PROBABILISTIC MODELING OF GLUED-LAMINATED TIMBER BEAMS

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ABSTRACT

An existing glued-laminated (glulam) timber beam simulation model was refined in this research. This refined model, referred to as PROLAM, incorporates results from recent research on localized lumber and end joint properties. Simulated beams are analyzed for strength using a transformed section method and analyzed for stiffness using a complementary virtual work technique. Other features of PROLAM include user options to proof test individual grades of lumber and to analyze the propagation of end joint and lamination failures that occur during a beam failure.

PROLAM was validated using independent glulam beam test data provided by the American Institute of Timber Construction. The simulated beam results were found to be in close agreement with the actual beam test results. In addition, predictions of allowable bending stress compared favorably with published design values.

Keywords: Simulation, glulam beams, reliability, modeling, glued-laminated.

INTRODUCTION

It is widely recognized that loads applied to structures and strengths of structural components are random variables. In the past, structural engineering design was based on deterministic methods using point estimates of loads and material properties. These loads and material properties are more accurately characterized by probability distribution functions, rather than deterministic values. Hence, reliability-based design techniques have evolved.

Glued-laminated timber beams (glulam) represent a category of wood products classified as engineered structural components. Glulam beams are made up of different layers of lumber, called laminations, that are glued together. Each lamination consists of individual

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pieces of sawn lumber joined end-to-end. This type of composite construction allows glulam beams to be configured so that higher grades of lumber can be used where the highest stresses occur. Some major advantages of glulam beams are that these engineered beams have higher strength and stiffness properties than solid timber beams and that they can be fabricated in almost any length, size, or structural shape. In addition, glulam beams exhibit less variability in strength and stiffness compared to solid-sawn timber. As a result, the use of these larger members as main, load-carrying structural components has increased the need to quantify their reliability.

One method of determining the strength properties of glulam timber is to destructively test enough beams to characterize the strength distribution accurately. However, physically testing enough beams to adequately define the strength distribution for each size and grade configuration is practically and economically infeasible. Research is needed to develop an accurate mathematical model to simulate glulam beam performance. Results of the simulated beam tests can be used to determine the strength and stiffness distributions needed for reliability-based design. A glulam beam model is also needed to develop specifications for new glulam combinations and to aid in planning future laboratory tests of promising glulam products.

BACKGROUND

Glulam beam models

There have been many attempts to model the performance of glulam beams. Several different approaches have been taken, ranging from early empirical techniques to more sophisticated stochastic methods. In recent years, more emphasis has been placed on the modeling of material properties.

Empirical I_k/I_g models.—The first efforts to predict glulam beam performance were made by Wilson and Cottingham (1947) and Freas and Selbo (1954), who used an empirical method, referred to as the I_k/I_g method. This method accounts for the strength-reducing influence of knots as a function of their moments of inertia. The term I_k is defined as the moment of inertia of all knots within 6 in. (15 cm) of the critical cross section, and I_g is the gross moment of inertia. The I_k/I_g method determines allowable design stresses in bending for glulam members by multiplying bending stress indices (clear wood design stresses) by strength ratios for each lumber grade. Bending strength ratios are a function of the ratio I_k/I_g. The I_k/I_g method is the basis for the current industry standard, ASTM D 3737 (1991). However, statistical distributions of glulam beam strength are not predicted with this method and the influence of end joints cannot be considered.

Finite element models.—Because of the recent shift to reliability-based design, the major focus of glulam research has been to model statistical distributions of beam strength accurately. Foschi and Barrett (1980) were among the first researchers to model the performance of glulam beams using a stochastic method. In their model, the laminations of the glulam beams were divided into elements, or cells. Their input consisted of generating clear wood densities and knot sizes and assigning them to each cell. Each cell was subsequently assigned a lumber modulus of elasticity (MOE) and tensile strength value that was correlated to the assigned density and knot size. The effect of end joints was not considered in this model because of the limited amount of data available on end joint strength. A finite element approach was then used to analyze the strength and stiffness of each beam. The probability distributions of beam strength and stiffness were characterized using Monte Carlo simulation.

Several models have stemmed from the original Foschi and Barrett (1980) model. Ehlbeck et al. (1985a, b, c) developed a glulam model similar to the Foschi and Barrett model, referred to as the "Karlsruhe calculation model." The two major improvements from the Foschi and Barrett model were the inclusion of end joint effects and the ability to simulate progressive failures. The properties of end joints were simulated using a regression approach that generated tensile strength of the joints as a function of the lower density of the two jointed boards. Progressive failures were simulated by checking if the remaining adjacent cells, after the first failure, were able to support the redistributed stresses. Colling (1990a, b, c) conducted several sensitivity analyses with the Karlsruhe calculation model to study size effects in glulam and to verify the simulation results through testing.

Another model that stemmed from the original of Foschi and Barrett was one by Govindarajoo (1989). Govindarajoo included a stochastic lumber properties model developed by Kline et al. (1986) to simulate correlated values of localized MOE within a piece of lumber. Regression models were added to simulate clear wood strength from the generated clear wood MOE values, and models developed by Burk and Bender (1989) were used to simulate end joint stiffness and strength from the localized MOE values of the two jointed boards. In addition to the finite element approach, Govindarajoo implemented a transformed section method to analyze the same beams. Results from this study indicated that the transformed section method of analysis provides similar results to those of the finite element method.

Transformed section models.—Another approach taken to analyze glulam beams was the transformed section method. This method transforms the composite glulam cross section so simple elastic formulas can be applied. Studies of glulam beams with lower quality inner laminations (Moody 1974, 1977) led to the incorporation of the transformed section method into the ASTM Standard D 3737 (1991). Brown and Suddarth (1977) developed a transformed section model to aid in the design of glulam beams. The input consisted of beam geometry and configuration as well as allowable fiber stresses for each lamination. A transformed section analysis was performed to calculate the allowable load-bearing capacity of the beam. An option in this model allowed the user to enter deterministic values of MOE or to randomly generate these MOE values.

Bender et al. (1985) developed a model based on generating actual lumber properties rather than clear wood properties. These distributions were obtained by fitting probability density functions to actual long-span lumber MOE and using a regression approach to simulate lumber tensile strength. The long-span lumber tensile strength values were adjusted for length by using an independent weakest-link approach. End joint strengths were modeled using test data collected during qualification of laminating plants by the American Institute of Timber Construction (AITC). This model is appealing because it used actual lumber properties to characterize the bending strength of glulam beams.

Richburg (1988) refined the Bender et al. (1985) model in a pilot study to observe the effects of spatial correlation between localized lumber properties. Spatially correlated localized lumber properties were simulated using a model developed by Taylor (1988). Models developed by Burk (1988) were used to simulate end joint properties as functions of the localized constituent lumber properties.

Models of localized lumber properties

Simulation of localized lumber properties.—At a given glulam beam cross section, the properties of each lamination are more accurately represented by short-span, or localized, lumber properties rather than long-span properties. Recent research has been targeted at characterizing localized lumber properties for such uses as glulam beam modeling. Kline et al. (1986) developed a model to simulate lengthwise variability in MOE of lumber. This model uses a second-order autoregressive, or Markov, model to generate serially correlated MOE values along 30-in. (76-cm) segments for a piece of lumber. Showalter et al. (1987) extended this research by adding a regression model to simulate correlated values of tensile strength in addition to MOE.

Taylor and Bender (1991) developed a model to simulate spatially correlated localized MOE and tensile strength (T) values for visually graded Douglas-fir laminating lumber. This method uses a transformation of the multivariate normal distribution to generate spatially correlated lumber MOE and T values along 24-in. (61-cm) segments. The advantage of the multivariate approach is that it exactly preserves the probability distributions of MOE and T, and closely approximates the spatial correlations. Richburg and Bender (in press) used a similar approach to characterize spatial variability of MOE and T for E-rated Douglas-fir laminating lumber.

Simulation of end-joint properties.—Glulam beam test results have shown that failures frequently initiate at end joints. For this reason, it is important to simulate the strength and stiffness properties of these joints accurately.

Bender et al. (1985) and Ehlbeck et al. (1985b) incorporated the effect of end joints into their glulam models. Bender et al. assigned the end-joint MOE values by averaging the MOE of the two boards on both sides of the joint. Because of limitations in available test data, it was not possible to generate correlated values of endjoint tensile strength. Therefore, Bender et al. simulated independent values of tensile strength from fitted distributions of actual end-joint test results. Ehlbeck et al. used individual tension tests of end-jointed lumber to develop a regression approach to simulate the joint properties as functions of the lower density of the two jointed boards.

Burk and Bender (1989) used a regression approach to relate the end-joint MOE to the MOE values of the two 2-ft (61-cm) lumber segments on each side of the joint. Another regression model was developed to relate the simulated end-joint T to the end-joint MOE. This method allows for the end-joint MOE and T properties to be generated as functions of the laminating lumber MOE. Hooper and Bender (1988) developed a similar regression approach for E-rated Douglas-fir lumber. They employed a recursive transformation to preserve the highly skewed statistical distributions of the end-joint MOE and T within each E-rated lumber grade.

Beam performance factors

The wood industry has long recognized that the bending strength of timber members decreases as beam size increases. This phenomenon can be explained by the increased probability of occurrence of strength-reducing characteristics, such as knots and slope-of-grain, as wood volume increases. Bohannon (1966) developed a "size effect" equation to account for this stress reduction:

$$C_f = \left(\frac{12}{d}\right)^{\frac{1}{9}} \tag{1}$$

where C_f is the size effect factor and d is beam depth in inches. Although this factor is only a function of depth, the effect of length is also included because the length-to-depth (L/d) ratios were fixed during testing.

Recently, Moody et al. (1988) developed a "volume effect" equation that explicitly considers beam depth, length, and width. This equation, which was based on a large sample of actual glulam beam tests, has the form:

$$C_v = k\left(\frac{12}{d}\right)^{\frac{1}{x}}\left(\frac{21}{L}\right)^{\frac{1}{y}}\left(\frac{5.125}{b}\right)^{\frac{1}{z}} \tag{2}$$

where:

- C_v is the volume effect factor,
- d is beam depth (in.),
- b is beam width (in.),
- L is beam length (ft),
- k is a form factor (1.0 for rectangular) and
- x, y, z are exponents for depth, length and width, respectively.

The volume effect equation was incorporated into ASTM Standard D 3737 (1991) with exponents equal to $x = y = 10$ and $z = 9$. The ASTM section working on D 3737 has recently adopted $x = y = z = 10$ for simplification purposes. AITC, on the other hand, has adopted the $x = y = z = 10$ exponents for all species groupings except southern pine. The parameters adopted for southern pine are $x = y = z = 20$.

OBJECTIVES

The objectives of this research were to refine an existing glulam beam model (Bender et al. 1985) and to validate the refined model using actual glulam beam test data. The refined glulam beam model is referred to as PROLAM throughout this manuscript.

DEVELOPMENT AND REFINEMENT OF PROLAM

PROLAM development

PROLAM simulates the assembly of glulam beams and properties of the laminating lumber and end joints, and it analyzes the load-carrying capacities of the simulated beams. Stochastic models are used to simulate the following random variables: 1) length of constituent lumber, 2) MOE and tensile strength of 2-ft (61-cm) lumber segments, and 3) MOE and tensile strength of end joints.

Lumber length.—The first step in the simulation process is to generate individual lengths of lumber. In glulam manufacturing, lumber length is influenced by the available laminating stock and trimming of defects and ends. Since these procedures vary between manufacturers, the PROLAM model requires the user to specify lumber length input parameters. Within PROLAM, the triangular distribution is used to simulate lumber length. Hence, the user only needs to specify the range of lumber length and the mode, which corresponds to the most likely occurring length.

Beam assembly.—PROLAM simulates the glulam beam assembly similar to the way beams are manufactured in industry. This procedure is important because it determines the end-joint locations within each beam. First, PROLAM generates a random length of lumber. PROLAM then tests if the length of the lamination is longer than the desired length of the beam. If not, the length of the lumber and the location of the end joint are stored for future use, and the next length of lumber is generated and placed end-to-end with the previous piece. If the total length of the lamination is longer than the desired length of the beam, the remaining piece is wrapped around to begin the next lamination. This procedure is repeated until the entire beam has been assembled.

Lumber MOE and tensile strength.—Once the beam is fully assembled, lumber and end-joint properties are generated for each corresponding component. The lumber properties are generated using the model developed by Taylor and Bender (1991), which simulates 2-ft (61-cm) MOE and T values using a modified multivariate normal approach. The multivariate approach treats each 2-ft (61-cm) MOE and T value along a piece of lumber as a spatially correlated random variable. Details on the lumber simulation algorithm are given by Taylor and Bender (1991). This approach preserves the original marginal distributions of each random variable as well as the correlations of the variables.

End-joint MOE and tensile strength.—The end-joint models developed by Burk and Bender (1989) are used to simulate the end-joint MOE and T properties. End-joint MOE is simulated using a regression approach that relates the end-joint MOE to the MOE values of the two 2-ft (61-cm) lumber segments on each side of the joint. A second regression model was developed to relate end-joint T to the simulated end-joint MOE. This method allows for the end-joint MOE and T to be generated as functions of the 2-ft (61-cm) lumber MOE values on each side of the joint. This method has been applied successfully for distributions that are fairly symmetric. For highly skewed distributions, the regression approach may not preserve the original marginal distributions. In this case, the recursive transformation method described by Hooper and Bender (1988) can be used.

Transformed section analysis.—PROLAM uses a transformed section method to analyze the assembled beam. This approach accurately predicts stresses in nonhomogeneous laminated beams without the rigorous computations required by other methods, such as the finite element method. The transformed section method analyzes a glulam beam by transforming the widths of each lamination in the composite cross section so simple elastic flexural formulas can be applied.

Ultimate moment-carrying capacity.—Once the cross section of the beam has been transformed, the ultimate moment-carrying capacity of the cross section is calculated. Because the majority of failures occur in the tension zone of the beam, the following relationships are used to calculate the ultimate moment-carrying capacity. The stresses for beam bending are represented by the equation

$$\sigma_b = \frac{Mc}{I} \tag{3}$$

where σ_b is the fiber bending stress at a distance c from the neutral axis, M is the moment at the location of the cross section, c is the distance from the neutral axis to the location of the fiber stress, and I is the gross moment of inertia of the cross section.

The ultimate moment-carrying capacity at each cross section is calculated by equating the short-span tensile strength of the lamination to the fiber bending stress as follows:

$$M_{ult} = \frac{\sigma_t I}{c} \tag{4}$$

where M_{ult} is the ultimate moment-carrying capacity and σ_t is the 2-ft (61-cm) tensile strength of the lamination, or end-joint.

The ultimate moment is calculated at the mid-depth of each lamination in the tension zone, and the ultimate moment-carrying capacity of the cross section is defined by the weakest lamination. This procedure is repeated along the entire beam length. The ultimate moment is then adjusted according to the moment diagram corresponding to the loading condition of the glulam beam. Finally, the minimum moment obtained from all of the analyses along the length of the beam defines the ultimate moment-carrying capacity of the entire glulam beam.

Apparent modulus of rupture.—Once the ultimate moment-carrying capacity of the glulam beam is determined, the apparent modulus of rupture (MOR) of the beam is calculated using the equation:

$$MOR = \frac{M_{ult}}{S} \tag{5}$$

where S is the gross section modulus.

The MOR calculated using this procedure determines the initial failure. Since glulam beams may have several internal failures of laminations or end joints before the entire glulam system fails, it is necessary to simulate the propagation of failures that occur within a beam. This feature is discussed in more detail in a later section.

PROLAM refinements

Calculation of beam stiffness.—Calculation of glulam beam stiffness is complicated by two factors: (1) MOE varies along the beam length and (2) deflection caused by shear may be significant, especially for beams with low span-to-depth ratios. The approach taken here was to use a complementary virtual work method applied by Hilson et al. (1988) to estimate glulam beam deflection. This method takes advantage of the incremental analyses performed along the length of the beam during the transformed section analysis. The complementary virtual work calculation method follows:

$$\Delta = \sum_{i=1}^{n} \left[\left(\frac{M_x m_x}{E_c I_i} + \frac{k V_x v_x}{A_i G_i} \right) \times \Delta_x \right] \tag{6}$$

where

- Δ is the total glulam beam deflection at x,
- M_x is the bending moment at x caused by actual loading,
- m_x is the bending moment at x caused by a unit load at the midspan of the beam,
- V_x is the shear at x caused by actual loading,
- v_x is the shear at x caused by a unit load at the midspan of the beam,
- k is a form factor (1.2 for rectangular section),
- E_c is a constant MOE value used in the transformed cross section,
- I_i is the moment of inertia of the i^{th} transformed cross section at x,

A_i is the transformed area at the i[th] transformed cross section at x,

G_i is the shear modulus of the i[th] transformed cross section at x,

Δ_x is the increment at which calculations are performed and

n is the total number of increments along the beam length.

Using the midspan beam deflection calculated with this method, an apparent beam MOE is then estimated using the usual elastic deflection formulas. The apparent beam MOE takes into account the effects of shear and is more closely related to published design values of MOE.

Simulation of progressive failures.—Several localized lamination or end-joint failures can occur before the entire glulam beam fails. Progressive failures are modeled in PROLAM by reanalyzing the critical cross section after the initial failure has occurred. The critical element of the cross section, whether a lamination or end joint, is assigned an MOE value of zero. A transformed section analysis is repeated at this cross section and a new ultimate moment is calculated. If this new moment value is greater than the previous value, then the beam is still capable of carrying more load and the next failure will occur at the cross section possessing the next lowest ultimate moment value. When the newly calculated ultimate moment is lower than all of the existing ultimate moments, then the final ultimate moment-carrying capacity of the glulam beam has been reached.

Handling of different loading conditions.—The loading condition is important when calculating the ultimate moment-carrying capacity of a glulam beam. The model developed by Bender et al. (1985) and refined by Richburg (1988) analyzes glulam beams under a symmetric two-point loading condition. Although this condition is common for laboratory testing of glulam beams, actual glulam beams used in residential and commercial buildings commonly experience a uniform loading condition. Therefore, an option to choose from both the symmetric two-point load and the uniformly distributed load was incorporated into PROLAM for its future development as a design and research tool. Choosing a uniformly distributed loading condition affects the calculations of ultimate moment-carrying capacity and beam deflection.

Calculation of summary statistics.—A subroutine was added to PROLAM to calculate summary statistics of the simulated glulam beam results. The summary statistics include average, coefficient of variation (COV), and 5th percentile of beam MOR, along with average and COV of apparent beam MOE. The percentage of lumber and end-joint failures occurring in each lamination is also calculated.

Other modifications.—Several modifications were made to PROLAM to improve the efficiency of the code and to make the program executable on a microcomputer. The microcomputer version of PROLAM is limited to approximately 3,500 Monte Carlo simulations. These limitations are a function of the memory capabilities of each individual computer. For computation time, 1,000 Monte Carlo simulations of an eight-lamination 21-ft (6.4-m) beam take approximately 3 minutes to execute on an IBM-compatible 33-Mhz 486 microcomputer. The computation time increases for larger beams. In addition, a user-friendly program was developed to assist the user in developing beam combinations, executing the simulation runs, calculating summary statistics on the simulation results and generating hard-copies of relative frequency histograms of beam MOR and MOE. Additional details on model modifications can be found in Hernandez (1991).

MODEL VALIDATION

Description of glulam beam test data

The AITC has undertaken an extensive glulam beam testing program for a variety of purposes, including validation of glulam beam models. As part of this program, lumber and end joints are sampled at the time of beam fabrication. Prior to beam fabrication, the

TABLE 1. *Probability distribution parameters for lumber tensile strength and MOE.*

Lumber grade	Distribution type	Distribution parameters		
		Location	Scale	Shape
Tensile Strength (10^3 psi)				
302-24	3-P Lognormal	0.5547	2.2629	0.4154
L1	3-P Lognormal	0.3142	2.0681	0.4167
L2D	3-P Lognormal	0.7647	1.8979	0.4685
L2	3-P Lognormal	0.5936	1.6331	0.4362
L3	3-P Lognormal	0.5503	1.4891	0.4274
Modulus of Elasticity (10^6 psi)				
302-24	3-P Weibull	1.5427	1.6038	3.7262
L1	3-P Weibull	1.3269	1.5339	3.5527
L2D	2-P Lognormal	0.0000	0.9387	0.1767
L2	3-P Weibull	1.0401	1.3062	3.1973
L3	2-P Lognormal	0.0000	0.6858	0.2018

lumber used in the beams is run through a continuous stress grading machine to obtain MOE profiles. Each piece of lumber is stamped with an identification number so it can be located within the beams after fabrication.

At the time the PROLAM research was being conducted, a group of thirty 16-lamination 24F-V4 Douglas-fir glulam beams (along with lumber and end joints) were tested. The 24F-V4 beam combination is comprised of visually graded lumber and has a design bending stress of 2,400 psi (16.5 MPa) (AITC 1987, 1988). The 24-in (61-cm) deep beams were manufactured to a total beam length of 40-ft (12 m) using nominal 2-in by 6-in (5-cm by 15-cm) Douglas-fir laminating lumber. Final beam width after planing was 5.125 in. (13 cm). After fabrication, the beams were tested to failure at a span of 38 ft (11.7 m) using a symmetric two-point loading configuration with the span between load points equal to 8 ft (2.5 m).

TABLE 2. *Serial and cross-correlations for lumber tensile strength (T) and MOE.*

Correlation	302-24	L1	L2D	L2	L3
Serial MOE					
lag-0	1.0000	1.0000	1.0000	1.0000	1.0000
lag-1	0.9581	0.9196	0.9196	0.7337	0.6781
lag-2	0.9277	0.8752	0.8752	0.6755	0.6949
lag-3	0.8902	0.8323	0.8323	0.6399	0.7078
lag-4	0.8559	0.7905	0.7905	0.5648	0.6083
lag-5	0.8220	0.7510	0.7510	0.5124	0.5942
lag-6	0.7897	0.7135	0.7135	0.4664	0.5646
lag-7	0.7586	0.6778	0.6778	0.4211	0.5200
lag-8	0.7288	0.6439	0.6439	0.3814	0.4956
lag-9	0.7001	0.6118	0.6118	0.3455	0.4662
lag-10	0.6725	0.5812	0.5812	0.3128	0.4368
lag-11	0.6460	0.5521	0.5521	0.2832	0.4127
lag-12	0.6206	0.5245	0.5245	0.2565	0.3881
Cross MOE-T					
lag-0	0.4015	0.5894	0.5894	0.3115	0.4844
lag-1	0.3847	0.5420	0.5420	0.2285	0.3285
lag-2	0.3724	0.5159	0.5159	0.2104	0.3366
lag-3	0.3574	0.4906	0.4906	0.1993	0.2633
lag-4	0.3436	0.4659	0.4659	0.1759	0.2946
lag-5	0.3300	0.4427	0.4427	0.1596	0.2878
lag-6	0.3171	0.4206	0.4206	0.1453	0.2735
lag-7	0.3046	0.3995	0.3995	0.1312	0.2519
lag-8	0.2926	0.3796	0.3796	0.1188	0.2401
lag-9	0.2811	0.3606	0.3606	0.1076	0.2258
lag-10	0.2700	0.3426	0.3426	0.0974	0.2116
lag-11	0.2594	0.3255	0.3255	0.0882	0.1999
lag-12	0.2492	0.3092	0.3092	0.0799	0.1880
Serial T					
lag-0	1.0000	1.0000	1.0000	1.0000	1.0000
lag-1	0.8008	0.8303	0.8303	0.7651	0.6410
lag-2	0.6413	0.6894	0.6894	0.5853	0.4108
lag-3	0.5136	0.5724	0.5724	0.4478	0.3428
lag-4	0.4113	0.4753	0.4753	0.3426	0.1688
lag-5	0.3294	0.3946	0.3946	0.2621	0.1082
lag-6	0.2638	0.3276	0.3276	0.2005	0.0693
lag-7	0.2113	0.2720	0.2720	0.1534	0.0444
lag-8	0.1692	0.2259	0.2259	0.1174	0.0285
lag-9	0.1355	0.1875	0.1875	0.0898	0.0183
lag-10	0.1085	0.1557	0.1557	0.0687	0.0117
lag-11	0.0869	0.1293	0.1293	0.0526	0.0075
lag-12	0.0696	0.1073	0.1073	0.0402	0.0048

Estimation of input lumber properties

It would be desirable to obtain PROLAM inputs from the same lumber used to fabricate the validation beams. In this way, the confounding effects of lumber sampling error would be minimized. However, it was impossible to obtain all input data needed from the lumber since destructive tests are required. As previously mentioned, MOE profiles were nondestructively measured for each piece of lumber used in the validation beams. A regression procedure was used to estimate 2-ft static bending MOE and 2-ft tensile strength from the MOE profiles. Parameters for the regression procedure were estimated in a pre-

TABLE 3. *Regression parameters for generating end joint MOE (10^6 psi) and T (10^3 psi).*

Grade	Regression parameters*					Resid. Std. Dev.	
	b_0	b_1	b_2	b_3	b_4	e_1	e_2
302-24	0.06748	0.5871	0.3851	1.8600	1.593	0.1190	1.000
L1	0.06748	0.5871	0.3851	1.2700	1.593	0.1190	0.800
L2D	0.06748	0.5871	0.3851	1.2700	1.593	0.1190	0.800
L2	0.06748	0.5871	0.3851	1.0800	1.593	0.1190	1.000
L3	0.06748	0.5871	0.3851	0.3100	1.593	0.1190	0.600

* $MOE_J = b_0 + b_1 MOE_{LO} + b_2 MOE_{HI} + z_1 e_1$; $T_J = b_3 + b_4 MOE_J + z_2 e_2$; where z_1, z_2 are independent standard normal deviates.

vious study on E-rated Douglas-fir laminating lumber (Richburg et al. 1991). Probability distributions were fit to the estimated 2-ft MOE and T data, and the distribution parameters are listed in Table 1.

Correlations between localized lumber properties were assumed to follow those reported by Taylor and Bender (1991) for 302-24 and L1 grades, and Richburg and Bender (in press) for L2 and L3 grades. Correlations for the L2D grade were assumed to be equal to the L1 grade. The assumed serial and cross-correlations for localized lumber tensile strength and MOE are given in Table 2.

Estimation of input end joint properties

The approach developed by Burk and Bender (1989) was used to simulate MOE and T of the end joints. Tests were conducted to determine if the regression parameters reported by Burk and Bender would result in simulated end-joint tensile strengths similar to those sampled during beam fabrication. It was observed that the simulated end joint tensile strengths were much greater than the actual strengths, suggesting that Burk and Bender's regression parameters are dependent on the manufacturer and that a different set of parameters should be used to simulate the joint strength for the validation beams.

In an effort to simulate end joints more representative of the AITC end joints, the original end-joint data from the Burk and Bender (1989) and Hooper and Bender (1988) studies were combined, and a single set of regression parameters were determined. Finally, the regression parameters of the end-joint tensile strength equation were manually adjusted until the simulated values were in agreement with the actual values of the end joints tested by AITC. The final regression parameters used to simulated end joint tensile strength and MOE in PROLAM are summarized in Table 3.

The lengths of each piece of lumber in the 30 validation beams were measured and the minimums, maximums, and modes are summarized in Table 4. These values were used to simulate lumber lengths in the PROLAM validation results. Lumber length is important because it determines the number and location of end joints.

PROLAM validation results

One-thousand 16-lamination 24F-V4 beams were simulated using the input data given in Tables 1 through 4. Previous research indicated a sample size of 1,000 gives stable statistical estimates of beam MOR and MOE (Govindarajoo 1989; Hernandez 1991). Simulated and actual beam results were compared using confidence interval and goodness-of-fit statistical approaches.

Beam strength predictions.—Empirical cumulative distribution functions (CDFs) were constructed for the simulated and actual beam MOR data (Fig. 1). The simulated CDF is much

TABLE 4. *Distribution parameters for lumber length.*

Lumber grade	Lumber length parameters (ft)		
	Minimum	Mode	Maximum
302-24	8.2	13.4	16.1
L1	12.1	15.6	22.5
L2D	8.3	15.2	20.0
L2	9.0	15.2	20.1
L3	6.5	15.1	20.4

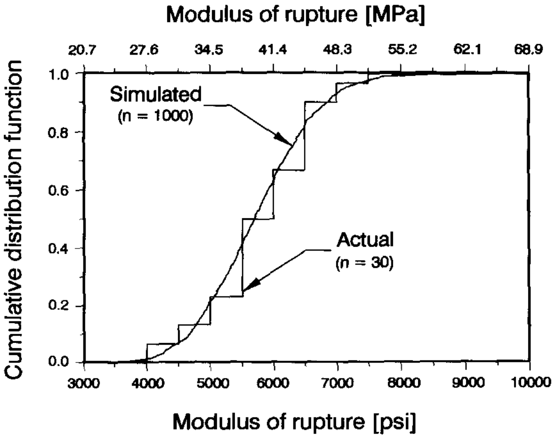

FIG. 1. Simulated and actual cumulative distribution functions of beam modulus of rupture.

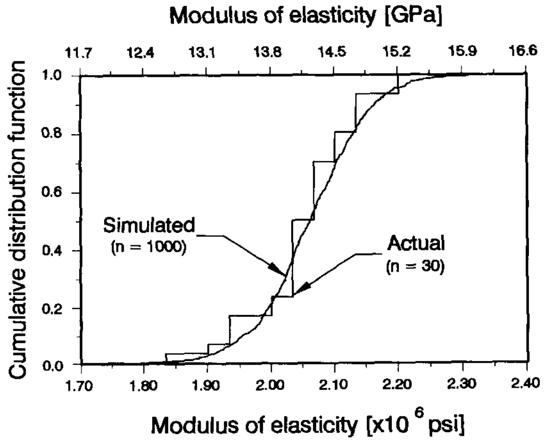

FIG. 2. Simulated and actual cumulative distribution functions of beam modulus of elasticity.

smoother than the actual CDF due to differences in sample sizes. The two CDFs shown in Fig. 1 are in excellent agreement. To supplement this visual appraisal, the Kolmogorov-Smirnov two-sample test was performed to test whether the simulated and actual CDFs were significantly different (D'Agostino and Stephens 1986). The K-S test failed to detect any significant difference between the two CDFs at a significance level of 20% (higher significance levels make it easier to detect differences).

As an additional check of the PROLAM model, 10 batches of 30 beams were simulated so confidence intervals could be constructed on beam MOR predictions (a batch size of 30 was chosen to match the sample size of actual beams). Summary statistics were calculated for both the actual and simulated data using the Lognormal distribution assumption, as recommended by ASTM Standard D 3737 (1991). To provide another benchmark for comparison, the ASTM D 3737 I_k/I_g method was used to predict the adjusted 5th percentile of beam MOR based on long-span MOE and knot properties measured on the lumber used to fabricate the 30 validation beams. These properties are summarized in Table 5, along with bending stress indices (clear wood design stresses) and strength ratios as specified in AITC 500 (1991).

TABLE 5. *Parmeters for I_k/I_g analysis.*

Lumber grade	Long-span MOE (10^6 psi)	Average knot size (% of area)	Maximum knot size (% of area)	Minimum strength ratio	Bending stress indices (psi)
L1	2.43	9.2	36.5	0.75	3,500
L2D	2.35	11.6	46.7	0.67	3,500
L2	1.98	11.4	54.0	0.67	3,000
L3	1.81	13.4	56.0	0.50	3,000

Actual and predicted beam MOR statistics (using PROLAM and I_k/I_g) are summarized in Table 6. The PROLAM predictions are in good agreement with the I_k/I_g prediction and actual beam results. For all three output statistics, confidence intervals from the simulated data covered the actual values. On the average, PROLAM under-predicted beam MOR by less than five percent.

Beam MOE predictions.—Predicted and actual beam MOE statistics are summarized in Table 7. The average beam MOE predicted using I_k/I_g was in excellent agreement with the value from the validation beams; however, the PROLAM prediction was 14.5% higher. The standard deviation of beam MOE predicted with PROLAM was in excellent agreement with the actual value.

PROLAM differs from the I_k/I_g method in that shear deflection is explicitly included in the beam MOE prediction. The I_k/I_g method uses long-span lumber MOE values as input,

TABLE 6. *Comparison of predicted and actual beam modulus of rupture (MOR).*

Beam group	Adjusted 5th percentile of MOR (psi)*		Mean MOR (psi)		Standard deviation of MOR (psi)	
	Average	95% conf. interval	Average	95% conf. interval	Average	95% conf. interval
Simulated	2,329	(1,988, 2,669)	5,748	(5,447, 6,049)	867	(558, 1,176)
I_k/I_g	2,395	**	**	**	**	**
Actual	2,440	***	6,045	***	920	***

* (5th percentile at 75% confidence) ÷ (2.1 × 0.879), where 2.1 is an adjustment factor to account for duration of load and end-use and 0.879 is the Moody et al. (1988) volume-effect adjustment for this beam size. Statistics were estimated assuming the Lognormal distribution.
** Not predicted by I_k/I_g method.
*** Insufficient data to calculate.

and then the final beam MOE prediction is multiplied by 0.95 to account for shear deflection. PROLAM is based on short-span, flexural MOE (with no shear effects). Then, shear deflection is predicted within PROLAM using a virtual work scheme based on the assumption of a shear modulus-to-MOE ratio of 1:16 (U.S. Forest Products Laboratory 1987). Recent research indicates localized values of shear modulus are highly variable, and can be much lower (with respect to MOE) than previously reported (Chui 1991). The shear modulus assumption may partially explain why PROLAM over-predicts beam MOE. More research is needed to test this hypothesis.

Up to this point, no calibration factors were used in the PROLAM model. However, it appears that a calibration factor of 0.87 is needed to adjust PROLAM predictions of beam MOE. Fig. 2 shows predicted and actual CDFs for beam MOE after the calibration factor is applied. The two CDFs are in excellent agreement, and no significant differences could be detected using the K-S two-sample test for a significance level of 40%.

SUMMARY AND CONCLUSIONS

A simulation model was developed to predict the probability distributions of glulam beam strength and stiffness. This model, referred to as PROLAM, differs from the current industry model (ASTM D 3737) in that PROLAM predicts entire statistical distributions rather than point estimates of beam strength and stiffness. Distributions are needed for reliability-based design. Additionally, PROLAM is based on actual lumber properties, rather than previously required clear wood stresses and knot information. Using actual lumber properties is desirable because input databases could be updated as part of the quality control programs already in place.

A group of thirty 16-lamination 24F-V4 Douglas-fir glulam beams were tested for the purpose of validating PROLAM. Predicted and actual beam strength and MOE were compared using confidence interval and goodness-of-fit statistical methods. PROLAM predicted beam strength with excellent accuracy, but over-predicted beam MOE by 14.5%. After PROLAM beam MOE predictions were calibrated using a factor of 0.87, the predicted distribution of MOE conformed closely to that of the validation beams. One possible source of error in the beam MOE prediction is the way shear deflection is modeled in PROLAM. Research is needed to test the robustness of the beam MOE calibration factor, and to further validate

TABLE 7. *Comparison of predicted and actual beam modulus of elasticity (MOE).*

Beam group	Mean MOE (psi)		Standard deviation of MOR (psi)	
	Average	95% conf. interval	Average	95% conf. interval
Simulated	2.362	(2.334, 2.391)	0.087	(0.054, 0.120)
I_k/I_g	2.086	*	*	*
Actual	2.063	**	0.081	**

* Not predicted by I_k/I_g method.
** Insufficient data to calculate.

PROLAM for other beam configurations, sizes and species groupings.

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