EFFECT OF CYCLIC HUMIDITY EXPOSURE ON MOISTURE DIFFUSION IN WOOD

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ABSTRACT

It is known that moisture content changes in wood strongly affect the mechano-sorptive effects, which in turn contribute significantly to the deformation and strength under long duration loads. Earlier studies have shown that the moisture changes due to the daily and annual variation in the surrounding air tend to be confined to the edges of practical size members. Therefore, prediction of the moisture content near the boundaries of a timber section becomes an important issue. This paper studies the mathematical solutions of the diffusion equation for the finite thickness element considering the effect of surface resistance. By introducing a surface attenuation factor, a simple approximate solution may be obtained. Procedures for evaluating the coefficients K and D are also discussed, together with some data on these values for radiata pine (*Pinus radiata*). The surface emission coefficient K is a function of air velocity V. A procedure for assessing this effect of the air velocity is also given.

A numerical example shows that the proposed approximate solution accurately predicts the cyclic range of moisture content for an ambient cyclic boundary condition. The procedures described herein can be easily extended to include the analysis of two-dimensional and three-dimensional elements.

Keywords: Moisture diffusion, cyclic humidity, surface attenuation.

INTRODUCTION

The moisture content changes in wood are known to have a strong effect on its mechano-sorptive behavior, which in turn contributes significantly to the deformation and strength under long duration loads. Earlier studies (Leicester and Lu 1992, 1994; Lu and Leicester 1993, 1994) have shown that for practical size members the moisture changes due to the daily and annual variations in the surrounding air tend to be confined to the edges. Therefore, it is important to know how the moisture content variation in timber in normal environmental conditions can be predicted and how the influence of air velocity affects the moisture diffusion, particularly the moisture content near the boundaries of a timber section. Although the diffusion coefficient D increases with increasing moisture content (Bramhall 1979a, b; Kouali and Vergnaud 1991; Toratti 1992) for normal climatic conditions, the range of moisture change is small, and for practical purposes, all parameters may be considered to be constant.

The commonly used diffusion equations are based on the assumption that the surface instantly comes to equilibrium with the air, i.e., they are based on neglecting the effect of surface resistance (Choong and Skaar 1969). However, Hart (1977) has stated that when wood is exposed to a change in ambient humidity, the effective wood surface relative humidity (RH) does not immediately change to the new ambient level; in fact, it does not reach this level until sorption ceases.

Regarding cyclic moisture changes in wood, Chomcharn and Skaar (1983) studied dynamic sorption and hygroexpansion of wood wafers exposed to sinusoidally varying humidity and investigated the interrelationships between moisture sorption, dimensional changes, time, and relative humidity. It was found that the moisture and dimensional changes were generally sinusoidal but lagged behind the imposed humidity and the computed curves assuming a constant diffusion coefficient, and sinusoidally varying boundary moisture conditions were qualitatively similar to those obtained experimentally.

This paper presents the mathematical solutions of the diffusion equation (Fick's Law) for cases of both zero and finite surface resistance. It should be noted that the mathematical solutions presented here are steady-state solutions applicable only to wood under cyclic ambient conditions after a long period of time when the initial transient moisture changes have become negligible. By introducing a surface attenuation factor, an approximate solution for the case of an ambient cyclic boundary condition is proposed. Some numerical examples are also presented. The effect of surface resistance is covered by introduction of a surface attenuation factor.

BASIC EQUATIONS AND BOUNDARY CONDITIONS

Fick's laws (Crank 1956) will be applied to estimate moisture concentration in the timber. For a one-dimensional problem, these are

$$F = -D\frac{\partial m}{\partial x} \tag{1}$$

$$\frac{\partial m}{\partial t} = D\frac{\partial^2 m}{\partial x^2} \tag{2}$$

where F is the rate of moisture transfer per unit area of section, m the concentration of moisture, x the space coordinate measured normal to the section (see Fig. 1), and D is the diffusion constant.
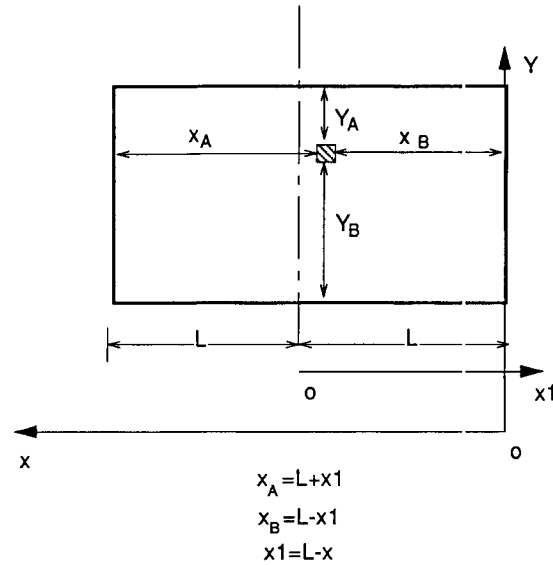
If the surface resistance is neglected, then

FIG. 1. Definition of coordinates x and x1 and notation for coordinates used in Eqs. (1), (16) and (B1).

$$x_A = L + x1$$
$$x_B = L - x1$$
$$x1 = L - x$$

the boundary condition at the air-wood interface is stated as follows:

$$m_{surface} = m_{air} \tag{3}$$

If the surface resistance is taken into account, then the boundary condition for the wood is

$$-D\frac{\partial m}{\partial x} = K(m_{air} - m_{surface}) \tag{4}$$

where $m_{surface}$ is the moisture concentration on the wood surface, K the surface emission coefficient, m_{air} the equilibrium moisture concentration for the temperature and relative humidity conditions of air around the timber (see Appendix A).

In the following analysis, we will consider the cyclic ambient conditions of the air surrounding the wood specimen to be given by:

$$m_{air} = m_{av} + m_A \sin pt \tag{5}$$

where m_{av}, a constant, denotes the mean, m_A is the cycle amplitude, and p is the cyclic frequency of the effective equilibrium moisture content of the ambient air surrounding the wood.

In the following, we will be concerned with the one-dimensional solution of an element with the finite thickness.

MATHEMATICAL SOLUTIONS FOR DIFFUSION IN A WOOD OF FINITE THICKNESS

Solution neglecting surface resistance

From Crank (1956), the solution to Eqs. (2), (3), and (5) is given by

$$m = m_{av} + m_A \sum_{n=1,3...} b_n \sin\left(\frac{n\pi\chi}{2L}\right) \quad (6)$$

where

$$b_n = \frac{4}{n\pi} \cdot \frac{Q}{Q^2 + 1} [Q \sin(pt) - \cos(pt)] \quad (7)$$

and

$$Q = \frac{\left(\frac{n\pi}{2L}\right)^2 \cdot D}{p} \quad (8)$$

Solution including the surface resistance

From Carslaw and Jaeger (1959), the solution to Eqs. (2), (4), and (5) is given by

$$m = m_{av} + m_A \cdot \beta(x1) \cdot \sin(pt + \gamma_0 - \gamma_1) \quad (9)$$

where

$$\beta(x1) = (K/D)(M_0/M_1) \quad (10)$$

$$\gamma_0 = \arctan\left(\frac{\sinh(bx_1)\sin(bx_1)}{\cosh(bx_1)\cos(bx_1)}\right) \quad (11)$$

$$M_0 = \frac{\cosh(bx_1)\cos(bx_1)}{\cos \gamma_0} \quad (12)$$

with

$$b = \sqrt{\frac{p}{2D}}$$

$$\gamma_1 = \arctan\big[[b \sinh(bL)\cos(bL) +$$

$$+ b \cosh(bL)\sin(bL) +$$

$$+ (K/D)\sinh(bL)\sin(bL)] \div$$

$$\div [b \sinh(bL)\cos(bL) -$$

$$- b \cosh(bL)\sin(bL) +$$

$$+ (K/D)\cosh(bL)\cos(bL)]\big] \quad (13)$$

$$M_1 = [b \sinh(bL)\cos(bL) -$$

$$- b \cosh(bL)\sin(bL) +$$

$$+ (K/D)\cosh(bL)\cos(bL)] \div$$

$$\div \cos \gamma_1 \quad (14)$$

where L is the half-thickness of the wood.

A surface attenuation factor of the cyclic component of the moisture content is defined as:

$$\alpha = |\beta(L)|. \quad (15)$$

The surface attenuation factor α is tabulated in Table 1 for several values of KL/D and bL. α for other values of KL/D and bL can be determined by linear interpolation. As may be expected, $\alpha \rightarrow 1.0$ as either $K \rightarrow \infty$, $D \rightarrow 0$, $p \rightarrow 0$ or $L \rightarrow 0$.

APPROXIMATE SOLUTIONS

It is seen that Eq. (9) is too complex to be applied in practice. For simplicity, by introducing the surface attenuation factor α in Eq. (9), an approximate solution (Eq. (16)) is obtained (Lu and Leicester 1994). This solution can be used to estimate the moisture content, considering the effect of surface resistance for the case of an ambient cyclic boundary condition. For a wood of finite thickness

$$m = m_{av} + \alpha m_A [e^{-bx_A}\sin(pt - bx_A) +$$

$$+ e^{-bx_B}\sin(pt - bx_B) -$$

$$- e^{-bx_A - bx_B}\sin(pt - bx_A) \cdot$$

$$\cdot \sin(pt - bx_B)] \quad (16)$$

where x_A and x_B are defined as shown in Fig. 1.

Equation (16) can be extended quite simply to two-dimensional and three-dimensional forms if required. For example, the two-dimensional solution is given in Appendix B.

EVALUATION OF DRYING PARAMETERS K AND D

The surface emission coefficient K is a function of air velocity V. For a given air velocity

TABLE 1. *Surface attenuation factor α.*

KL/D $\sqrt{p/2D} \cdot L$	0.1	0.5	1	2	4	10	100	∞
0.1	0.979	0.999	0.9997	0.9999	1.000	1.000	1.000	1.000
0.3	0.475	0.924	0.974	0.991	0.996	0.999	1.000	1.000
0.5	0.194	0.662	0.845	0.936	0.974	0.991	1.000	1.000
1	0.061	0.265	0.442	0.641	0.8	0.918	1.000	1.000
2	0.034	0.153	0.271	0.439	0.623	0.815	1.000	1.000
4	0.017	0.083	0.156	0.277	0.447	0.687	1.000	1.000
10	0.007	0.034	0.067	0.128	0.232	0.447	1.000	1.000
50	0.001	0.007	0.014	0.028	0.054	0.128	1.000	1.000

L is the half thickness of wood, D is the diffusion constant, K is the surface emission coefficient.

V, the effective values of surface emission coefficient K and the diffusion coefficient D can be found experimentally by the technique for separating the internal and external resistance to moisture removal in wood drying (Choong and Skaar 1969; Liu 1989; Siau and Avramidis 1996) from experimentally measured drying curves. In this paper, Choong and Skaar's technique was adopted to determine D and K. The details of the technique and the coefficients D and K evaluated from the experimental results are given in Appendix C.

Some typical values obtained for radiata pine of D and K (average) are: $D_R = D_T = 0.71$ mm²/h, $D_L = 5.3$ mm²/h, and $K = 1.127$ mm/h and $K = 0.63$ mm/h for air velocities of 2.6 m/sec and 0.3 m/sec, respectively (L = longitudinal, T = tangential, R = radial movement). These coefficients are of the same order

of magnitude at the values shown in the literature (Choong and Fogg 1968; Choong and Skaar 1969; Toratti 1992; Siau and Avramidis 1996).

In addition, the effects of air velocity on K were investigated by modifying Bramhall's model (1979a, b). To do this, it will be assumed that for the practical range of air velocity the wood surface heat transfer coefficient U_h is related to air velocity V by (private communication with Dr John Sutherland, CSIRO Division of Forest Products, Clayton, Victoria 3168, Australia, 1992):

$$U_h = 0.68784 \ V^{0.67} \ (\text{cal} \cdot \text{Cm}^{-2} \cdot \text{h}^{-1} \ {}^\circ\text{C}^{-1})$$
(17)

where U_h is the boundary layer heat transfer coefficient (cal/cm² h °C) and V is the air speed (m/sec).

A comparison of experimental measurements and theoretical prediction for one particular case is shown in Fig. 2. It can be seen that the simulated curves closely follow the experimental results; this implies the accuracy and feasibility of the modified model. It therefore gives us sufficient confidence to apply the model to obtain drying curves for other values of air velocity.

The modified Bramhall's equations were used to compute drying curves for various thicknesses and air velocities, and then the method of Choong and Skaar was applied to these curves to obtain the effective values of K. These values are plotted against the air velocities in Fig. 3. For comparison, the test data obtained for radiata pine are also included in
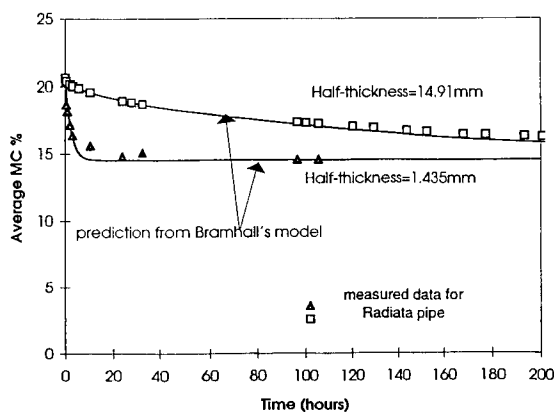

FIG. 2. Comparison of the experimental drying curves with those from computer simulation for air velocity of 2.6 m/sec.
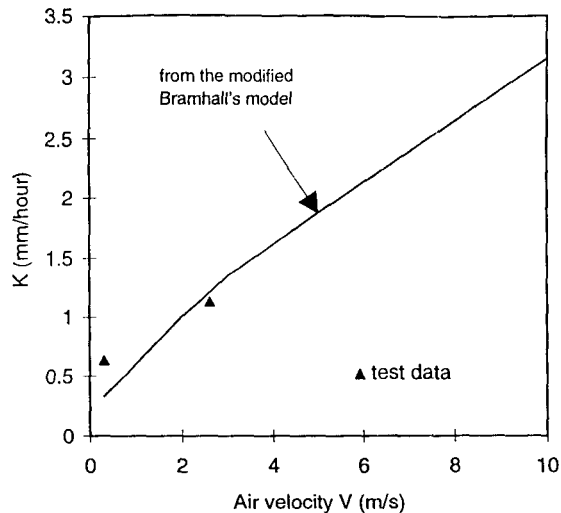
FIG. 3. Relationship between surface emission coefficient K and air velocity V.

FIG. 4. Comparison of mathematical solutions with/without the effect of surface resistance and approximate solutions (thickness 2.87 mm).

this figure.

NUMERICAL EXAMPLES

The following numerical example is given to illustrate the effects of surface resistance and to compare the mathematical and approximate solutions.

The parameters assumed for the example are as follows; L = 1.435 mm, K = 1.127 mm/h, D = 0.71 mm^2/h, p = $2\pi/T$, T = 2 h, m_{av} = 17.6%, m_A = 3.1%. The parameters chosen were intended to match some of the available experimental data. These data lead to KL/D = 2.31, $\sqrt{[(p)/(2D)]} \cdot L$ = 2.15 and the surface attenuation factor α being about 0.46.

Figure 4 shows the results calculated from both mathematical solutions and approximate solutions for cyclic moisture conditions considering the effect of surface resistance. For comparison purposes, the results without considering the effect of surface resistance are also included. Fifty terms were used in evaluating Eq. (6). It is seen that, for this example, the effect of surface resistance should not be neglected. The good agreement between the two curves computed from Eqs. (9) and (16) suggests that the proposed approximate solutions (Eq. (16)) can predict the cycle range of moisture content with sufficient accuracy. If a time phase factor γ_1 (Eq. (13)) is added in Eq. (16),
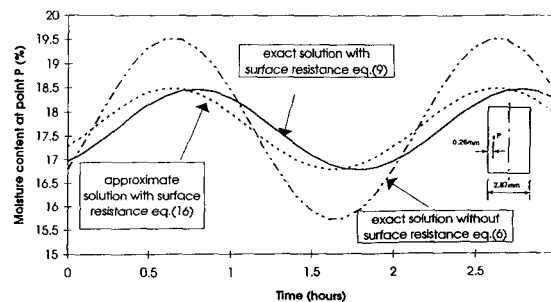
the approximate equation gives very close solutions to those calculated from the exact Eq. (9).

CONCLUSIONS

By studying the mathematical solutions of the diffusion equation for the finite thickness element considering the effect of surface resistance for different boundary conditions, it is found that for the parameters used herein, the effect of surface resistance on moisture content predictions should not be neglected for thin specimens or high speed cycling. For ease in application, particularly for two- and three-dimensional problems, a simple approximate solution is proposed. The numerical examples show that the proposed approximate solution can accurately predict the cycle range of moisture content for an ambient cyclic boundary condition.

Practical procedures for evaluating the coefficients K and D, including the effects of ambient air velocity, are also discussed, together with some data on these values for radiata pine.

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APPENDIX A

Formula for equilibrium moisture concentration

An approximate formula (Simpson 1973) for the equilibrium moisture concentration, denoted by 'm_{air}', within a piece of wood is given by

$$m_{air} = \ln\{[\ln(rh/100) - K_3]/K_2\}/(\ln K_1) \qquad (A1)$$

where

$$K_1 = 1.0327 - 0.000674 \, Te$$

$$K_2 = 17.884 - 0.1432 \, Te + 0.0002362 \, Te2$$

$$K_3 = 0.0251$$

where rh denotes the relative humidity of the air and Te denotes the air temperature in °K.

APPENDIX B

Approximate solution for two-dimensional problems

$$m = m_{av} + \alpha m_A(m_x + m_y - m_x m_y) \qquad (B1)$$

where

$$m_x = e^{-b_x x^A} + e^{-b_x x^B} - e^{-b_x x^A} e^{-b_x x^B} \qquad (B2)$$

$$m_y = e^{-b_y y^A} + e^{-b_y y^B} - e^{-b_y y^A} e^{-b_y y^B} \qquad (B3)$$

where the notation $b_x = \sqrt{p/2Dx}$, $b_y = \sqrt{p/2D y}$ and the distances x_A, x_B, y_A, y_B are defined as shown in Fig. 1.

APPENDIX C

D and K evaluation

Approach.—The technique for separating the internal and external resistance to moisture removal in wood drying was first described by Choong and Skaar (1969) to evaluate the diffusion coefficient D and the surface emission coefficient K from two drying curves. More recently, Liu (1989) developed an analytical procedure to separate the diffusion and surface emission coefficients from a single drying curve. Choong and Skaar's technique is based on Newman's graphical solution (1931) to obtain both the internal and external resistance to drying if the half-drying

TABLE C1. *Coefficients calculated from test data (radiata pine).*

Direction of movement	Specimens	D (mm²/h)	K (mm/h)	Air speed (m/s)
Side grain	A1 and B1	0.556	1.169	
	A2 and B2	0.664	1.159	2.6
	Average	0.61	1.164	
	A3 and B3	0.596	0.559	
	A4 and B4	0.511	0.647	0.3
	Average	0.554	0.593	
Radial	C1 and D1	1.056	1.05	
	C2 and D2	0.643	1.484	2.6
	Average	0.85	1.267	
	C3 and D3	0.814	0.593	
	C4 and D4	0.78	0.608	0.3
	C5 and D5	0.633	0.733	
	Average	0.742	0.645	
Tangential	C8 and D8	0.876	0.483	
	C9 and D9	0.742	0.567	0.3
	Average	0.809	0.525	
Longitudinal	C10 and D10	7.15	0.84	
	C11 and D11	6.14	1.018	2.6
	Average	6.645	0.929	
	C12 and D12	4.986	0.631	
	C13 and D13	2.935	0.892	0.3
	Average	3.961	0.762	

Dimensions for A1 to A4: 5 cm × 5 cm × 0.3 cm;
   B1 to B4; 5 cm × 5 cm × 3 cm;
   C1 to C13: 3 cm × 3 cm × 0.3 cm;
   D1 to D13: 3 cm × 3 cm × 3 cm.

times are known for two samples of different thickness dried under identical conditions. Assume that two different one-dimensional drying experiments are carried out on matched material of two different thicknesses, $2a_1$ and $2a_2$, whose half-drying times are respectively, t_1 and t_2. The diffusion coefficient D and the surface emission coefficient K can be determined according to the following formulae:

$$D = \frac{0.2(a_2 - a_1)}{\dfrac{t_2}{a_2} - \dfrac{t_1}{a_1}} \qquad \text{(C1)}$$

$$K = H \cdot D \qquad \text{(C2)}$$

where

$$H = \frac{0.7}{D\dfrac{t_1}{a_1} - 0.2a_1} \qquad \text{(C3)}$$

or

$$H = \frac{0.7}{D\dfrac{t_2}{a_2} - 0.2a_2} \qquad \text{(C4)}$$

Procedure. — Radiata pine (*Pinus radiata*) samples, are shaped like parallelepiped of two different nominal thickness, 0.3 cm and 3 cm, and other dimensions 5 cm × 5 cm for group I and 3 cm × 3 cm for group II.

The samples are cut out so that the transverse diffusion is conducted through their thickness by protecting the other four faces from the moisture with an impermeable foil. The samples were equilibrated to a uniform equilibrium moisture content of about 20%. Half of the samples were placed in an experimental drying chamber where the environment was controlled at 25°C and 75% humidity (the equilibrium moisture content at this environment is 14.5%), with a low air velocity less than 0.3 m/sec. Another half of the samples were placed in a duct in a high airflow with an air velocity of 2.6 m/sec. The weight of each sample was measured at periodic time intervals during drying until equilibrium was attained.

Results. — Substitution of the half-drying times and half-thicknesses from the experimental drying curves into Eqs. (C1) to (C4) results in the diffusion coefficients D and surface emission coefficients K, which are given in Table C1.

Substitution of the half-drying times and half-thicknesses from the simulated curves into Eqs. (C1) to (C4), the diffusion coefficients D and surface emission coefficients K for any desired air velocity can be determined.