

EFFECT OF JUVENILE WOOD AND CHOICE OF PARAMETRIC PROPERTY DISTRIBUTIONS ON RELIABILITY-BASED BEAM DESIGN

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ABSTRACT

A comparison is made of the effect of choice of the S_b distribution, Weibull distribution, or log-normal distribution on reliability-based design of a 2×4 southern pine beam, No. 2 grade. The S_b distribution provided most flexibility in describing the lumber properties.

The presence of juvenile wood in lumber may affect the distributional characterization of lumber properties and in turn affect reliability-based design results. This study shows that juvenile wood had a significant effect on the reliability-based design results when stiffness was the limiting state. Unless juvenile wood lumber is separated from mature wood lumber in the grading process, a considerable loss in efficiency in utilizing lumber from fast-grown trees will occur where stiffness is critical.

Keywords: Reliability-based design, S_b distribution, Weibull distribution, lognormal distribution, juvenile wood, mechanical properties, beam design.

INTRODUCTION

Engineering design attempts to achieve the optimum balance of safety and economy. Design procedures have to allow for variation in service conditions, materials, workmanship, and other contingencies that cannot be predicted accurately. The traditional design procedure applies a factor of safety to cover these uncertainties, i.e., the computed stresses are lowered by a deterministic amount to provide a reserve of strength in the structure to enable it to sustain likely overloads. Reliability-based design has been developed from statistical estimates of these uncertainties and uses the probability of failure as a measure of risk. The advantage of reliability design is that it provides a more rational, logically consistent, and reliable design procedure.

If the stress due to applied loads, S , and the

available strength of a structure, R , are assumed to be independent in a given case, the probability of failure, P_f , can be defined as (Thoft-Christensen and Baker 1982)

$$P_f = P(f \leq 0) = \int_{-\infty}^{+\infty} f_s(X) \cdot f_R(X) \cdot dx \quad (1)$$

where $f = R - S$ is called a failure function, and $f_s(X)$ and $f_R(X)$ are the probability density functions of S and R , respectively. The reliability of the structure, $P_r = 1 - P_f$, is commonly measured by a so-called reliability index β in order to keep the design procedure simple, compact, and consistent. In the case of a linear failure function and standard normal basic variables, the relation between P_f and β can be determined by

$$P_f = \Phi(-\beta) \quad (2)$$

where Φ is the standard normal probability distribution function. The larger the value of β , the lower is the probability of failure of the structure.

The reliability of the structure depends on the characteristics of the distributions of both the stress variable S and the strength variable R . However, there is no general agreement so far on the type of distribution that gives the best fit to the material properties. In load and resistance factor design for steel and concrete construction, a normal distribution is commonly adopted for the strength properties (Ravindra et al. 1978; Mirza et al. 1979), and lognormal and Beta distributions have been used for some reinforced concretes (MacGregor et al. 1983). Foschi et al. (1989) described mechanical properties of lumber data in terms of lognormal and Weibull distributions and used a 2-parameter Weibull distribution to develop a reliability-based design code. Pearson (1980) discussed the applicability of the 4-parameter S_B distribution to describe mechanical properties of lumber data. Pellicane (1984) used the S_B distribution to correlate the distribution of a destructively evaluated parameter (MOR) from a nondestructively evaluated parameter (MOE). However, the applicability of the S_B distribution in reliability design has not been studied.

The changing lumber resource due to trees being grown faster has caused the wood industry great concern about lumber quality and end use. The presence of juvenile wood in lumber may affect distributional characterization of lumber properties, and in turn affect reliability-based design results. A significant difference between the mechanical properties of defect-free juvenile and mature wood has been well documented (Bendtsen 1978). Possible differences between the distributions of juvenile and mature wood lumber and their effect on reliability-based design have not been investigated yet. Understanding the influence of juvenile wood on design results may lead to further improvement in the structural lumber grading system. The purpose of this paper is:

(1) to compare the lognormal, Weibull and

S_B distributions for fitting the bending properties of both juvenile and mature wood lumber of No. 2 grade southern pine, and

(2) to study the effect of the different distributions and the presence of juvenile wood in the lumber on reliability-based beam design.

MATERIAL AND TESTS

A total of 120 loblolly pine (*Pinus taeda* L.) trees were selected from a plantation about 25 years old, operated by Federal Paper Company near Boulton, North Carolina. The diameter of the trees at 10 ft from the butt ranged from approximately 8 to 16 in. Two logs from each tree were sawn to produce a diametral plank from which 2×4 boards 8 ft long were sawn adjacent to each other. The boards sawn adjacent to the pith contained mainly juvenile wood and are called "juvenile lumber," and the boards sawn further from the pith contained substantially more mature wood and for simplicity are called "mature lumber" (Fig. 1). The term "combined lumber" is used for the mix of juvenile lumber with mature lumber. Before being dried, the boards were machined to uniform thickness and width, and visually graded by an inspector from the Southern Pine Inspection Bureau.

The green lumber was tested flatwise for modulus of elasticity (MOE) by using the Metriguard Model 1201 Stress Wave Lumber Grading Machine and Metriguard Model 3300E Computer. The MOE test was repeated three times for each board and the average value was used. Of the total of 632 pieces of No. 2 grade lumber tested for MOE, 317 pieces were juvenile lumber and 315 pieces were mature lumber.

Static bending tests were carried out on 360 pieces of the No. 2 grade 2×4 lumber after drying, half the boards being juvenile lumber and half being mature lumber. These 360 pieces of lumber were randomly selected from the total 632 pieces, and the lumber left was used for another study. The boards were tested on edge in third-point static bending over a span of 5 ft. They were oriented so that, as far as

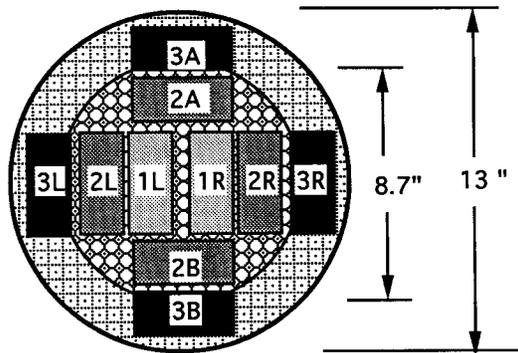


FIG. 1. Cutting plan showing maximum number of 2×4 s for logs of various diameters. Only boards labelled 1L or 1R were called "juvenile."

possible, the apparent worst defect was in the tension zone between the load points. The 6,000-lb range of a 30,000-lb capacity Tinius-Olsen universal testing machine was used, the rate of head movement being 0.2 in./min. The maximum load was recorded. The moisture content of the boards at test was obtained by weighing, oven-drying, and reweighing a small coupon cut from each board near the fracture. Modulus of rupture (MOR) was computed and corrected to 12% moisture content according to ASTM D2915-90 (1990).

COMPARISON OF DISTRIBUTIONS FOR FITTING MECHANICAL PROPERTIES

Lognormal and Weibull distributions have often been used to describe the results of mechanical tests on lumber. The Weibull distribution, which has been increasingly preferred in recent years, has the advantage of being able to represent both negatively and positively skewed data, whereas the lognormal is restricted to positively skewed data. One problem with these two distributions is that their skewness ($\sqrt{\beta_1}$) and kurtosis (β_2) are functionally related so that theoretically they are very limited in the values of skewness and kurtosis that they can accommodate. The S_B distribution is much less restricted in this regard, as shown in Fig. 2 (Johnson 1949a). It arises from a transformation of the data into a normal distribution so that normal distribution theory may be ap-

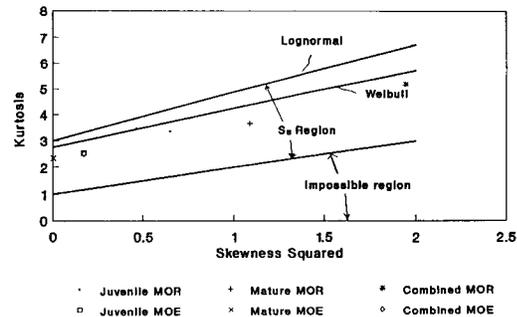


FIG. 2. Space showing where the lognormal, Weibull and S_B distributions are valid (Johnson 1949a). The data points for MOR and MOE were plotted from this study.

plied to the transformed data. The S_B distribution was first described by Johnson (1949a, b) and later used by Schreuder and Hafley (1977) and by Hafley and Schreuder (1977) to fit tree heights and diameters with considerable success. Pearson (1980) applied the S_B distribution successfully to fit the MOR and MOE of mixed grades of 2×8 southern pine lumber. He concluded that a four-parameter, univariate distribution, S_B , has the potential to fit a much wider range of naturally occurring frequency distributions than the lognormal or Weibull. In evaluating goodness-of-fit of 96 data sets of structural lumber, Pellicane (1985) also concluded that the S_B distribution generally provided the best fit to the data among normal, lognormal, Weibull and S_B distributions. The density function of the S_B distribution is

$$f(X) = \frac{\delta}{\sqrt{2\pi}u(1-u)\lambda} \cdot \exp\left[-\frac{1}{2}\left(\gamma + \delta \ln \frac{u}{1-u}\right)^2\right] \quad (3)$$

where δ and γ are shape parameters; $u = (X - \xi)/\lambda$; ξ is a location factor, being the low bound, the λ is the range of the population, and so is the scale factor.

A suitable computer program has been developed to calculate the parameters for the lognormal, Weibull and S_B distributions based

on the maximum likelihood method (Pearson 1980).

Recent literature provides little guidance for conducting goodness-of-fit tests of lumber distributions. The four test-statistics that are used here to compare the goodness-of-fit of the three distributions were proposed by Christensen (1981, 1984). They are: (1) the log likelihood statistic; (2) Pearson's T sum of the normalized squared differences statistic; (3) Kolmogorov-Smirnov's D_n maximum absolute difference test-statistic; and (4) Cramer-Von Mises-Smirnov's W^2 average squared deviation statistic.

The comparison of the goodness-of-fit of the assumed distributions for fitting the test data is given in Table 1 for stress wave MOE (green lumber) and MOR (dry lumber). The rank of the distributions for each test is given in parentheses in Table 1. Except for Pearson's T statistic for mature lumber, the S_B distribution gave the best fit for the MOE of juvenile, mature, and combined lumber. The S_B distribution also gave the best fit to the MOR data of mature and combined lumber. For the MOR data of juvenile lumber, the S_B distribution gave the best fit only for the log-likelihood statistic, the lognormal distribution giving the best fit according to the other three statistics. However, the differences between the S_B and lognormal statistics were very small in this case. Figures 3 and 4 also show that the S_B distribution gave the best fit to the MOE and MOR data at both the lower tail and the whole distribution for the juvenile lumber. A similar goodness-of-fit was observed for the data of the mature and combined lumber. That the S_B distribution would give the best fit is to be expected because the six plotted points in Fig. 2 for skewness and kurtosis of the MOE and MOR data lie well below the lognormal and Weibull lines.

The S_B distribution, which provides the potential for fitting a wide range of test data, appears to have considerable potential for better describing lumber distributions. A more precise description of the population of lumber properties would lead to higher confidence in a reliability design code.

COMPARISON OF DISTRIBUTIONS IN BEAM DESIGN

Both choice of the distribution and the differences between juvenile and mature lumber affect reliability-based design results. In this part of the study, a simply supported beam is used to investigate these influences.

Design for ultimate-strength limit state

Consider an ultimate-strength limit state design of a simply supported beam under a uniformly distributed load. The design equation is (Foschi et al. 1989)

$$\frac{(1.25 \cdot D_n + 1.5 \cdot Q_n)L^2}{8Z} = \varphi \cdot R_0 \quad (4)$$

The failure function is given by

$$f = R - S = R - \frac{(D + Q)L^2}{8Z} \quad (5)$$

Substitution from (4) leads to

$$f = R - \frac{\varphi \cdot R_0 \cdot (\gamma \cdot d + q)}{1.25 \cdot \gamma + 1.5} \quad (6)$$

where

- R = bending strength;
- d = D/D_n ;
- q = Q/Q_n ;
- D = uniformly distributed dead load;
- Q = uniformly distributed live load;
- D_n = nominal dead load;
- Q_n = nominal live load;
- $\gamma = D_n/Q_n$;
- L = beam span, 144 inch;
- Z = section modulus of the beam;
- R_0 = characteristic bending strength, usually the 5th percentile from the distribution of the variable R;
- φ = performance factor applied to R_0 for the population of lumber.

Design for serviceability limit state

A serviceability limit state design is determined by the specified stiffness requirements. Deflection is commonly limited to span/360

TABLE 1. Comparison of goodness-of-fit of distributions of modulus of elasticity and modulus of rupture.*

Statistic:	Modulus of elasticity				Modulus of rupture			
	Log likelihood	Pearson's T	Kol-Sim's D _n	Mises' W ²	Log likelihood	Pearson's T	Kol-Sim's D _n	Mises' W ²
Juvenile: Lognormal	6.656 (2)	11.649 (2)	0.0314 (2)	0.1449 (2)	-290.045 (2)	4.513 (1)	0.0265 (1)	0.0304 (1)
Weibull	2.063 (3)	19.962 (3)	0.0373 (3)	0.1912 (3)	-301.706 (3)	18.529 (3)	0.0861 (3)	0.4105 (3)
S _B	17.301 (1)	3.016 (1)	0.0169 (1)	0.0283 (1)	-288.937 (1)	5.055 (2)	0.0370 (2)	0.0597 (2)
Mature: Lognormal	-100.506 (3)	38.528 (3)	0.0749 (3)	0.4591 (3)	-382.925 (2)	4.375 (2)	0.0410 (2)	0.0887 (2)
Weibull	-85.157 (2)	11.571 (1)	0.0313 (2)	0.0886 (2)	-396.478 (3)	20.601 (3)	0.0833 (3)	0.4284 (3)
S _B	-80.433 (1)	13.233 (2)	0.0241 (1)	0.0395 (1)	-378.893 (1)	2.210 (1)	0.0268 (1)	0.0403 (1)
Combined: Lognormal	-200.700 (2)	30.755 (2)	0.0357 (2)	0.3420 (3)	-692.175 (2)	10.185 (2)	0.0352 (2)	0.1650 (3)
Weibull	-201.691 (3)	38.414 (3)	0.0385 (3)	0.3362 (2)	-733.860 (3)	91.111 (3)	0.0876 (3)	1.2626 (2)
S _B	-178.143 (1)	8.657 (1)	0.0142 (1)	0.0240 (1)	-687.367 (1)	2.825 (1)	0.0138 (1)	0.0425 (1)

* The smaller number in parentheses indicates the better fit, being the ranking of the 3 distributions given by the statistical test. Sample size for MOE: Juvenile, 317; Mature, 315; Combined, 632. Sample size for MOR: Juvenile, 180; Mature, 180; Combined, 360.

under the live load, so we have the maximum deflection given by

$$\Delta_{\max} = \frac{5}{384 \times 12} \cdot \frac{Q_n \cdot L^4}{\varphi \cdot \bar{E} \cdot I} \leq \frac{L}{360} \quad (7)$$

where φ = performance factor applied to the mean modulus of elasticity \bar{E} for the population of lumber; I = moment of inertia.

The following failure function f can be constructed (Foschi et al. 1989):

$$f = 1.0 - \varphi \cdot \frac{\bar{E}}{E} \cdot q \quad (8)$$

where E = random variable MOE of the beam.

In structural design applications, the probability of failure given by Eq. (1) is difficult to calculate because it is usually impossible to derive the exact distributions for stress due to load, $f_s(x)$, and material strength $f_R(x)$. The probability of failure, however, can be estimated by some approximate methods. The Extended Level 2 approximate method with Rackwitz and Fiessler's transformation for nonlinear basic variables, derived by Thoft-Christensen and Baker (1982), is used herein

to determine the relationship between the reliability index β and the performance factor φ for given statistical information on the random variables R , E , d and q in Eqs. (6) and (8). The important difference between φ and β is that φ is based on tradition, experience, and consensus of professional judgment, and β is based on statistical measurement of uncertainties. Conversion of the reliability index to a corresponding performance factor can add clarity and consistency to a design procedure. After the factor φ is determined for a given index β , the required section modulus and the moment of inertia of a beam can be calculated from Eqs. (4) and (7), respectively.

The load variables used here are based on Foschi's study (1989). The residential occupancy load q for the maximum combination of sustained and extraordinary loads for periods of 30 years was fitted with a Type I extreme-value distribution specified as having mean $\mu_q = 0.812$ and coefficient of variation $COV = 0.272$. The nominal live load was $Q_n = 40$ lb/ft (for 1 ft width) and $\gamma = 0.25$. The design dead load d was modelled as a random

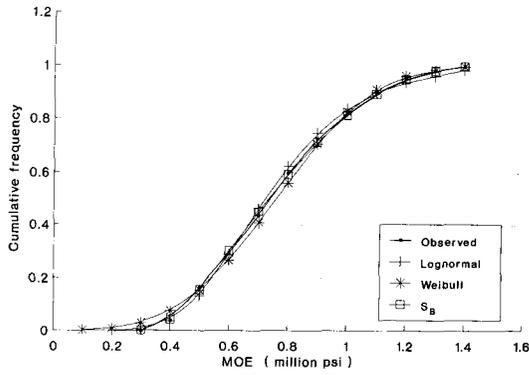


FIG. 3. Comparison of fits of cumulative distributions to MOE of 317 pieces of 2 x 4 southern pine juvenile wood lumber.

normal variable with $\mu_d = 1.0$ and $COV = 0.10$.

A PASCAL computer program was developed to perform the above reliability analyses for the properties of the juvenile, mature and combined lumber.

RESULTS AND DISCUSSION

The effect of the assumed distributions for MOR on ultimate-strength limit state design is represented in Fig. 5 for the juvenile lumber. The curves for the mature and combined lumber were similar and so are not shown. The design results for the juvenile, mature, and combined lumber were not significantly different from each other because the 5th percentile estimates and other values in the lower

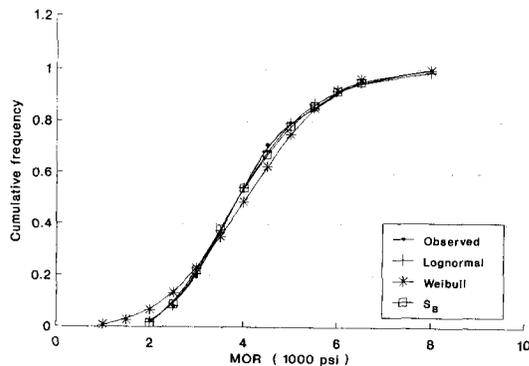


FIG. 4. Comparison of fits of cumulative distributions to MOR of 180 pieces of 2 x 4 southern pine juvenile wood lumber.

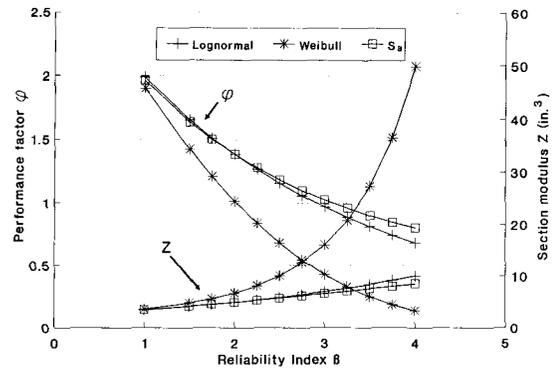


FIG. 5. Effect of distributions on the β - ϕ and β -Z relation for the juvenile wood lumber.

tails of the distributions for the three types of lumber did not differ significantly. As the reliability index β increases, Fig. 5 shows that the influence of the distributions on the design performance factor ϕ and the required section modulus Z dramatically increases. Within the target range of β from 2.0 to 4.0, the Weibull distribution was very sensitive to β and always gave the most conservative results. The S_B and lognormal distributions were less sensitive and gave similar results, the S_B being the least conservative. Irrespective of the distribution, the presence or absence of juvenile wood in the lumber did not affect the size of member for a given value β .

Figure 6 shows the effect of the distributions on the performance factor ϕ and the required moment of inertia I obtained from serviceability limit state design for the juvenile lumber. As for strength limit state design, the influence of the distribution type on ϕ and I significantly increases as β increases. In serviceability limit state design, the target reliability index β is usually set up to be either 1.50 (Zahn 1977) or 2.00 (Foschi et al. 1989). From $\beta = 1.5$ to 2.0, the Weibull distribution always produced a much lower performance factor ϕ and higher required moment of inertia I than those obtained from the S_B distribution for all three types of lumber. The lower sensitivity of the S_B and lognormal distributions to β may be related to the better fit of those distributions.

The fit of the assumed distribution to the

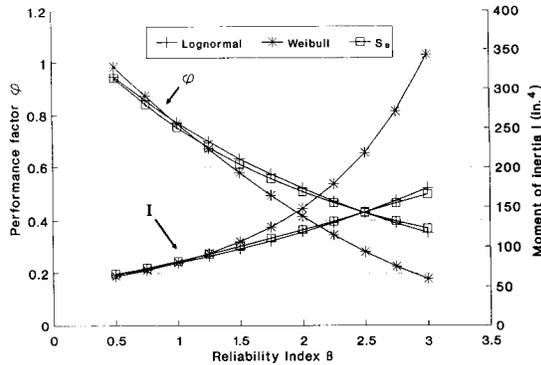


FIG. 6. Effect of distributions on the β - ϕ and β -I relation for the juvenile wood lumber.

data, especially to the lower tail of the data, will affect the reliability design results. A pre-standard report of load and resistance factor design for engineered wood construction (Murphy 1988) recommended using a 2-parameter Weibull distribution to fit the lower tail only of the distribution of test results because the Weibull distribution, when fitted to the complete population, did not fit the lower tail well. Fitting parameters to the lower tail rather than to the complete distribution certainly adds more weight to the lower values, but may also produce reliability results quite different from the true distribution. Since the reliability index is computed based on the distributional parameters (fitting either the lower tail or the complete population), the distribution that better fits both the lower tail and the whole population could provide higher confidence in reliability design results.

In serviceability limit state design, the presence of juvenile wood had a significant effect on the design results irrespective of the distribution used. Although the Weibull distribution produced a more significant effect than the S_B and lognormal distributions, similar percentage differences between the juvenile, mature, and combined lumber were observed within the range of a target β irrespective of the distribution used. Figure 7 shows that the difference between the design I for the juvenile and mature lumber rapidly increases as β increases. At a target $\beta = 2.0$, the curve for the

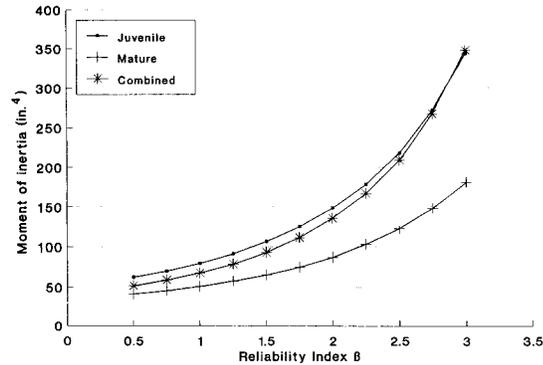


FIG. 7. Comparison of design moment of inertia between juvenile, mature and combined lumber with Weibull distribution.

combined lumber deviates from that for the mature lumber and approaches the curve for the juvenile lumber. Consequently, we would have to use the stiffness properties of juvenile wood in serviceability limit state design unless the juvenile lumber is separated from the mature lumber. It is obvious that the population of mature lumber is not effectively used in such a case. Since the stiffness requirements govern design in most cases, the separation of juvenile lumber from mature lumber would reduce by at least 15% the volume of lumber required to achieve a target reliability index $\beta = 2.0$, based on these data of No. 2 visually graded 2×4 lumber.

Complete separation of the more juvenile wood lumber from the more mature wood lumber is difficult in practice, both on the production line and in the grading process. Visual inspection is the predominant grading method used in the lumber industry in the United States today, but the current visual grading rules focussing on strength reducing effects of knots and slope-of-grain, rather than stiffness, are inadequate for sorting lumber from rapidly grown stands into structurally efficient end-use categories. In this study, all lumber was visually graded as No. 2 grade, but about half of it contained a substantial amount of juvenile wood. Barrett and Kellogg (1986) found that over 80% of the lumber cut from second growth Douglas-fir stands met requirements for select

structural visual grade, but the stiffness of the combined mature and juvenile wood specimens was substantially lower than currently assigned values. If juvenile lumber is not distinguished from mature lumber in the grading process, a considerable loss in efficiency will occur in utilizing the future lumber resource. As increasing quantities of lumber from plantation resources come on the market, and with the future introduction of reliability-based design, improvements of the grading system are essential. A potential solution to this problem is to machine-stress-rate the lumber. Machine grading involves sorting according to stiffness, so whether the lumber is juvenile or mature becomes much less important.

SUMMARY

In this paper, a comparison has been made between three types of distribution for fitting the stress-wave values of MOE for green lumber and MOR values for dry lumber, all lumber being No. 2 grade 2×4 southern pine. The effect of the distributions and of juvenile wood on reliability-based beam design have been discussed. Based on this study, the following conclusions were drawn:

1. Although the differences between the lognormal, Weibull, and S_B distributions were small in a few cases, the S_B distribution gave the best overall fit to the MOE and MOR data at both the lower tail and the distribution as a whole. The S_B distribution appears to provide an additional degree of flexibility to describe mechanical properties data of lumber.

2. The effect of the particular distribution type on reliability-based design of a simple beam was very significant within the range of a target reliability index. For both ultimate-strength and serviceability limit state designs, the S_B distribution always gave much less conservative results than those of the Weibull distribution. The lognormal distribution usually gave results quite close to the S_B distribution. Further studies are needed to compare the design results of the S_B and Weibull distribution when fit to the lower tail only of the distribution.

3. A comparison of the ultimate-strength limit state design results among juvenile, mature, and combined lumber did not show that the juvenile lumber had a significant effect because the MOR values were similar for the tails of the distribution of all three types of lumber. However, in serviceability limit state design, the results indicated that the juvenile lumber almost dominated the total population of combined lumber at a target reliability index. Unless the juvenile lumber is separated from the mature lumber, a considerable loss in efficiency of utilizing the lumber from fast-grown trees would occur where stiffness requirements govern. Sorting juvenile lumber from mature lumber could result in a considerable saving of wood material in the design. A potential way of solving this problem is to modify the current visual grading rules and to increase the use of machine stress-grading.

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