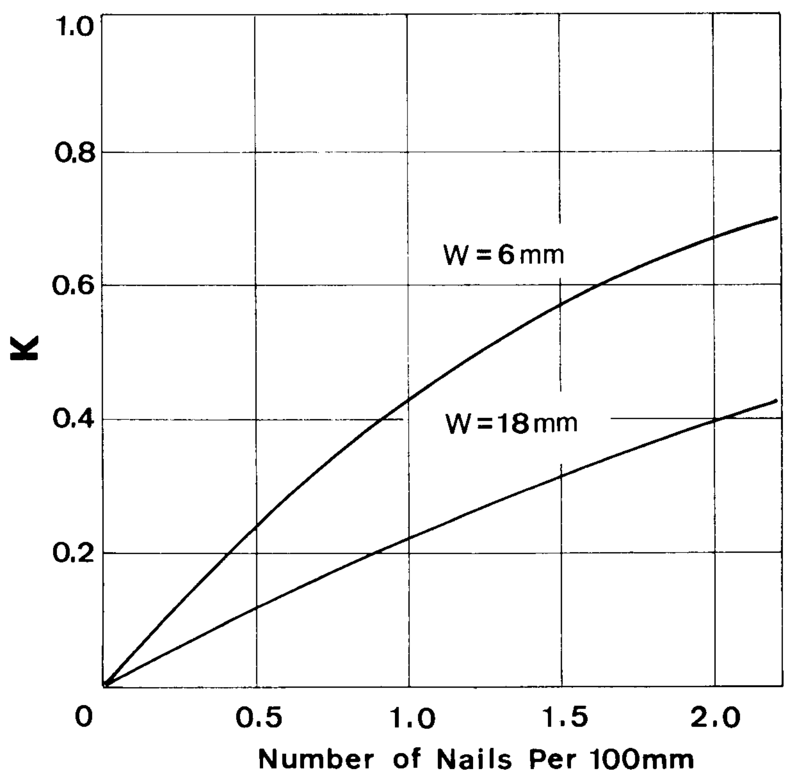
FIG. 10. Strain distribution, for D290F.

FIG. 11. Effect of number of nails on stiffness.



P=100kgf
P=200kgf
P=300kgf
X=0.475L
1000μ strain
X=0.225L
—— ; Theoretical plot
• ; Experimental point

1.0
0.8
0.6
K
0.4
0.2
W=6mm
W=18mm
0
0.5
1.0
1.5
2.0
Number of Nails Per 100mm

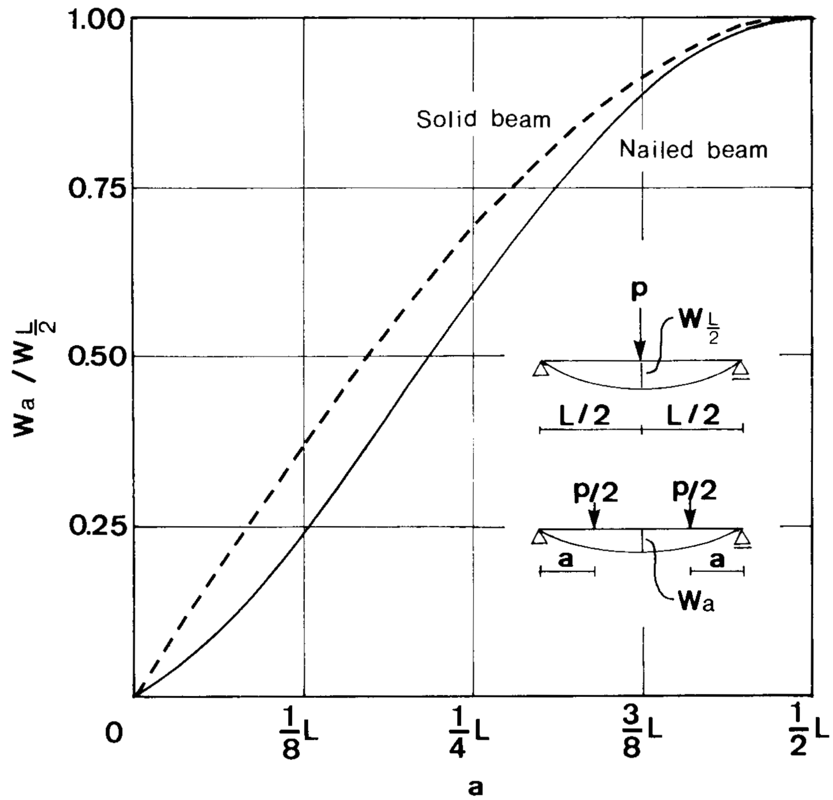



FIG. 12. Effect of loading method on stiffness, at P = 200 kgf.

this experiment, and the same layers and the q $-$ $\gamma$ curve for the test specimen, D190F, were used.

Figure 11 shows the effect of the number of nails on the stiffness. The nailed layered beam has an intermediate stiffness between the solid beam and no connections beam. Therefore, the following factor might express well the contribution of the connectors.

$$K = \frac{P - Po}{Ps - Po} \tag{13}$$

where P, Po and Ps are, respectively, the applied load of nailed layered beam, the no connections beam, and the solid beam corresponding to the same deflection. When there is no connection, K is zero. When there is an infinite multitude of connectors, K takes its maximum value, 1. In Fig. 11, K is taken as the coordinate axis. We can see that, even if the number of nails doubles, the K value does not double, i.e., the rate of increase of K decreases gradually as the number of nails increases. The K value corresponding to the deflection, 6 mm, is greater than that to the deflection, 18 mm. This means that the contribution of connectors is as great as the slip is small. This is due to the nonlinear characteristic of the nailed joint and is of particular interest.

Figure 12 shows the effect of the loading method on the stiffness. The stiffness of the solid beam or the no connections beam is independent of the loading method. In this figure, the curve for the nailed beam is different from that for the solid beam, which means that the stiffness of the nailed layered beam is affected by the loading method. This is also due to the nonlinear characteristic of the nailed joint. In this figure, we can see that the nailed layered beam has the minimum stiffness for the central point loading.

## CONCLUSIONS

An analytical procedure that predicts the nonlinear bending behavior of the nailed layered beams has been developed. The input data required are the beam geometry, the Young's moduli of the layers, the nailing schedule, and the load-slip characteristic for a single nail.

As the calculation is the numerical repetitive procedure and needs the electronic computer, a simple procedure that can predict the approximate stiffness has also been developed.

The experiments were conducted to examine the validity of the procedures. The comparisons showed that both solutions give excellent agreements with the experimental results.

The theory given here is expected to be extended to the stressed-skin components.

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