RESPONSE ANALYSIS OF WOOD STRUCTURES UNDER NATURAL HAZARD DYNAMIC LOADS[1]

Greg C. Foliente[2]
Senior Research Scientist
CSIRO Division of Building, Construction and Engineering
P.O. Box 56, Highett, Victoria 3190, Australia

Mahendra P. Singh
Professor
Department of Engineering Science and Mechanics
Virginia Polytechnic Institute and State University
Blacksburg, VA 24061-0219

and

J. Daniel Dolan
Associate Professor
Department of Wood Science and Forest Products
Virginia Polytechnic Institute and State University
Blacksburg, VA 24061-0503

(Received September 1994)

ABSTRACT

The basic requirements needed for response analysis of wood structures against natural hazards are reviewed. A method for stochastic dynamic analysis of wood structures, which allows investigations into their performance and safety under natural hazards such as earthquakes and severe winds, is presented. To illustrate the method, earthquake ground motions are modeled as a stochastic process with Gaussian white noise properties. A single-degree-of-freedom wood structural system is modeled by a hysteretic constitutive law that produces a smoothly varying hysteresis. It models previously observed behavior of wood joints and structural systems, namely, (1) nonlinear, inelastic behavior, (2) stiffness degradation, (3) strength degradation, and (4) pinching. The constitutive law takes into account the experimentally observed dependence of wood joints' response to the input and response at an earlier time (known as memory). Hysteresis shapes produced by the proposed model compare favorably with common wood joints. The hysteresis model can produce a wide variety of hysteresis shapes, degradations, and pinching behavior to model a whole gamut of possible combinations of materials and joint configurations in wood construction. The nonstationary response statistics of a single-degree-of-freedom wood building subjected to white noise excitations are obtained by Monte Carlo simulation and stochastic equivalent linearization. The latter is shown to give a reasonably accurate prediction of the system's response statistics, which may be used in calculating design response values. The method of analysis is general and may be used to study the response of various kinds of structural systems, including multi-degree-of-freedom systems, as long as appropriate structural models are available and appropriate hysteresis model parameters for these systems are known.

Keywords: Wood structures, hysteresis modeling, dynamic analysis, random vibrations.

---

[1] The Commonwealth Scientific and Industrial Research Organisation (CSIRO) has granted permission to the Society of Wood Science and Technology to publish this paper.

[2] Formerly, Research Specialist, Forest Products Laboratory, University of California at Berkeley 1301 South 46th Street, Richmond, CA 94804.

Wood and Fiber Science, 28(1), 1996, pp. 110–127

INTRODUCTION

Natural phenomena such as earthquakes, severe winds (including hurricanes and tornadoes), snow, flood, storm surge, and landslides can cause significant losses of lives and properties, and adversely impact the national economy. Response analysis and structural design to mitigate losses are, however, difficult because of a high level of uncertainties in these events. Newmark and Rosenblueth (1971) remarked, "It is our task to design engineering systems—about whose pertinent properties we know little—to resist future earthquakes and tidal waves (and severe winds)—about whose characteristics we know even less." This paper reviews the requirements needed for response analysis of wood structures against natural hazards, introduces key concepts of stochastic structural dynamics, and presents a method of computing response statistics of a simple nonlinear, inelastic system under natural hazard loads modeled as stochastic or random processes. Only natural hazards involving dynamic action such as earthquakes and winds are considered, with emphasis given to the former in the example computation. The long-term view is to pave the development of realistic and reliable analysis and design procedures for wood structures under natural hazards. This is achieved by bringing together current know-how in the areas of stochastic structural dynamics and wood engineering.

There is little understanding of the dynamic behavior of wood structures under natural hazards. Most of what we know about wood structural behavior under dynamic loading comes from qualitative field data and/or limited experimental data with little theoretical understanding of actual behavior. Difficulties in characterizing wood system behavior (e.g., sensitivity of material properties to the rate and duration of loading, and inelastic and nonlinear behavior) have hindered investigations into their performance under dynamic loading. Because of this, wood structures are treated unfavorably in seismic design codes. Stringent and unclear code requirements put wood at a disadvantage in competing with other construction materials for the engineered structures market. Efforts to revise the requirements for wood structures have been met with resistance because of a lack of technical information on the structural response of wood joints and systems to dynamic loads. Methods for inelastic dynamic analysis of wood structures are needed to investigate the performance and safety of engineered wood systems against natural hazards and to demonstrate quantitatively that wood is a competitive structural engineering material. Without this ability, opportunities for new, innovative structural wood products may be lost, the current market share of wood in the engineered structures market may dwindle, and present nonengineered wood structures may be compromised.

BACKGROUND

A dynamic analysis or dynamic response analysis problem is one in which the dynamic action (i.e., force) on a structural system—which is modeled mathematically by the assumed (or measured) mass, damping, and stiffness properties of the actual system—is known and the corresponding system response is sought. The system model should provide as realistic a description of the actual structure's behavior as possible, and the random nature of natural hazard loadings should be considered in the analysis.

Constitutive modeling

With static monotonic loading, an appropriate load-displacement relation is normally sufficient to predict system response. Under cyclic loading, the load-displacement trace produces hysteresis loops caused by damping and/or inelastic deformation. Figure 1, for example, shows experimental hysteresis response of wood buildings and subassemblies. (The area contained in the loop represents the energy dissipated by the system.) Analytical modeling of an inelastic structure under dynamic loading ideally requires a force-displacement relation, or hysteresis model, that can produce the true behavior of the structure

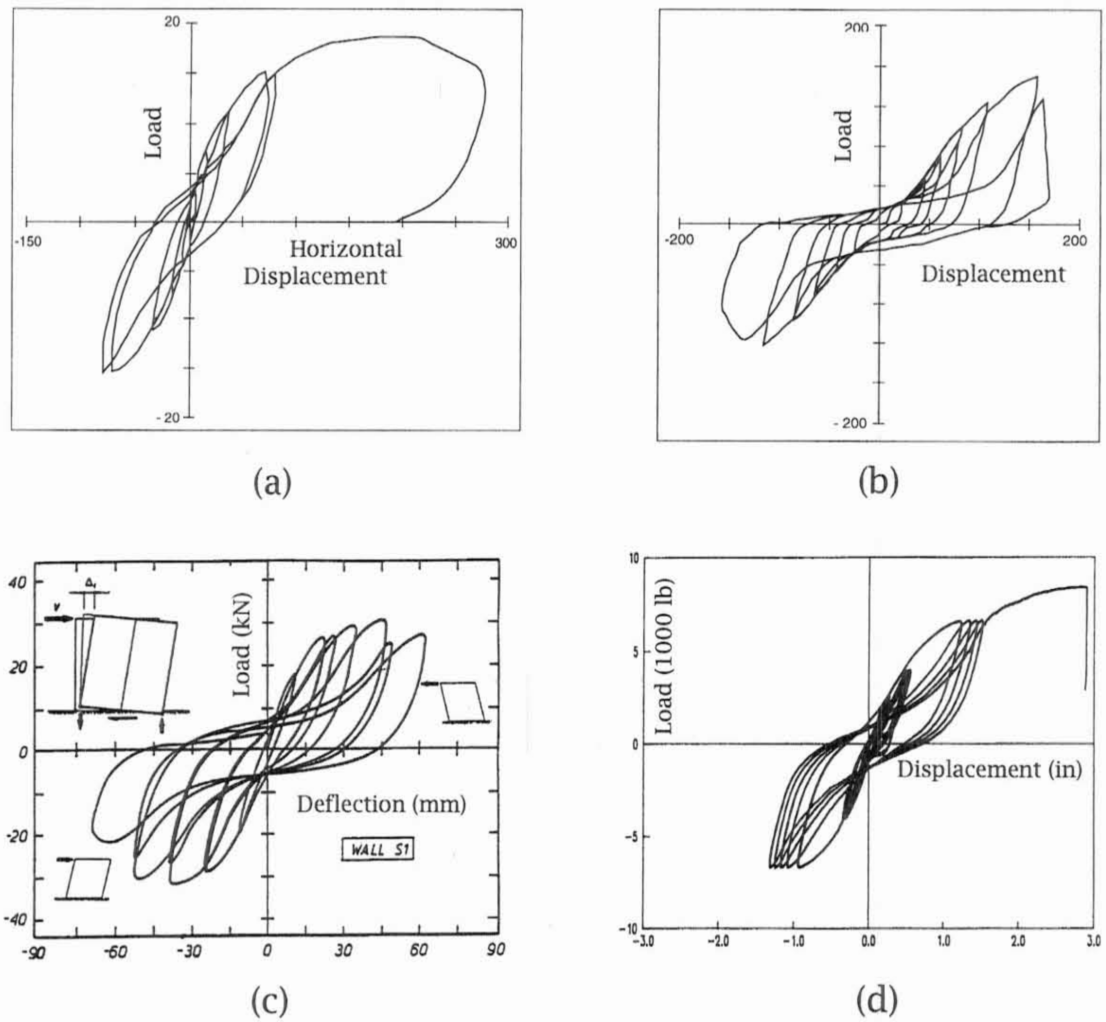

FIG. 1. Typical hysteresis of wood structures and subassemblies: (a) three-story wood building (from Yasumura et al. 1988); (b) braced glulam frame (from Yasumura 1990); (c) plywood shear wall (from Stewart 1987); (d) oriented strand board diaphragm (from Hanson 1990).

at all displacement levels and strain rates (Sozen 1974). Consequently, the energy dissipation mechanisms of wood joints and structural systems must be known and the hysteretic behavior modeled properly before we can accurately predict the overall system response of wood structures to dynamic loads.

Several researchers have proposed hysteresis models for wood joints and/or structural systems: Ewing et al. (1980) for wood diaphragms; Kivell et al. (1981) for moment-resisting nailed timber joints used in glulam construction in New Zealand; Lee (1987), Chou (1987), Stewart (1987), and Dolan (1989) for nailed plywood-to-wood connections typically used in shear wall construction; Ceccotti and Vignoli (1990) for moment-resisting semirigid joints used in glulam portal frames in Europe; Kamiya (1988) for wood-sheathed shear walls; Sakamoto and Ohashi (1988) and Miyazawa (1990) for Japanese wood houses; Gavrilović and Gramatikov (1991) for truss-frame wood systems; Foschi and associates for dowel-type fasteners (UBC 1993). These models, however, use either a complex set of force-history rules or limited empirical relations. [The interested reader is referred to Foliente (1994, 1995) for a review of these and other nonwood hysteresis models.]

While current hysteresis models satisfied some of the specific features of the joints or structural systems that they meant to model, they may be inappropriate for joints or systems with different configurations and material components (Foliente 1995). Furthermore, they are given in forms that are difficult to use in stochastic dynamic analysis (this will be described shortly). Since there are hundreds of combinations of materials and joint configurations in wood systems, and since wood-based products, fasteners, and use of wood-based products continue to evolve, a general constitutive model is preferred over models derived from specific configurations. A completely empirical model will not only be expensive to obtain but may also be of limited use in dynamic analysis. A general constitutive model that simulates the general hysteretic features of wood systems and that is mathematically tractable in stochastic dynamic analysis is preferred. Parameters of the new hysteresis model may be estimated from data obtained from previous tests of specific wood joints and assemblies.

Stochastic dynamic analysis

There are two different approaches in evaluating structural response to dynamic loads: deterministic, and nondeterministic or stochastic or random. The type of loading considered in the analysis determines the kind of approach to use. When the loading is assumed as a known function of time (i.e., its time variation is completely known at each time instant, also called deterministic dynamic loading), the method of analysis used to evaluate the response is called deterministic dynamic analysis. When the loading is not completely known a priori but can be defined in a statistical sense (also called random loading or excitation), the corresponding analysis is defined as stochastic dynamic analysis. It is most popularly referred to as random vibration analysis in the literature.

The orthodox viewpoint in engineering design "maintained that the objective of design was to prevent failure; it idealized variables as deterministic" (Newmark and Rosenblueth 1971). The traditional approach was to make convenient assumptions that allow the use of "equivalent" static loadings and analysis (Corotis 1982) in place of the actual random dynamic characteristics of natural hazard loadings, such as earthquakes and high winds. Although actual recorded data of past earthquake and wind events have been used to analyze structural properties and behavior, this approach is still strictly deterministic. A structure that has been analyzed and designed based on only one or two earthquake or wind records may behave very differently when an earthquake or wind event with different characteristics occurs; gross errors in analysis may lead to unsatisfactory design, e.g., the structure can collapse during an intense earthquake or have excessive sways in severe wind.

In seismic analysis, a large number of strong motion earthquake records is necessary to estimate response statistics. This approach is limited, however, by a relatively small number of available records of strong motion earthquakes. Even if artificial accelerograms are used, the cost and effort needed to perform these time history analyses may be prohibitive. The random characteristics of natural hazard loadings and the corresponding structural response should be represented by stochastic mathematical models. Random vibration analyses proved useful in estimating response statistics of structures subjected to loadings modeled as random processes [e.g., Amin and Ang (1968); Wen (1980); Baber and Noori (1986); Soong and Grigoriu (1993); among others]. Kareem (1987) and Branstetter et al. (1988) provide excellent reviews of the probabilistic framework needed to compute the response statistics of structures to wind and earthquake, respectively, using random vibration methods. With a reliable estimate of response statistics, one may then design a structure based on accepted levels of safety, measured in terms of probability of failure.

Research on the dynamic analyses of wood structures has, so far, been limited to deterministic approaches (Ceccotti 1989; Gupta and Moss 1991) and has lagged behind advances in general structural dynamics. An appropriate hysteresis model for wood structures that is suited for both deterministic and random vibration analyses is clearly needed.

Summary of basic requirements

Response analysis of nonlinear dynamic systems subjected to stochastic excitations basically involves three elements: (1) a structural model with elements incorporating constitutive or hysteresis relations that best represents behavior under cyclic loading, (2) a stochastic process model that simulates natural hazard loadings, and (3) a solution technique that allows practical estimates of response of the structural system, modeled by (1), subjected to stochastic excitations, modeled by (2). These three elements are discussed next.

DYNAMIC MODELING OF STRUCTURAL SYSTEMS

Structures are continuous systems and as such have an infinite number of degrees of freedom (DOF). For analytical purposes, the structure is simplified by means of spatial discretization of the continuum. The following discretization methods can be used in the dynamic modeling of structures (Clough and Penzien 1993): (1) concentrated mass method, (2) generalized displacements method, and (3) finite element method. Use of any of these methods results in a discretized structural model with a finite number of DOF. The concentrated mass method and the finite element method are most commonly used [see Foliente (1994) for a review of their use in dynamic analysis]. The resulting structural model should incorporate elements that exhibit the appropriate hysteretic behavior.

Hysteresis modeling

Behavior of wood joints and structural systems.—Dowrick (1986) collected cyclic test data of wood joints and structural systems in New Zealand, Japan, and North America and examined common hysteresis loops for timber structures. He noted that the hysteresis behavior of wood systems normally follows that of their primary connections. Stewart (1987) and Dolan (1989) corroborated this observation. (Compare, for example, the plywood shear wall behavior in Fig. 1c with the nailed plywood sheathing joint behavior in Fig. 2b.) Thus, for analytical purposes, Dowrick classified the hysteresis loops for timber structures, based on their shape characteristics, into joints with: (1) yielding plates, (2) yielding nails, and (3) yielding bolts (Fig. 2). Similarities in the hysteresis shapes of the dowel-type fasteners, i.e., nails and bolts, can be seen in Figs. 2b and c.

Characteristic features of cyclic response typically observed in wood structural systems (e.g., Fig. 1) have been summarized (Foliente 1995) as follows: (1) nonlinear, inelastic load-displacement relationship without a distinct yield point; (2) progressive loss of lateral stiffness in each loading cycle (will be referred to as stiffness degradation); (3) degradation of strength when cyclically loaded to the same displacement level (will be referred to as strength degradation); and (4) pinched hysteresis loops (i.e., thinner loops in the middle than near extreme ends). The response of wood joints (and wood structures, in general) at a given time depends not only on instantaneous displacement but also on its past history (i.e., the input and response at an earlier time). This is known as memory. Whale (1988) observed that nailed or bolted timber joints under irregular short or medium term lateral loading have memory.

Proposed model.—Any hysteresis or constitutive model for timber structures should incorporate experimentally observed characteristics such as those given above. All the available models for wood systems use either a complex set of force-history rules or very limited empirical relations. While these models satisfied some of the specific features of the joints or structural systems that they meant to model, they may be inappropriate for joints or

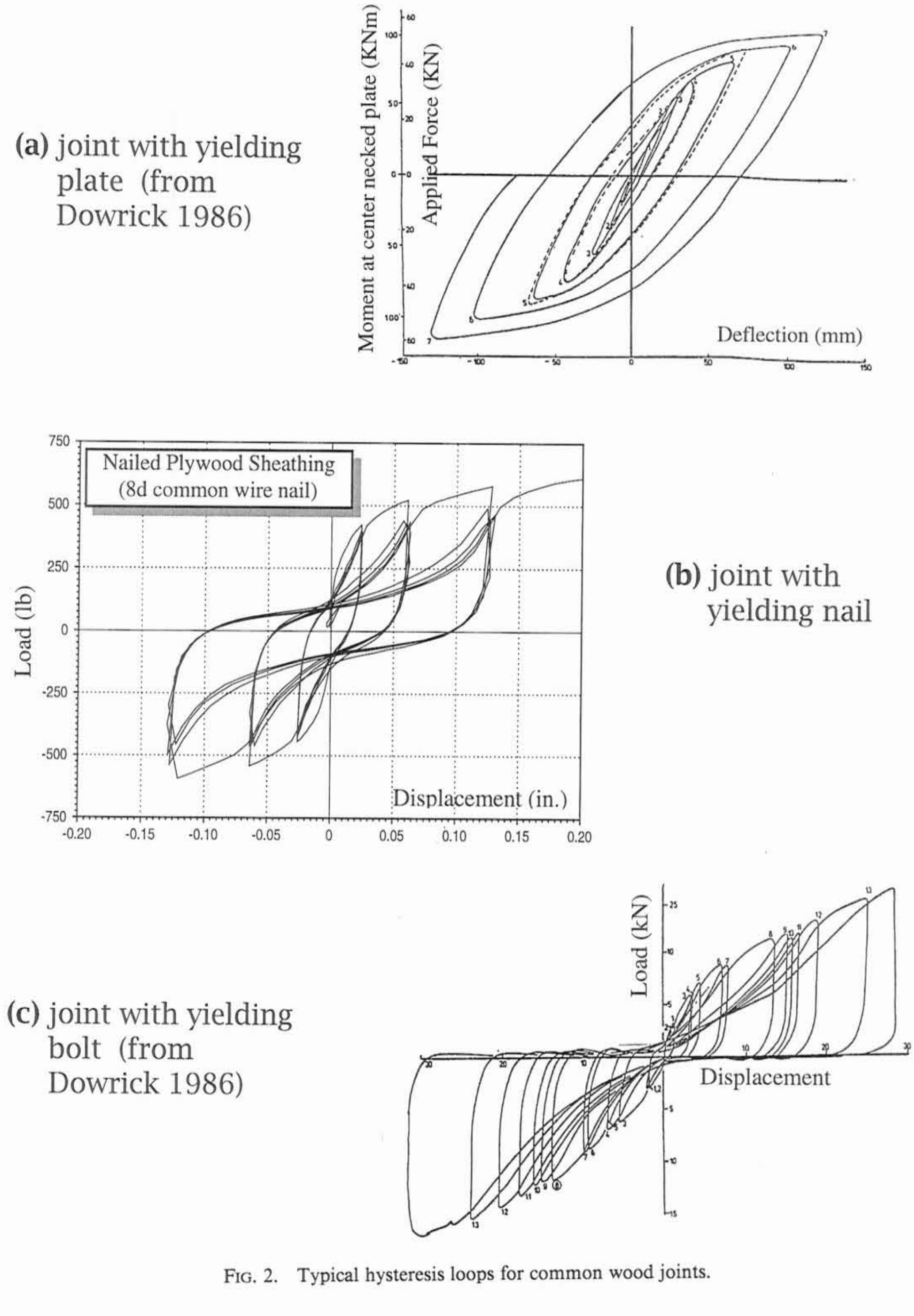

FIG. 2. Typical hysteresis loops for common wood joints.

systems with different configurations and material components. A mathematical model, meeting all the criteria given above, is preferred. Model parameters may be estimated from tests of representative wood joints or assemblies. The hysteresis model proposed by Foliente (1995) is described next.

The essence of the hysteresis model may be described with a single-degree-of-freedom (SDOF) system. The equation of motion of the SDOF system in Fig. 3 is generally written as

$$m\ddot{u} + c\dot{u} + \mathcal{F}_T[u(t), z(t); t] = F(t) \qquad (1)$$

where u = relative displacement of the mass m with respect to the ground motion (dots designate derivatives with respect to time t, i.e., $\dot{u}$ = velocity and $\ddot{u}$ = acceleration), c = linear viscous damping coefficient, F(t) = forcing function, $\mathcal{F}_T[u(t), z(t); t]$ = non-damping restoring force consisting of the linear restoring force αku and the hysteretic restoring force $(1 - \alpha)kz$, α = rigidity ratio, and z = hysteretic displacement. Dividing both sides of Eq. (1) by m, the following standard form is obtained:

$$\ddot{u} + 2\xi_0\omega_0\dot{u} + \alpha\omega_0^2u + (1 - \alpha)\omega_0^2z = f(t) \qquad (2)$$

where ξ_0 = system's linear damping ratio, ω_0 = system's linear natural frequency, and f(t) = mass-normalized forcing function. The hysteretic restoring force is given by the fourth term in Eq. (2) as $(1 - \alpha)\omega_0^2z$. Since $[(1 - \alpha)\omega_0^2]$ is a time-invariant system property, the hysteretic restoring force will also be referred to as z from here on. The constitutive law that relates the hysteretic restoring force z to displacement u is given by the following first-order nonlinear differential equation

$$\dot{z} = h(z)\left\{\frac{\dot{u} - \nu(\beta|\dot{u}||z|^{n-1}z + \gamma\dot{u}|z|^n}{\eta}\right\} \qquad (3)$$

with pinching function

$$h(z) = 1.0 - \zeta_1\exp\left[-\frac{z\ \mathrm{sgn}(\dot{u}) - qz_u)^2}{\zeta_2^2}\right] \qquad (4)$$

where sgn(·) is the signum function [i.e., sgn(a) gives −1, 0 or 1 depending on whether a is

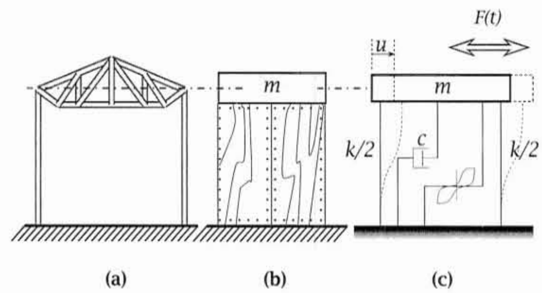

FIG. 3. Single-degree-of-freedom (SDOF) idealization of wood structural systems (from Foliente 1995): (a) truss frame, (b) shear wall, (c) SDOF mechanical model.

negative, zero or positive, respectively], z_u is the ultimate value of z given by

$$z_u = \left[\frac{1}{\nu(\beta + \gamma)}\right]^{1/n} \qquad (5)$$

and

$$\zeta_1(\epsilon) = \zeta_{1o}[1.0 - \exp(-p\epsilon)] \qquad (6)$$

$$\zeta_2(\epsilon) = (\psi_0 + \delta_\psi\epsilon)(\lambda + \zeta_1). \qquad (7)$$

The hysteresis model parameters are summarized in Table 1, and their effect on hysteresis shape was discussed by Foliente (1993). The constitutive law given by Eq. (3) is based on a modified "endochronic" model of the force-displacement relations. The hereditary restoring force model satisfies the requirement that the response depends not only on instantaneous displacement but also on its past history (referred to earlier as memory).

Strength and stiffness degradation are modeled, respectively, by

$$\nu(\epsilon) = 1.0 + \delta_\nu\epsilon$$

$$\eta(\epsilon) = 1.0 + \delta_\eta\epsilon. \qquad (8)$$

Pinching and strength and stiffness degradation are controlled by the hysteretic energy dissipation

$$\epsilon = (1 - \alpha)\omega_0^2\int_{t_0}^{t_f} z\dot{u}\,dt. \qquad (9)$$

The foregoing model is a generalization of the models proposed by Bouc (1967), Wen

TABLE 1. *Tabulated summary of hysteresis model parameters.*

Parameter	Definition
System properties and hysteresis shape parameters	
ω_0 (rad/sec)	natural frequency of the structural system
ξ_0	damping ratio of the structural system
α	rigidity ratio; a weighting constant representing the relative participations of the linear and nonlinear terms ($0 < \alpha < 1$)
β, γ	parameters that control the basic hysteresis shape ($\beta > 0$)
n	parameter that controls hysteresis curve smoothness
Degradation parameters	
δ_ν	parameter that controls strength degradation
δ_η	parameter that controls stiffness degradation
Pinching parameters	
ζ_1	parameter that controls the severity of pinching; depends on the values of ζ_{1o} and p
ζ_2	parameter that controls the rate of pinching; depends on the values of ζ_1, Ψ_0, δ_Ψ and λ
ζ_{1o}	measure of total slip (e.g., $\zeta_{1o} = 0.98$ means a high pinching system and $\zeta_{1o} = 0.70$ means a low pinching system)
q	percentage of ultimate restoring force z_u where pinching (or slipping) occurs
p	parameter that controls the rate of initial drop in slope
Ψ_0	parameter that contributes to the amount of pinching
δ_Ψ	parameter specified for the desired rate of change of ζ_2 based on ϵ
λ	parameter that controls the rate of change of ζ_2 as ζ_1 changes

(1980), Baber and Wen (1981), and Baber and Noori (1986), and is called the modified Bouc-Wen-Baber-Noori (BWBN) model (Foliente 1995). It satisfies all the experimentally observed features of hysteretic behavior of wood joints and structural systems, namely, (1) nonlinear hysteresis, (2) stiffness degradation, (3) strength degradation, and (4) pinching.

The model is very flexible and can actually produce a wide variety of hysteresis shapes to model the behavior of hysteretic degrading systems with general pinching behavior (such as reinforced concrete structures, braced steel frames, and laterally loaded piles), if the parameters in Table 1 are varied. Figure 4, for example, shows some possible hysteresis shapes for various combinations of β and γ values.

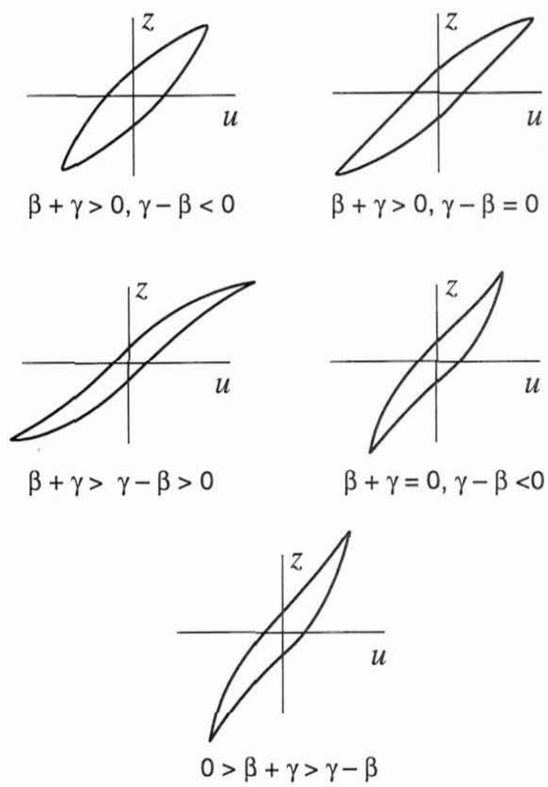

FIG. 4. Possible hysteresis shapes, n = 1.

Model verification.—Foliente (1995) estimated the model parameters of three common wood joints to represent the major hysteresis types for timber structures that Dowrick (1986) identified. Figures 5a, b, and c show the hysteresis shapes produced by the model when f(t) is taken as a sinusoidal function of the form $f(t) = (a_1 + a_2t)\sin(\omega t)$, where the a_is are specified constants and ω is the excitation frequency. Note that to get the exact hysteresis shapes and response values as shown in Fig. 2, complete information about connection materials, test set-up, and the forcing function that was used in testing is needed. Since most of this information is not known, the focus should be

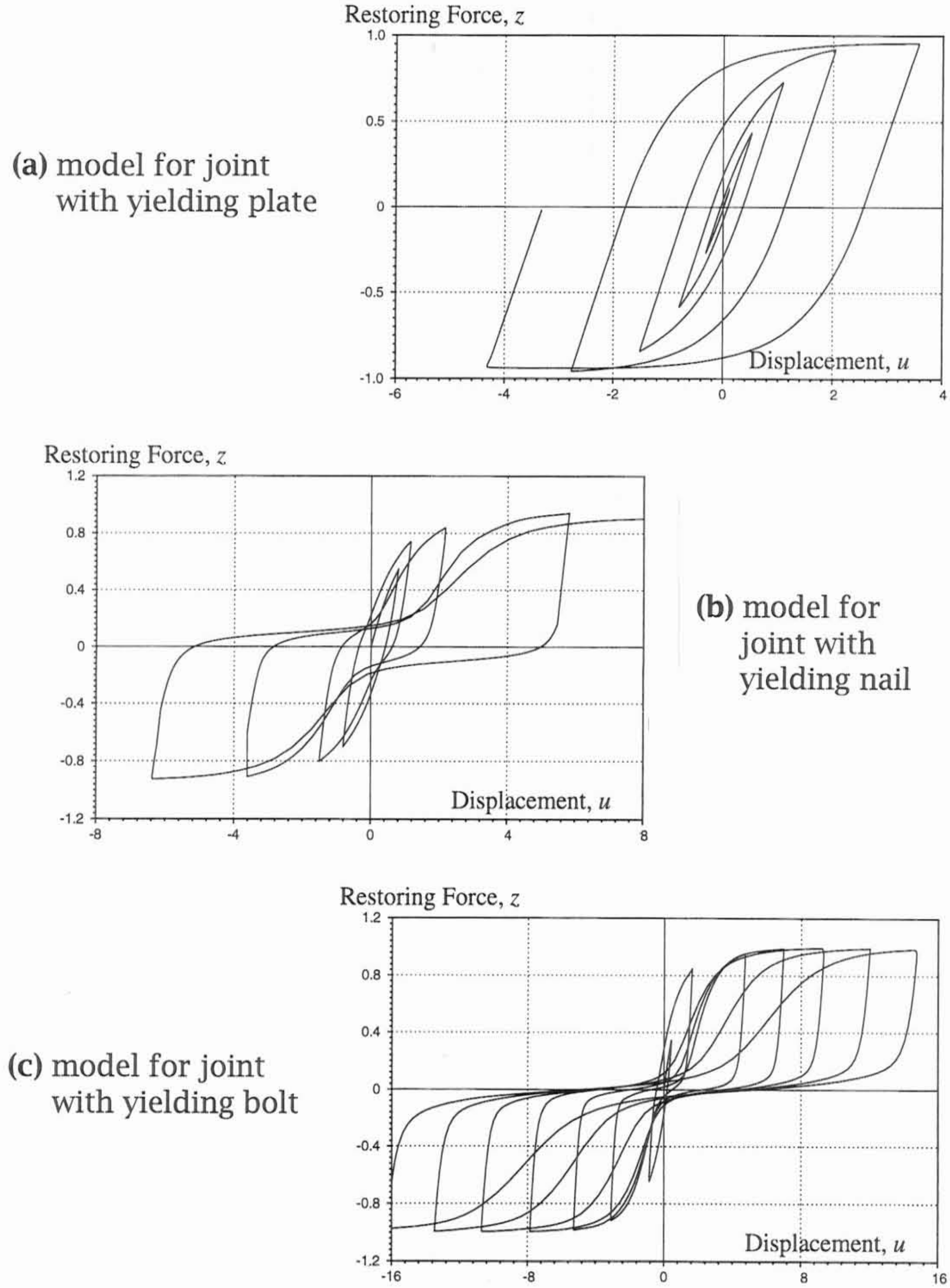

FIG. 5. Hysteresis loops produced by the modified BWBN model (from Foliente 1995).

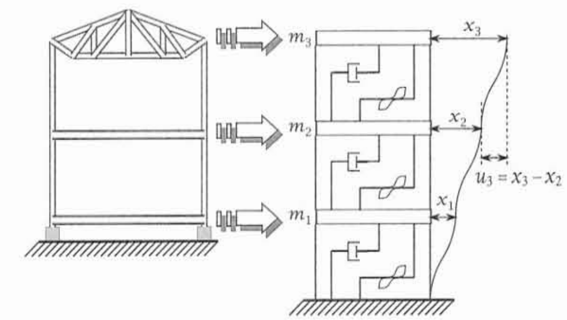

FIG. 6. MDOF idealization of a multi-storey timber building.

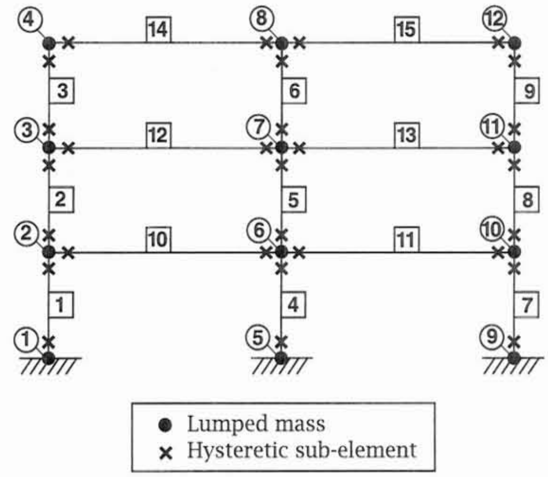

FIG. 7. Hysteresis frame discrete hinge model.

only on modeling the basic hysteresis shape of the joints in Fig. 2; thus, no specific force and displacement units are considered.

Comparison of model hysteresis (Fig. 5) with experimental hysteresis (Fig. 2) shows that the proposed model reasonably mimics the basic hysteresis shape of test data. The slight discrepancy in the hysteresis shapes of the yielding bolt joints (Fig. 2c vs. Fig. 5c) may be attributed largely to different forcing functions used in the test and the analysis. Even then, the basic behavior of the bolt joint can be observed in the model hysteresis.

Incorporation of the hysteresis model into a nonlinear dynamic analysis computer program for SDOF systems is relatively straightforward. System response from arbitrary dynamic loading, such as cyclic or earthquake-type loadings, can be computed (Foliente 1995).

Structural modeling

A single-degree-of-freedom model is, in some cases, sufficient to obtain a basic understanding of the dynamic behavior of a structural system. Figure 3 shows an SDOF dynamic model idealization of a trussed wood-frame and a wood shear wall. Stewart (1987) and Kamiya (1988) performed time history analyses of wood-sheathed shear walls using an SDOF model that incorporates their hysteresis models. Stewart obtained hysteresis parameters from full-scale cyclic tests of the walls, while Kamiya obtained model parameters from pseudo-dynamic tests. Gavrilović and Gramatikov (1991) also used an SDOF model to compute the dynamic response of a trussed-frame wood structure. The SDOF model for simple timber systems with the modified BWBN restoring force model is completely described by Eqs. (2) to (9).

A more complex wood structure may be discretized, to form a multi-degree-of-freedom (MDOF) system, as a weak column system ("shear-beam"), a strong column system, or a more general discrete hinge system where plastic hinges, due to local plastic deformation, are allowed to form only at columns, only at beams, or both.

In a shear-beam or shear building model, the simplest MDOF model possible, there are as many DOFs as it has lumped masses at the floor levels. Rotation at the girder-to-column joint is suppressed, and the rigid beam/girders remain horizontal during ground motion. Despite these limitations, shear building models provide enhancements that make them preferable to alternative SDOF models. A two-story timber building shown in Fig. 6a, for example, is better represented by a three-degree-of-freedom (3-DOF) shear building than by an SDOF model. Sakamoto and Ohashi (1988) used a shear building model to compute the seismic response of one-, two- and three-story conventional Japanese wood houses. Foliente (1995) formulated an MDOF shear building model with the modified BWBN hysteresis model as story restoring force.

In a strong column system, the hysteresis model can be used as overall restoring moment. In discrete hinge idealization, yielding is confined at certain regions, e.g., joints, that incorporate the modified BWBN constitutive relations (Fig. 7). Since rotation of the joints is allowed, in addition to story displacements, this is a better representation of the structural frame. More DOFs are, however, required. Ceccotti and Vignoli (1991), Kikuchi (1994), and Komatsu et al. (1994), among others, used frame models with semirigid joints in deterministic static and dynamic analyses. Foliente is currently modeling hysteretic timber frames, e.g., glulam and heavy timber frames, using the discrete hinge concept for random vibration analysis.

Wood structures and structural systems have been modeled using finite elements in deterministic dynamic analysis. Dolan (1989), for example, used a combination of plate, framing, and connector elements to model wood shear walls. Lee (1987) used a composite-beam finite element as a key component of a wood-framed building model for seismic analysis. Recent attempt has been made to develop a three-dimensional finite element model of light-frame wood buildings for deterministic dynamic analysis (Tarabia and Itani 1994). The parent form of the modified BWBN hysteresis model proposed herein has also been used in nonlinear random vibration analysis using finite elements (Simulescu et al. 1989).

The relative advantages and disadvantages of the preceding structural modeling techniques are discussed by Foliente (1994). Each modeling technique plays an important role in obtaining a better understanding of the dynamic behavior of structures, in general, and timber structures, in particular. The choice of an appropriate structural model should be made to meet clearly defined analysis objectives. Careful considerations in the discretization of, assigning DOFs in, and selecting an effective method of reducing dynamic matrices for wood structures are necessary to strike a balance between accuracy and computational efficiency. This is especially important if—going beyond deterministic solutions—we are interested in studying the nonstationary response statistics of the structure under stochastic excitations.

STOCHASTIC MODELING OF NATURAL HAZARD LOADS

Earthquake ground motions are generated through numerous random phenomena—seismic waves from the hypocenter undergo a very large number of reflections and refractions, which are influenced by the unordered locations of geological stratifications (Augusti et al. 1984). Winds of random characteristics are produced when the atmospheric flow system—influenced by the earth's rotation, topography, reflective and thermal properties of the earth's surface, cloud cover, precipitation, etc.—interacts with other factors that lead to energy cascades from large- to small-scale motion (Kareem 1987). Thus, earthquakes and winds may be modeled as random processes. A random process is a parametered random variable. A random process, say $X(t)$, represents a large number of possible time functions, none of which are exactly alike. A particular realization (or time history) of this process is a sample function of the underlying random process; an ensemble of the process is a collection or family of such sample functions (Ang 1974) as shown in Fig. 8.

A random process is normally described by its probabilistic nature, which is normally limited to the "two point" probability law, i.e., probabilistic information based on a pair of random variables $X(t_1)$ and $X(t_2)$ at any two time instants t_1 and t_2. These descriptors are further limited to the mean value function $E[X(t)]$ and the autocorrelation function $R_{XX}(t_1, t_2) = E[X(t_1)X(t_2)]$, where $E[\cdot]$ is the expected value. A stationary random process has an autocorrelation function that depends only on time lag $\tau = t_2 - t_1$ and not on actual time instants t_1 and t_2. Its mean remains constant with time.

For a stationary Gaussian random process,

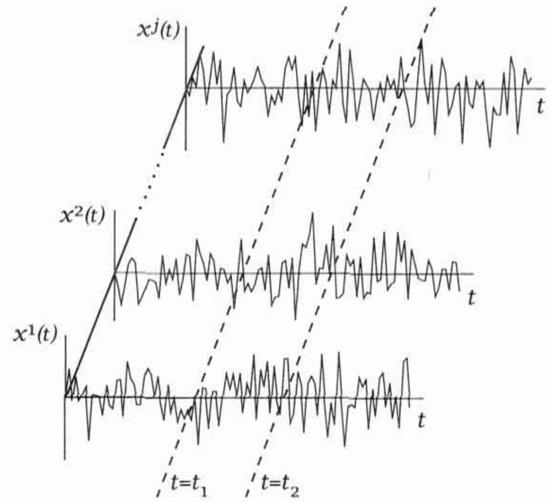

FIG. 8. Ensemble of a random process, X(t).

the mean value and autocorrelation functions are sufficient to completely describe the process. Since earthquake motions are usually assumed as a zero mean Gaussian process, the information about the autocorrelation function is adequate. In seismic analysis, the use of the Power Spectral Density (PSD) function, a representation of the same random process in the frequency domain, rather than the autocorrelation function is preferred. PSD function, $\Phi_X(\omega)$, and autocorrelation function, $R_{XX}(\tau)$, are Fourier transform pairs:

$$R_{XX}(\tau) = \int_{-\infty}^{\infty} \Phi_X(\omega)e^{i\omega\tau}\, d\omega$$

$$\Phi_X(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{XX}(\tau)e^{-i\omega\tau}\, d\omega \quad (10)$$

where ω is the frequency of the random process X(t).

A stationary Gaussian process, also known as white noise, is most often used in stochastic modeling of seismic ground motions and winds. It has the following characteristics:

$$R_{XX}(\tau) = 2\pi S_0\delta(\tau)$$

$$\Phi_X(\omega) = S_0 \quad (11)$$

where δ is the Dirac delta function, giving a sharp impulse at $\tau = 0$. Although actual earthquakes are nonstationary and do not have a flat power spectrum like a white noise, it may be satisfactory for wide band excitation (i.e., when the excitation spectrum varies slowly in the vicinity of the structures's natural frequency). Filtering and modulation of the input excitation can be easily incorporated in the model [e.g., Baber and Wen (1981)].

The most common filter transfer function for stationary filtered white noise excitation is that proposed by Kanai (1957) and Tajimi (1960). Application of this filter gives the following Kanai-Tajimi power spectral density:

$$\Phi(\omega) = S_0 \frac{1 + \left[2\xi_g \dfrac{\omega}{\omega_g}\right]^2}{\left[1 - \left(\dfrac{\omega}{\omega_g}\right)^2\right]^2 + \left[2\xi_g \dfrac{\omega}{\omega_g}\right]^2} \quad (12)$$

where ω_g and ξ_g are the dominant frequency and damping, respectively, of the ground through which the seismic wave propagates. This gives a better approximation of the spectral power distributions in actual earthquakes. To obtain a nonstationary filtered white noise random process, the stationary filtered white noise may be multiplied by a specified time varying function [e.g., Amin and Ang (1968); Shinozuka and Sato (1967), among others].

A number of other stochastic models for artificial ground acceleration has been developed and successfully applied to a variety of structural dynamics problems. Shinozuka and Deodatis (1988) and Kozin (1988) provide extensive reviews of currently available models, including those based on: (1) filtered white noise processes, (2) filtered Poisson processes, (3) spectral representation of stochastic processes, (4) stochastic wave theory, and (5) auto-regressive moving average (ARMA) models. Shinozuka and Deodatis (1988) provided the mathematical expressions for the first four models along with comments on their usefulness, advantages, and disadvantages, while Kozin (1988) discussed the features and current developments in ARMA model procedures for earthquake engineering applications.

Kareem (1987) reviewed strategies for numerical simulation of wind effects using a Gaussian white noise process, fast Fourier transform techniques, and ARMA models.

NONLINEAR RANDOM VIBRATION ANALYSIS

An analytical solution of differential equations, representing nonlinear dynamic systems, is difficult. Powerful methods of linear system theory, such as the normal mode approach and the convolution integral, cannot be applied because the principle of superposition does not apply to nonlinear problems. Hysteretic systems, which have multivalued functions in the equations of motion, make an exact solution even more difficult to obtain. Thus, many researchers use approximate solutions, which may fall under one of the following broad categories (Branstetter et al. 1988; Foliente 1993; Soong and Grigoriu 1993): (1) Markov methods, (2) perturbation and functional series methods, (3) moment closure, (4) statistical or stochastic equivalent linearization, (5) equivalent nonlinear equations, and (6) simulation methods. Other methods (e.g., associate linear system approach, and decomposition method of Adomian) have been proposed, but the equivalent linearization technique is the one that has been most extensively used and is probably the most useful from an engineering point of view. This technique gives reasonably good results for even strongly nonlinear systems, be it of the geometric or material source, and is easily extended into the analysis of MDOF systems (Wen 1988). These make equivalent linearization the clear choice over other approximate methods in nonlinear random vibration analysis of complex MDOF systems. The other methods tend to involve severe analytical difficulty and/or excessive computational requirements in dealing with these types of problem.

Herein, Monte Carlo simulation (MCS) and the equivalent linearization technique are used to obtain the response statistics of an SDOF wood system with a hysteresis behavior given by Eqs. (2) to (9) and subjected to a Gaussian white noise process (Eq. 11). In the former, a large number of sample functions of the excitation process is generated; then the corresponding sample function of the response is computed for each sample excitation. [Each deterministic solution follows that given in Foliente (1995).] Response statistics are computed from the ensemble of response functions. MCS results are typically used as the basis in evaluating the accuracy of other approximate solutions.

Stochastic equivalent linearization

Although the equivalent linearization technique had been extensively used to solve nonlinear random vibration problems, solution for highly nonlinear cases—as seen in seismic loadings—requires an iterative procedure and assumption of slowly varying response parameters (known as the Krylov-Bogoliubov, KB, assumption). The special form of Eqs. (2) to (9) allows the closed form linearization of the equations, without resorting to the KB assumption. This is ideal because this assumption: (1) is equivalent to assuming a narrow band process (i.e., a random process with significant power spectral density values over a narrow frequency band around a central frequency) "while it is known that the response of hysteretic systems is wide band" (Baber and Wen 1981); (2) prohibits drifts in the system; and (3) may seriously underestimate the root mean square (RMS) response (Wen 1988)—or in the present case, the response standard deviation.

A brief overview of the equivalent linearization procedure is described next. Details of the solution procedure for systems with the modified BWBN restoring force model can be found in Foliente (1993) and Foliente et al. (1996).

The nonlinear equation of motion, Eq. (1), is replaced by an equivalent linear system as follows:

$$m\ddot{u} + c_e\dot{u} + \alpha k_e u + (1 - \alpha)k_e z = F(t) \quad (13)$$

where c_e and k_e are determined by the requirement that the mean square error $E[e^2]$ is minimized, where

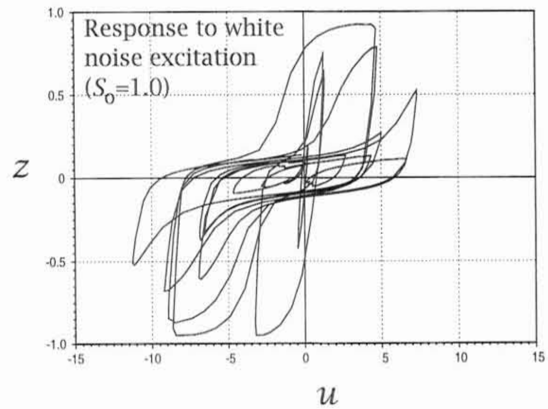

FIG. 9. Sample hysteresis plot of a SDOF wood system under white noise excitation.

$$e = m\ddot{u} + c\dot{u} + \mathcal{F}_T[u(t), z(t); t] - $$
$$- [m\ddot{u} + c_e\dot{u} + \alpha k_e u + (1 - \alpha)k_e z]. \qquad (14)$$

This error is a random process. If the mean square error is minimized, we obtain

$$\frac{\partial}{\partial c_e} E[e^2] = 0$$
$$\frac{\partial}{\partial k_e} E[e^2] = 0. \qquad (15)$$

By substituting Eq. (14) into Eq. (15), the coefficients of the equivalent linear system can be solved. Since c_e and k_e are functions of the response statistics, an iterative procedure is required to solve the covariance equation that is derived from Eq. (13), after further mathematical manipulation. The iterative process is repeated until a desired convergence is achieved. The nonstationary mean square responses (or variances) are elements of the zero mean time lag covariance matrix, i.e.,

$$\sigma^2{}_u = E[u^2]$$
$$\sigma^2{}_{\dot{u}} = E[\dot{u}^2]$$
$$\sigma^2{}_z = E[z^2]. \qquad (16)$$

Numerical studies

Consider an inelastic SDOF wood building whose response is governed primarily by plywood shear walls and with hysteresis similar to that in Fig. 5b. Three levels of white noise excitation (S_0 = 0.1, 0.5 and 1.0; see Table 2) are used to obtain the zero time lag covariance matrix response, starting with zero initial conditions. Figure 9 shows a sample response of an SDOF high degrading, high pinching wood system to a white noise input. Figures 10a to d show comparisons of MCS and linearization results. The nonstationary response statistics from MCS are computed based on two hundred response samples.

TABLE 2. *White noise excitation intensity levels [g = acceleration of gravity (32.2 ft/sec²); 1 ft = 0.305 m].*

White noise level S_0	Average peak acceleration (based on 500 simulation samples)
1.0	0.96 g
0.5	0.68 g
0.1	0.31 g

Figure 10a shows that linearization results generally agree with MCS results. The former, however, tends to slightly underestimate root mean square (RMS) displacements, σ_u, from time t = 15 to 45 seconds at high excitation level (S_0 = 1.0). On the other hand, RMS velocities, $\sigma_{\dot{u}}$, RMS restoring forces, σ_z, and mean dissipated energy, μ_ϵ, are estimated very closely at all excitation levels as shown in Figs. 10b to d.

Thus, for practical purposes, the equivalent linearization technique may be used to obtain RMS responses of a hysteretic system under white noise excitation, in lieu of the Monte Carlo simulation. This is important since Spanos (1981) has estimated that for typical engineering applications involving SDOF systems, the computational efficiency of the equivalent linearization method is of the order of one hundred (10^2) to one thousand (10^3) times greater than that of the approach based on simulation. The significance of the computational superiority of equivalent linearization increases with (1) increasing number of response samples generated for the Monte Carlo study, (2) decreasing values of the damping, and (3) increasing dimension of the structural system.

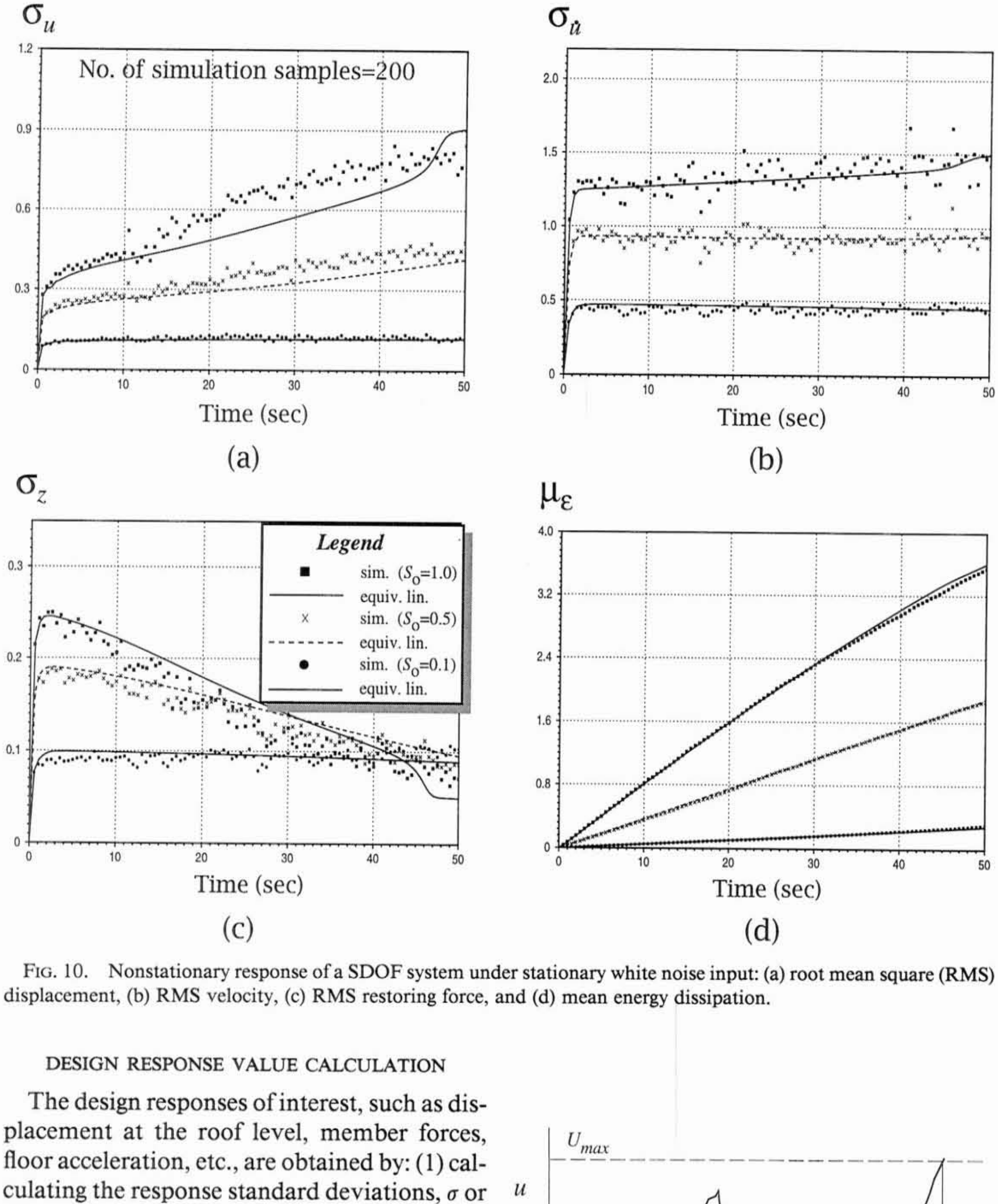

FIG. 10. Nonstationary response of a SDOF system under stationary white noise input: (a) root mean square (RMS) displacement, (b) RMS velocity, (c) RMS restoring force, and (d) mean energy dissipation.

DESIGN RESPONSE VALUE CALCULATION

The design responses of interest, such as displacement at the roof level, member forces, floor acceleration, etc., are obtained by: (1) calculating the response standard deviations, σ or RMS, and (2) amplifying the standard deviations by a peak factor, F_p. Two methods of calculating RMS responses were demonstrated in the previous section. Here, a simple method of obtaining a maximum design response value is presented. It should be emphasized that this represents the probable maximum response of

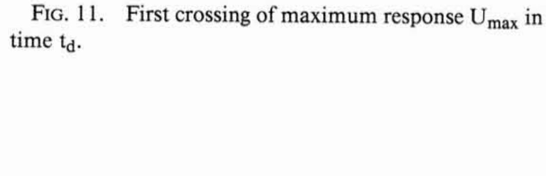

FIG. 11. First crossing of maximum response U_{max} in time t_d.

the system from an ensemble of possible ground motion histories.

Consider mass displacement, u, of the SDOF wood system as the response of interest. If a maximum response level, $\pm U_{max}$, is assumed and a small probability, p_0, of exceeding this value in time t_d (Fig. 11) is allowed, we can obtain U_{max} as

$$U_{max} = F_p \sigma_u \tag{17}$$

where F_p is the peak factor, given by

$$F_p = \left\{2 \ln\left[-\frac{1}{\pi}\frac{\sigma_{\dot{u}}}{\sigma_u}\frac{t_d}{\ln(1 - p_0)}\right]\right\}^{1/2}, \tag{18}$$

if the excursions are assumed as a Poisson process. If the calculation results in $F_p = 3$, the design response value U_{max} is 3 standard deviations higher than the mean response.

If, on the other hand, the interest is to know the probability p_0 that the system displacement will exceed a specified level $\pm U_s$ in the time interval $0 \le t \le t_d$, then Eqs. (17) and (18) can be recast to obtain

$$p_0(t_d) = 1 - \exp\left[\frac{1}{\pi}\frac{\sigma_{\dot{u}}}{\sigma_u} t_d \exp\left(-\frac{U_s^2}{2\sigma^2_u}\right)\right]. \tag{19}$$

This shows that the distribution of the maximum displacement in t_d is of the double exponential form and, therefore, is a Type I extremal distribution (Ang 1974).

Other approaches in system performance and safety evaluation using random vibration results are discussed in Foliente (1993) and Soong and Grigoriu (1993).

SUMMARY

Key concepts of stochastic structural dynamics were introduced, and basic requirements needed for response analysis of wood structures against natural hazards were reviewed. A method for stochastic dynamic analysis of wood structures, which allows investigations into their performance and safety under natural hazard loadings such as earthquakes, was presented.

Single-degree-of-freedom wood structural systems were modeled by a hysteretic constitutive law that produces a versatile, smoothly varying hysteresis. It models previously observed behavior of wood joints and structural systems, namely, (1) nonlinear, inelastic behavior, (2) stiffness degradation, (3) strength degradation, and (4) pinching. The constitutive law takes into account the experimentally observed dependence of wood joints' response to their past history (or memory). Hysteresis shapes produced by the proposed model compare favorably with experimental hysteresis of wood joints with: (1) yielding plates, (2) yielding nails, and (3) yielding bolts. The hysteresis model can produce a wide variety of hysteresis shapes, degradations, and pinching behavior to model a whole gamut of possible combinations of materials and joint configurations in wood systems. Continued evolution of wood-based products, fasteners, and use of wood-based products need not be a problem, as long as hysteresis data from tests of representative wood joints or structural systems are available, from which model parameters can be estimated.

Subjected to stochastic excitations modeled by a Gaussian white noise process, the nonstationary response statistics of an SDOF wood building with plywood shear walls were obtained by Monte Carlo simulation and stochastic equivalent linearization. It was shown that the use of equivalent linearization technique is sufficient in obtaining relevant response statistics that can be used in calculating design response values. The response analysis technique is general and can be applied not only in random vibration analysis of wood structural systems but also in the analysis of a wide variety of hysteretic systems with general pinching behavior. It can also be applied to multi-degree-of-freedom systems, as long as appropriate structural models are available and appropriate hysteresis model parameters for these systems are known.

This is the first time random vibration techniques have been used in studying the response of wood structures under natural hazards. This will hopefully help narrow the gap between advances in general structural dynamics and those in structural wood engineering. This numerical tool can be used to perform various kinds of sensitivity and parametric studies of wood structural systems subjected to an ensemble of ground motion histories, even if we have a only limited set of experimental data. Future work includes the application of mechanical control theory in systematically identifying hysteresis model parameters from test data, extension of the present work to multi-degree-of-freedom systems, dynamic reliability analysis and first-passage studies, and seismic damage analysis of timber structures. Thus, with the proposed hysteresis model for wood structural systems and the successful application of stochastic equivalent linearization in nonlinear random vibration analysis of these systems, a number of future research opportunities in the area of analysis and design of timber structures against natural hazards has opened.

ACKNOWLEDGMENT

This research was funded by the USDA National Research Initiative Competitive Grants Program.

REFERENCES

AMIN, M., AND A. H.-S. ANG. 1968. Nonstationary stochastic model of earthquake motions. J. Eng. Mech. ASCE 94(EM2):559–583.

ANG, A. H.-S. 1974. Probabilistic concepts in earthquake engineering. Pages 225–259 *in* W. D. Iwan, ed. Applied mechanics in earthquake engineering. ASME, New York, NY.

ATALIK, T. S., AND S. UTKU. 1976. Stochastic linearization of multidegree of freedom nonlinear systems. Earthquake Eng. Struct. Dyn. 4:411–420.

AUGUSTI, G., A. BARATTA, AND F. CASCIATI. 1984. Probabilistic methods in structural engineering. Chapman and Hall, New York, NY.

BABER, T., AND M. N. NOORI. 1986. Modeling general hysteresis behavior and random vibration application. J. Vibr., Acoust, Stress Reliab Design ASME. 108:411–420.

———, AND Y.-K. WEN. 1981. Random vibration of hysteretic degrading systems. J. Eng. Mech. ASCE 107(EM6):1069–1089.

BOUC, R. 1967. Forced vibration of mechanical systems with hysteresis, Abstract. Proc. Fourth Conference on Nonlinear Oscillation, Praque, Czechoslovakia.

BRANSTETTER, L. J., G. D. JEONG, AND J. T. P. YAO. 1988. Mathematical modelling of structural behaviour during earthquakes. Prob. Eng. Mech. 3(3):130–145.

CECCOTTI A., ED. 1989. Proc. Workshop on Structural Behavior of Timber Construction in Seismic Zones, Florence, Italy.

———, AND A. VIGNOLI. 1990. Engineered timber structures: An evaluation of their seismic behavior. Pages 946–953 *in* Proc. 1990 International Timber Engineering Conference, Tokyo, Japan.

———, AND ———. 1991. Seismic behavior of low-rise portal frames. XV-1–22 *in* Proc. Workshop on Full-scale Behavior of Wood-Framed Buildings in Earthquakes and High Winds, Watford, UK.

CHOU, C. 1987. Modeling of nonlinear stiffness and non-viscous damping in nailed joints between wood and plywood. Ph.D. thesis, Oregon State Univ., Corvallis, OR.

CLOUGH, R. W., AND J. PENZIEN. 1993. Dynamics of structures, 2nd ed. McGraw-Hill Book Co., New York, NY.

COROTIS, R. B. 1982. Design of timber structures for natural hazards. Pages 327–352 *in* R. W. Meyer and R. M. Kellog, eds. Structural use of wood in adverse environments. Van Nostrand Reinhold Co., New York, NY.

DOLAN, J. D. 1989. The dynamic response of timber shear walls. Ph.D. thesis, Univ. of British Columbia, Vancouver, BC, Canada.

DOWRICK, D. J. 1986. Hysteresis loops for timber structures. Bull. NZ Natl. Soci. Earthquake Eng. 19(20):143–152.

EWING, R. D., T. J. HEALEY, AND M. S. AGBABIAN. 1980. Seismic analysis of wood diaphragms in masonry buildings. Pages 253–276 *in* Proc. Workshop on Design of Horizontal Wood Diaphragms, Applied Technology Council, Berkeley, CA.

FOLIENTE, G. C. 1993. Stochastic dynamic response of wood structural systems. Ph.D. dissertation, Virginia Polytechnic Institute and State University, Blacksburg, VA.

———. 1994. Modeling and analysis of timber structures under seismic loads. Pages 87–110 *in* G. C. Foliente, ed. Analysis, design, and testing of timber structures under seismic loads, Proceedings of a research needs workshop, University of California Forest Products Laboratory, Richmond, CA.

———. 1995. Hysteresis modeling of wood joints and structural systems. J. Struct. Eng. ASCE 121(6):1013–1022.

———, M. P. SINGH, AND M. N. NOORI. 1996. Equivalent linearization of generally pinching hysteretic, degrading systems. Earthquake Eng. Struct. Dyn. (to appear).

GAVRILOVIĆ, P., AND K. GRAMATIKOV. 1991. Experimental and theoretical investigations of wooden truss-frame structures under quasi-static and dynamic loads. XXVI-1-37 *in* Proc. Workshop on Full-scale Behavior of Wood-Framed Buildings in Earthquakes and High Winds, Watford, UK.

GUPTA, A. K., AND P. J. MOSS, eds. 1991. Proc. Workshop on Full-scale Behavior of Wood-Framed Buildings in Earthquakes and High Winds, Watford, UK.

HANSON, D. 1990. Shear wall and diaphragm cyclic load testing, cyclic shear fastener testing, and panel durability performance testing of Weyerhaeuser sturdi-wood oriented strand board. Weyerhaeuser Company, Federal Way, WA.

KAMIYA, F. 1988. Nonlinear earthquake response analysis of sheathed wood walls by a computer-actuator on-line system. Pages 1:838–847 *in* Proc. 1988 International Conference on Timber Engineering, Seattle, WA.

KANAI, K. 1967. Semi-empirical formula for the seismic characteristics of the ground. Bull. Earthquake Res. Inst. Univ. of Tokyo, Tokyo, Japan. 35:308–325.

KAREEM, A. 1987. Wind effects on structures: A probabilistic viewpoint. Prob. Eng. Mech. 2(4):166–200.

KIKUCHI, F. 1994. Earthquake resistance of multistorey timber frame structures. Pages 1:205–214 *in* Proc. Pacific Timber Engineering Conference (PTEC '94), Gold Coast, Australia.

KIVELL, B. T., P. J. MOSS, AND A. J. CARR. 1981. Hysteretic modeling of moment resisting nailed timber joints. Bull. NZ Natl. Soc. Earthquake Eng. 14(4):233–245.

KOMATSU, K., M. HARADA, AND T. INOUE. 1994. Development of glulam moment-resisting joints for multistorey timber buildings. Pages 2:36–43 *in* Proc. Pacific Timber Engineering Conference (PTEC '94), Gold Coast, Australia.

KOZIN, F. 1988. Autoregressive moving average models of earthquake records. Prob. Eng. Mech. 3(2):58–63.

LEE, C.-S. 1987. A composite-beam finite element for seismic analysis of wood-framed buildings. Ph.D. thesis, Oregon State Univ., Corvallis, OR.

MIYAZAWA, K. 1990. Study on nonlinear static and dynamic structural analysis of wooden wall-frame buildings subjected to horizontal force. Proc. Thirteenth Symposium on Computer Technology of Information, Systems and Applications, A.I.J., Japan.

NEWMARK, N., AND B. ROSENBLUETH. 1971. Fundamentals of earthquake engineering. McGraw-Hill Book Co., New York, NY.

SAKAMOTO, I., AND Y. OHASHI. 1988. Seismic response and required lateral strength of wooden dwellings. Pages 2:243–247 *in* Proc. 1988 International Conference on Timber Engineering, Seattle, WA.

SHINOZUKA, M., AND G. DEODATIS. 1988. Stochastic process models for earthquake ground motions. Prob. Eng. Mech. 3(3):114–123.

———, AND Y. SATO. 1967. Simulation of nonstationary random processes. J. Eng. Mech. Div. ASCE 93(EM1): 11–40.

SIMULESCU, I., T. MOCHIO, AND M. SHINOZUKA. 1989. Equivalent linearization method in nonlinear FEM. J. Eng. Mech. ASCE 115(3):475–492.

SOONG, T. T., AND M. GRIGORIU. 1993. Random vibration of mechanical and structural systems. Prentice-Hall, Englewood Cliffs, NJ.

SOZEN, M. A. 1974. Hysteresis in structural elements. Pages 63–98 *in* W. D. Iwan, ed. Applied mechanics in earthquake engineering. ASME, New York, NY.

SPANOS, P. D. 1981. Stochastic linearization in structural dynamics. Appl. Mech. Revi. ASME 34(1):1–8.

STEWART, W. G. 1987. The seismic design of plywood-sheathed shear walls. Ph.D. thesis, Univ. of Canterbury, Christchurch, NZ.

TAJIMI, H. 1960. A statistical method of determining the maximum response of a building structure during an earthquake. Pages 2:781–797 Vol. 2 *in* Proc. Second World Conf. on Earthquake Engineering, Tokyo and Kyoto, Japan.

TARABIA, A. M., AND R. Y. ITANI. 1994. Seismic analysis of light-frame wood structures. Proc. Second International Workshop on Full-scale Behavior of Low-rise Buildings, Townsville, Australia. 15 pp.

UBC. 1993. Timber engineering software. Dept. of Civil Engineering, Univ. of British Columbia, Vancouver, BC, Canada.

WEN, Y.-K. 1980. Equivalent linearization for hysteretic systems under random excitation. J. Appl. Mech. ASME 47:150–154.

———. 1988. Equivalent linearization methods. Appendix I *in* L. J. Branstetter, L. J., G. D. Jeong, and J. T. P. Yao, 1988, Mathematical modelling of structural behaviour during earthquakes. Prob. Eng. Mech. 3(3):130–145.

WHALE, L. R. J. 1988. Deformation characteristics of nailed or bolted timber joints subjected to irregular short or medium term lateral loading. Ph.D. thesis, Polytechnic of the South Bank, CNAA, UK.

YASUMURA, M. 1990. Seismic behavior of arched frames and braced frames. Pages 3:863–870 *in* Proc. 1990 International Timber Engineering Conference, Tokyo, Japan.

———, I. NISHIYAMA, T. MUROTA, AND N. YAMAGUCHI. 1988. Experiments on a three-storied wooden frame building subjected to horizontal load. Pages 2:262–275 *in* Proc. 1988 International Conference on Timber Engineering, Seattle, WA.