INFLUENCE OF MEAN FREE PATH ON GAS FLOW IN WOOD

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ABSTRACT

It is shown theoretically that the existence of more than one structural component in the flow path through wood causes the gradient of a graph of superficial gas permeability against reciprocal mean pressure not to be directly proportional to the mean free path of the gas. This is unlike the behaviour of homogeneous media and agrees with previously published experimental data.

Keywords: Softwoods, gas permeability, slip flow, mean free path, tracheids, bordered pits.

INTRODUCTION

When laminar flow of a gas occurs through a permeable medium, slip flow takes place if the mean free path of the gas molecules is an appreciable fraction of the radius of the capillaries within the medium.

The superficial gas permeability K of a sample through which slip flow occurs is given by:

$$K = (Qp\eta)/(\bar{p}\Delta p) = A + (B/\bar{p}), \quad (1)$$

where Q is the volumetric flow rate of a gas through a sample measured at pressure p when a pressure difference Δp is maintained across the sample. The mean pressure in the sample is $\bar{p}$, A and B are constants, and η is the viscosity of the gas. Equation (1) is essentially the relationship derived by Klinkenberg (1941) and applied to wood by, for example, Comstock (1967) and Wiley and Choong (1975).

For a cylindrical capillary with any ratio of length L to radius r the constants in Eq. (1) are given by (Petty 1974):

$$A = \frac{(\pi r^4)}{[8(L + 3\pi r/8)]}$$

and

$$B = \frac{(\eta \pi r^2 \gamma K_c)}{(\sqrt{2\pi M/RT})}, \quad (2)$$

where γ is a constant usually taken to be 0.9, K_c is the Clausing factor, which is a function of the ratio L/r, M is the molecular weight of the gas, R is the gas constant and T is the absolute temperature. A and B are respectively the intercept and gradient of a graph showing the variation of permeability K with $1/\bar{p}$. A is not affected by the nature of the gas and is a constant for a given sample. However, B does depend on which gas is being used.

Since the mean free path of the gas molecules λ is given by (Carman 1956):

$$\lambda = (\eta/\bar{p})(\sqrt{\pi RT/2M}), \quad (3)$$

therefore $B = r^2 \gamma K_c \lambda \bar{p}$. (4)

The ratio of the values of B for two gases at the same temperature will thus be the ratio of their mean free paths at the same mean pressure. Wiley and Choong (1975) report that this relationship was not observed when the longitudinal permeability of wood to nitrogen and helium was measured. Instead of being 2.95, the ratio of the values of B for the two gases was found to vary between 0 and 2.32. They report that Comstock, working with *Tsuga canadensis*, found values for the same quantity in the range 1.51–2.35.

PERMEABILITY OF TWO-COMPONENT SYSTEMS

The above analysis refers to a permeable medium containing capillaries of a single type, as is commonly found in porous rocks. However, in recent years it has been shown that in wood two or more distinct types of capillary make significant contributions to the total resistance to fluid flow (Petty 1970; Petty and Puritch 1970; Smith and Banks 1971; Bolton and Petty 1975; Siau 1976). The two principal components causing resistance to flow in conifer wood have been identified as the tracheid lumina and the bordered pits. These constitute two very different types of capillary. If two structural components arranged in series in wood have permeabilities of K_1 and K_2, then the observed permeability K of the wood is given by:

$$1/K = (1/K_1) + (1/K_2), \qquad (5)$$

where $K_1 = A_1 + B_1/\bar{p}$

and $K_2 = A_2 + B_2/\bar{p}$. (6)

This is analogous to the calculation of conductances in electrical networks.

By combining Eqs. (5) and (6) it may be shown that:

$$K = \frac{[(A_1 + B_1/\bar{p})(A_2 + B_2/\bar{p})]}{[A_1 + A_2 + (B_1 + B_2)/\bar{p}]}. \qquad (7)$$

It will be seen that K does not vary linearly with $1/\bar{p}$ as might be expected for a homogeneous permeable medium, although in practice the variation is close to linear at mean pressures above one atmosphere. Inspection of Eq. (7) shows that although B_1 and B_2 vary in proportion to the mean free path of the gas concerned (Eq. 4) the rate of change of K with $1/\bar{p}$, that is, the effective value of B for the wood as a whole, will in general not vary in the same way.

It is possible to derive the gradient B and intercept A of each straight line component defined by Eq. (6) from the experimentally determined variation of K with $1/\bar{p}$. Examples of this type of analysis are given in the papers referred to above. In particular, Smith and Banks (1971) have shown that the experimental variations of the constants for each component with η and M are consistent with Eq. (2). This implies that Eq. (4) applies to each component.

Published data concerning the permeabilities of individual components in a wood sample for a particular gas may be used to predict the permeabilities to other gases. A sample of air-dried *Picea sitchensis* sapwood (specimen no. 10 of Petty 1970) will be taken as an example. Dry air at mean pressures between 10 and 750 mm Hg was used for these measurements.

In this case component 1 is the tracheid lumina and component 2 the bordered pits. Using the notation of Eq. (6), these components may be expressed as:

$$K_1 = 1{,}400 + (56.3 \times 10^3)/\bar{p}$$

and $K_2 = 548 + (264 \times 10^3)/\bar{p}$, (8)

where K is expressed in μm^3 and $\bar{p}$ in mm Hg. The dimensions of K are $(\text{length})^3$ because the permeability is calculated for the particular specimen. If values are calculated for unit volume of wood, the dimensions become $(\text{length})^2$. Using Eq. (7) K for air is found to be 708 μm^3 at $\bar{p}$ = 350 mm Hg and 569 μm^3 at $\bar{p}$ = 700 mm Hg. These permeability values differ by 139 μm^3.

At a temperature of 20 C and at 750 mm Hg pressure, the mean free path of air molecules is approximately 9×10^{-5} mm, and that of helium molecules is approximately 27×10^{-5} mm. Thus if Eq. (4) applies, the equations of the two components for helium are:

$$K_1 = 1{,}400 + (169 \times 10^3)/\bar{p}$$

and $K_2 = 548 + (792 \times 10^3)/\bar{p}$, (9)

when, as before, K is in μm^3 and $\bar{p}$ is in mm Hg. The values of B for air and helium differ by a factor of 3, corresponding to the ratio of the mean free paths. Using Eq. (7) again, K for helium is found to be 1,130 μm^3 at $\bar{p}$ = 350 mm Hg and 833 μm^3 at $\bar{p}$ = 700 mm Hg. These values differ by 292 μm^3. This difference gives a measure of the average effective value of the gradient B for the wood over this range of pressure (Fig. 1). The corresponding differences for air and helium differ by a factor of 2.1, and not by 3.0, which is the ratio of the mean free paths.

CONCLUSION

It is evident that the existence of two or

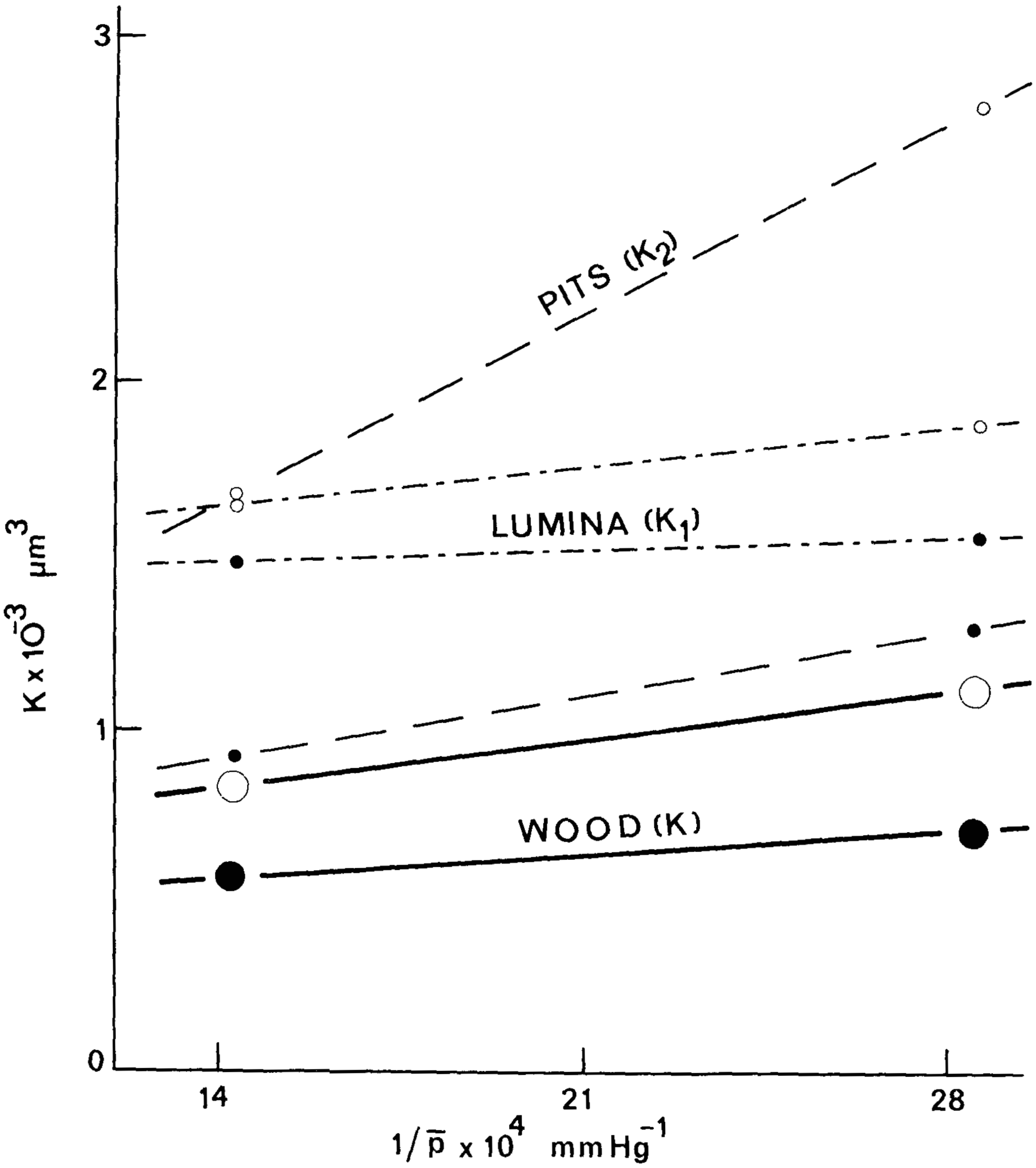

FIG. 1. The variation with reciprocal mean pressure of the superficial gas permeability to helium (open circles) and air (solid circles) of a sample of *Picea sitchensis* sapwood (K) and the two components causing resistance to flow, the tracheid lumina (K_1) and the bordered pits (K_2). Calculated values are shown for mean pressures of 350 and 700 mm Hg and $K = K_1 K_2/(K_1 + K_2)$ for each gas. The gradients are proportional to the mean free path for each component but not for the wood as a whole.

more distinct structural components arranged in series in conifer wood causes the effect of gaseous mean free path on permeability to differ from the effect obtaining for media containing capillaries of only one type. The extent to which the ratio of the mean gradients of graphs of K against $1/\bar{p}$ differs from the ratio of the mean free paths will depend on the nature of the components and will vary from sample to sample. This theoretical prediction is confirmed by the observations of Comstock (1967) and Wiley and Choong (1975). More than one component is present even in hardwoods (Siau 1976) so that this effect will apply also to these timbers. An additional factor, in addition to slip flow and turbulence, which affects the flow of gases through wood (Wiley and Choong 1975) is thus the existence of more than one type of capillary in the flow path. In future experimental investigations of the influence of mean free path on permeability, the permeabilities of the individual components in the flow path must be calculated.

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