MOISTURE CHANGES INDUCED IN RED OAK BY TRANSVERSE STRESS

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ABSTRACT

The effect of tensile and compressive stresses on northern red oak, in the radial and tangential directions, was investigated at a number of initial moisture contents. Tensile stresses were found to increase moisture content, while compressive stresses decreased moisture content. The size of the stress-induced change in moisture content approaches 1% at high stress levels and high initial moisture contents and increases in an approximately exponential way with initial moisture content. A thermodynamic relationship used to calculate the change in moisture content under stress produced values in reasonably close agreement with observed changes.

INTRODUCTION

An understanding of sorption isotherms and of the factors affecting them is fundamental to understanding wood-moisture relations. The temperature dependence of sorption isotherms can be explained in fundamental terms, but the effect of mechanical stress on sorption isotherms is not as well known. Fundamental reasons for the effect have been proposed by Barkas (1949) and Barkas, Hearmon, and Rance (1953), but not tested quantitatively to any extent for wood over a wide moisture range. The purposes of the study reported here were: (1) to determine the effect of stress on sorption isotherms, that is, the magnitude of the effect and the effect of some of the variables on the magnitude; and (2) to test the validity of the fundamental reasons that have been proposed to explain the effect.

THEORY

Porter (1907) derived the following equation to describe compressible solutions:

$$(\partial V/\partial m)_p = \nu(\partial h/\partial p)_m, \qquad (1)$$

where h is vapor pressure, ν is the specific volume of solvent, p is hydrostatic pressure, V is the volume of solution per gram of solute, and m is the mass of solvent per unit mass of solute.

The theory for the effect of stress on the moisture content of wood and other hygroscopic gel materials has been developed by Barkas (1949) and Barkas et al. (1953) by extending the thermodynamics of compressible solutions to the sorption of water vapor by rigid gels. He defines a gel in the following manner:

1. Gels adsorb moisture from the surrounding atmosphere and swell. The adsorbed moisture exerts a vapor pressure less than that of the saturation vapor pressure of free water.
2. Gels are rigid materials and can therefore withstand directional stresses.
3. Gels show limited sorption in a saturated atmosphere; that is, they do not adsorb water to the point of dissolution.
4. For the purpose of the theoretical treatment, the gel is assumed to be perfectly elastic.

Equation 1 can be modified for directional stress in rigid gels. For a stress in one direction only, equation (1) becomes

$$(V/x)\,(\partial x/\partial m)_\sigma = \nu(\partial h/\partial \sigma)_m, \qquad (2)$$

where σ is stress in the x direction.

In this study we were concerned with measuring the change in moisture content caused by a constant uniaxial stress in an environment of constant temperature and vapor pressure. Equation (2) can be written to describe this experimental approach and make possible an approximate

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solution to the differential equation (Treloar 1952). The derivation is given in the Appendix. The final equation is

$$(\partial m/\partial \sigma)_h = -[(m+1)/(\nu \rho h_o)] \times (1/x)\,[\partial x/\partial(h/h_o)]_\sigma, \quad (3)$$

where x = dimension in the stress direction, m = mass of water per unit mass of dry material, σ = stress, ν = specific volume of water vapor at vapor pressure h, h = vapor pressure, h_o = saturation vapor pressure, and ρ = swollen density.

The sign convention for stress is positive for compression and negative for tension (Porter 1907; Barkas 1949; Barkas et al. 1953). Since the individual terms on the right side of equation (3) are all positive, the equation predicts a decrease in moisture content under compressive stress and an increase under tensile stress.

To verify equation (3), it is necessary to measure the rate of change of dimension with vapor pressure, $[\partial x/\partial(h/h_o)]_\sigma$. An exact solution to equation (3) would require a knowledge of the functional relationship between dimension, relative vapor pressure, and stress, which is considered beyond the scope of this study. Instead, the solution to equation (3) is approximated in the form:

$$(\Delta m/\Delta \sigma)_h = -[(m+1)/(\nu h_o \rho)] \times (1/x)\,(\lambda_o), \quad (4)$$

where $\lambda_o = [\partial x/\partial(h/h_o)]_{\sigma=0}$ is the stress-free rate of change of dimension with relative vapor pressure.

PREVIOUS WORK

Treloar (1952, 1953) measured the increase in the moisture content of human hair and viscose filaments under tensile stress. One of his objectives was to verify equation (3), which requires the experimental conditions of a constant stress applied at constant relative vapor pressure. For this calculation, it was necessary to make an approximation to the term $\lambda = [\partial x/\partial(h/h_o)]_\sigma$ from a separate experiment. With a small stress on the fiber specimen to keep it straight, he measured the change in dimension with respect to relative vapor pressure. He observed increases in moisture content with tensile stress that ranged from approximately 0.1 to 1.5%, depending upon the material and the stress level. He also found fairly good agreement between observed increases in moisture content and those calculated from equation (3).

Kubát and Nyborg (1962) studied the influence of a constant tensile stress on the equilibrium moisture content (EMC) of kraft paper and found increases ranging from 0.10 to 0.23%. They calculated the increase in moisture content in the same manner as Treloar (1952, 1953) and found close agreement between the measured and calculated values.

Libby and Haygreen (1967) found that the EMC of Douglas-fir was increased by a tensile stress applied in the tangential direction. The increased ranged from 0.02 to 0.20%, depending on the experimental conditions.

EXPERIMENTAL

Goals of the experimental work were to determine the effects of stress on the EMC of wood—specifically, the effects of tensile and compressive stresses in both the radial and tangential directions for red oak while under a constant stress at constant relative humidity and temperature. Further objectives were to determine the effects of initial moisture content on the magnitude of the stress-induced moisture change and to make quantitative comparisons between experimental values and those calculated from equation (3).

Sample preparation

All of the test specimens were cut from one northern red oak (*Quercus rubra*) log. Three flat- and three quarter-sawn boards were cut, and each of the boards provided all of the radial or tangential specimens for one of the three stress levels used in the study (40, 55, and 70% of the maximum tensile strength). Each of the six boards was further cut into six sections to represent six levels of relative humidity (43, 64, 75, 86, 93, and 98%). These sections were cut into specimen blanks 7 by 1½ inches by ½ inch along the grain. The blanks were dried gradually to equilibrium with the approximate relative humidity at which they would be tested, and then cut to the final specimen size—6 inches by ½ inch by ½ inch along the grain for the tension specimens and 2 inches by ½ inch by ½ inch along the grain for the compression specimens.

Environmental control

Tests were made in an environmental cabinet in which relative humidity was controlled by saturated salt solutions. A fan provided circulation within the cabinet, and the entire room housing the cabinet was temperature controlled. After the specimens were cut, they were stored in desiccators over the saturated salt solutions corresponding to the particular test conditions. The salt solutions, relative humidities, and nominal moisture contents are listed in Table 1. All tests were conducted at 25 C.

The test chamber was equipped with tension grips, compression gages, and a 5:1 lever arm system for loading the specimens with dead weights. The system was calibrated with a load cell.

Moisture content measurement

The equilibrium change in moisture content under stress was measured gravimetrically. However, the time required for the moisture content to reach a new equilibrium under stress was determined by a technique similar to that described by Simpson and Skaar (1968). In this technique the change in moisture content as a function of time was approximated by measuring the change in electrical resistance with time after loading. The measurements showed that a new moisture equilibrium could be attained essentially within 24 hr after loading. Therefore, the specimens were stressed for at least 24 hr.

Two replicates were made at each experimental condition. Matched, unstressed controls were included in each test run so that corrections could be made for any change in moisture content due to changes in the temperature or relative humidity. The specimens were allowed to equilibrate in the test cabinet before stressing. Immediately before stressing, each specimen was weighed to the nearest 0.0001 gram to determine its moisture content. To do this, each pair of specimens (one tension and one compression) in one test run, and the unstressed control, were individually wrapped in a vinylidene chloride vapor barrier, and then in aluminum foil, before they were removed from the cabinet. All wrapping was done through portholes in the cabinet, fitted with rubber gloves so that conditions in the cabinet were disturbed as little as possible. After weighing, the specimens were returned to the cabinet, unwrapped, and loaded. After 24 hr, the specimens were unloaded and wrapped as before within several seconds, so that no significant change in moisture content was presumed to occur before the final weighing.

Entire compression specimens were weighed before and after loading. With tension specimens, there were problems associated with this method. Because portions of the tension specimens that were in the grips were not subjected to the same tensile stress as the portions between the grips, the final moisture content was determined by clipping the center portion of the tension specimens for weighing. Since the initial moisture content was based on the entire specimen, this method had the disadvantage of assuming that the initial moisture content was uniform throughout.

TABLE 1. *Saturated salt solutions used to control relative humidity at 25 C*

Salt solution	Relative humidity (%)	Equilibrium moisture content (%)
Potassium carbonate ($K_2CO_3 \cdot 2H_2O$)	43	9
Sodium nitrite ($NaNO_2$)	64	12
Sodium chloride ($NaCl$)	75	14
Potassium chloride (KCl)	86	17
Potassium nitrate (KNO_3)	93	20
Potassium dichromate ($K_2Cr_2O_7$)	98	24

TABLE 2. *Average values of measured moisture changes per unit stress after 24 hr of stress, values calculated from equation (4), and initial equilibrium moisture content values*

Relative humidity %	Tangential Compression Calculated	Tangential Compression Experimental	Tangential Tension Calculated	Tangential Tension Experimental	Radial Compression Calculated	Radial Compression Experimental	Radial Tension Calculated	Radial Tension Experimental
	Change in moisture content (%) per unit stress (PSI $\times 10^3$)							
43	−0.141	−0.0933	+0.0967	+0.0440	−0.0948	−0.0751	+0.0687	+0.0172
64	−0.195	−0.119	+0.235	+0.202	−0.133	−0.0798	+0.139	+0.113
75	−0.220	−0.130	+0.243	+0.203	−0.144	−0.0805	+0.153	+0.122
86	−0.398	−0.189	+0.715	+0.375	−0.215	−0.209	+0.381	+0.0504
93	−0.596	−0.384	+1.358	+1.000	−0.339	−0.182	+0.818	+0.470
98	—	−0.624	—	+1.620	—	−0.335	—	+0.647
	Initial moisture content (%)							
43	—	9.08	—	8.94	—	9.08	—	9.10
64	—	11.48	—	11.35	—	11.75	—	11.65
75	—	13.51	—	13.40	—	13.19	—	13.23
86	—	16.67	—	17.00	—	17.95	—	18.08
93	—	19.32	—	19.32	—	19.70	—	19.66
98	—	24.03	—	23.81	—	24.13	—	24.46

Shrinkage measurement

The method used to determine λ_o of equation (4) was to measure shrinkage for the compression specimens and swelling for the tension specimens between three relative humidities. For example, to determine λ_o for compression specimens at 64% relative humidity, the dimension of a matched specimen was measured at 75, 64, and 43% relative humidity. The dimension, x, was fitted by least squares to a second degree polynomial, with x the dependent variable and relative vapor pressure the independent variable. The first derivative of this equation at $h/h_o = 0.64$ was taken as λ_o. The specimens used in this test were taken from a position in the board adjacent to the specimens that were stressed. The density and moisture content of the matched specimens were used in equation (3). The specific volume of water vapor, v, was taken from Barkas (1949) and checked with another source (Marvin 1941).

Results

The experimental results, summarized in Table 2, show that the EMC of wood is decreased by compression stress and increased by tension stress. The change ranged from 0.02 to 0.62% moisture content, depending on the level of the variables.

Since it was assumed in deriving equation (4) that the rate of change of dimension with respect to relative vapor pressure is independent of stress level, the moisture change per change in stress is also assumed to be independent of stress level. The experimental results gave no evidence that this assumption was invalid. Therefore, this independence was assumed and each stress-induced moisture change was divided by the change in stress ($\triangle\sigma = \sigma - o$), and the moisture change per unit stress was averaged over the three stress levels at each experimental condition. These average values are listed in Table 2, with the corresponding average initial moisture contents. Values calculated from equation (4) are listed also.

These data were analyzed by linear regression techniques. Plots of the logarithm of the change in moisture content per unit stress versus initial moisture content are approximately linear, leading to the model

$$Q = a \exp(bM), \qquad (5)$$

where Q is the absolute value of the moisture change per unit stress (per cent-pounds per square inch^{-1}), a and b are constants,

TABLE 3. *Results of regression analyses of the stress-induced change in moisture content on initial moisture content. The least-squares equation is of the form* Q = a exp(bM), *where* Q *is absolute value of the change in moisture content per unit stress,* a *and* b *are constants, and* M *is per cent moisture content.* R *is the correlation coefficient and* F *is the variance ratio*

Item	Tangential				Radial			
	Compression		Tension		Compression		Tension	
	Calculated	Experimental	Calculated	Experimental	Calculated	Experimental	Calculated	Experimental
$a \times 10^5$	4.494	2.564	1.179	1.050	3.926	2.433	0.9535	0.9228
b	0.12180	0.12770	0.24487	0.22247	0.09638	0.10822	0.22044	0.16729
R	0.9817	0.9859	0.9840	0.9411	0.9903	0.9570	0.9942	0.7276
F	79.68	138.7	91.37	31.01	151.9	43.57	257.0	4.500
Level of significance (%)	99	99	99	99	99	99	99	90

and M is the initial per cent moisture content.

The regression analysis was necessarily a weighted one since the sample size varied. The study was designed for equal sample sizes, but experimental difficulties resulted in the loss of some data. Since each value in Table 2 is the average of a number of specimens, the weighting factor was taken as the number of values making up the average (Steele and Torrie 1960). The results of the regression analysis are shown in Table 3, where the constants a and b, the correlation coefficient R, and the F value for the test of statistical significance are listed. The results are plotted in Figs. 1 to 4.

A number of comparisons were made between the eight regressions. The purpose was to compare both the slopes and levels of the regression equations between calculated and experimental results, radial and tangential results, and tension and compression results. The 12 paired comparisons are listed in the Appendix.

The procedure for these covariance analyses is that outlined by Freese (1964). The results can be summarized by three general observations:

1. The moisture changes per unit stress calculated from equation (4) are consistently greater than those measured experimentally. The dependence of the calculated and measured results on initial moisture content was the same.

2. While the moisture change per unit

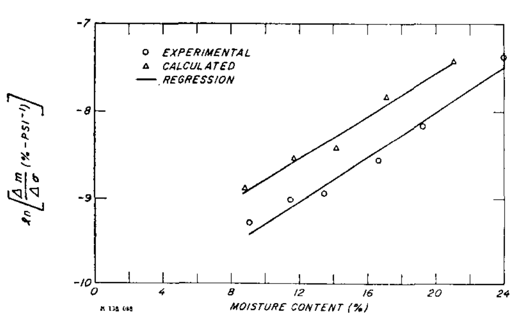

FIG. 1. Regression of the absolute value of the change in moisture content per unit stress on initial moisture content for compression in the tangential direction.

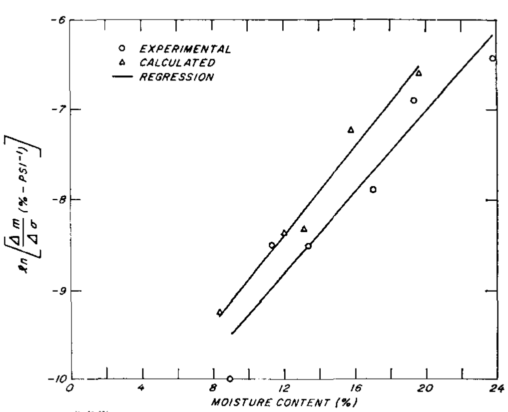

FIG. 2. Regression of the absolute value of the change in moisture content per unit stress on initial moisture content for tension in the tangential direction.

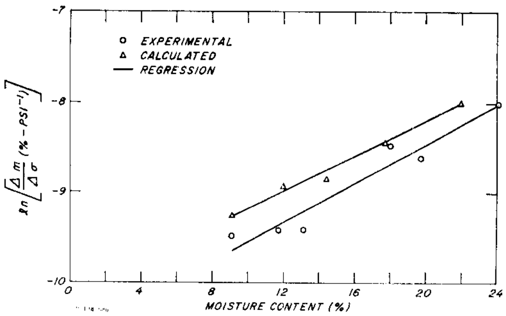

FIG. 3. Regression of the absolute value of the change in moisture content per unit stress on initial moisture content for compression in the radial direction.

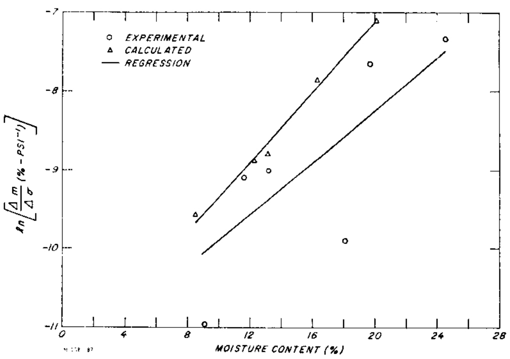

FIG. 4. Regression of the absolute value of the change in moisture content per unit stress on initial moisture content for tension in the radial direction.

stress increased with initial moisture content for both tension and compression specimens, the rate of increase with initial moisture content was greater in tension than in compression. The magnitude of the moisture content change was greater in tension than in compression.

3. The moisture change per unit stress was greater for a stress in the tangential direction than for a radial stress. The dependence on initial moisture content was the same in both directions.

DISCUSSION

Calculated vs. experimental results

There are several possible reasons why the calculated values are greater than the experimental values. Equation (3) is a differential equation, and at this point no analytical solution is known to the author. Therefore, it is necessary to deal with measurable changes in m, σ, x, and h, and to assume that the relationships obtained between the variables using finite increments approximate the continuous relationships given in equation (3). Furthermore, Barkas' development assumes perfect elasticity, i.e., that Hooke's law is obeyed and that all changes are reversible. The errors that may be involved in these assumptions sound formidable, but Treloar (1952, 1953) and Kubát and Nyborg (1962) made calculations similar to those in this study and found reasonable agreement between calculated and experimental results—better, in fact, than the agreement found in this study.

Kubát and Nyborg (1962) point out that the role of Poisson's ratio is not included in the mathematical treatment of Barkas. A derivation that includes the effect of Poisson's ratio is given in the Appendix. The final equation for a radial stress is:

$$\begin{aligned}(\partial m/\partial \sigma)_h = - &[1 - \mu_{RT}(\delta R/R)] \\ \times &[1 - \mu_{RL}(\delta R/R)] \\ \times &[TL/(W_o v h_o)] \\ \times &[\partial R/\partial (h/h_o)]_{\sigma_R}, \qquad (6)\end{aligned}$$

where μ is Poisson's ratio, R, T, and L are the radial, tangential, and longitudinal dimensions, and W_o is the mass of dry wood.

The effect of this correction on the calculated results was tested and found to make a very small reduction (less than 1%) in the calculated results. Total strain at the end of the test runs was used for $\delta R/R$ and $\delta T/T$ (total strain was measured for the specimens at 64 and 75% relative humidity in a separate phase of the study), and the Poisson ratios for oak were taken from Hearmon (1948).

The differences between the calculated and experimental values may also be resident in the term λ. This term was measured at $\sigma = 0$ in the study. Treloar (1952, 1953) measured λ at $\sigma \approx 170$ pounds per square inch (psi), while he measured the changes in moisture content at stresses that

TABLE 4. *Results of regression analyses of the rate of change of dimension with respect to relative vapor pressure,* λ_o, *on per cent initial moisture content,* M. *The least-squares equation is of the form* $\lambda_o = p\,exp(qM)$, *where* p *and* q *are constants.* R *is the correlation coefficient and* F *is the variance ratio.*

Item	Tangential		Radial	
	Compression (desorption)	Tension (adsorption)	Compression (desorption)	Tension (adsorption)
p	0.04434	0.01283	0.0409	0.01026
q	0.05760	0.1710	0.03087	0.1475
R	0.8376	0.9457	0.7822	0.9462
F	7.055	25.41	4.728	25.64
Level of significance (%)	90	97.5	80	97.5

ranged from approximately 800 to 7,000 psi. Kubát and Nyborg (1962) measured λ at $\sigma \approx 15$ psi, while the change in moisture content was measured at stresses of 3,000 to 6,000 psi. Compared to the actual stresses used to measure the change in moisture content, these measurements of λ are essentially stress-free shrinkage or swelling and may not be the same as shrinkage or swelling at higher stresses. Treloar (1952) made a brief study of this problem and found that λ was lowered by almost 20% when the stress was increased from 170 to 3,300 psi. If this effect of stress on λ is real, then the calculated values of the change in moisture content per unit stress are actually lower than the values calculated using λ_o, which would bring the calculated and experimental values obtained in this study into closer agreement.

The slopes of the regression lines of the moisture change per unit stress on initial moisture content have been shown to be the same for the calculated and experimental results, and this lends support to the principles underlying Barkas' theory. The terms in equation (4) that cause the moisture change per unit stress to increase with initial moisture content are $m + 1$, ν, ρ, and λ_o. Both $m + 1$ and ρ change very little with moisture content. The density, $1/\nu$, of water vapor increases with moisture content, but does so at a decreasing rate. Therefore, the term $1/\nu$ cannot account for the nonlinear increase in the moisture change per unit stress with initial moisture content. Table 4 shows the results of a regression analysis of λ_o on initial moisture content, using the same model as equation (5). Although the results do not conclusively show an exponential relationship, they do suggest that this term increases with initial moisture content at a rate that is greater than linear and therefore it is the most important factor in causing the moisture change per unit stress to increases exponentially with initial moisture content. The variation in λ_o with initial moisture content can also be traced to the sorption isotherm. This term can be written as:

$$\partial x/\partial(h/h_o) = (\partial x/\partial m)\,[\partial m/\partial(h/h_o)]. \quad (7)$$

The first term on the right side of equation (7) is essentially constant with moisture content, i.e., swelling and shrinking are nearly linear with moisture content. The second term is the slope of the sorption isotherm, which increases with moisture content above the inflection point in the isotherm, i.e., moisture content increases with relative vapor pressure at a rate that is greater than linear.

Tension vs. compression results

The results shown in Figs. 1 to 4 indicate that the moisture change per unit stress increases with initial moisture content at a greater rate in tension (absorption) than in compression (desorption). This is reflected in the regression analyses of λ_o on initial moisture content. Paired comparisons showed that λ_o increased with moisture content faster in absorption than in desorption.

Tangential vs. radial results

The results indicate that the moisture change per unit stress is greater for a tangential than for a radial stress, and that the dependence on initial moisture content is the same. Again, this can be traced to λ_o, where paired comparisons between tangential and radial results showed that λ_o increased with moisture content at the same rate in both directions, but that the level of λ_o was greater in the tangential direction.

CONCLUSIONS

1. The sorption isotherm of wood is stress dependent. The equilibrium moisture content of wood is increased by a tensile stress and decreased by a compressive stress.
2. The moisture change per unit stress increased approximately exponentially with initial moisture content. The rate of increase was greater in tension than in compression, and the magnitude of the change was greater in tension than in compression.
3. The moisture change per unit stress was greater for a tangential than for a radial stress. The dependence on initial moisture content was the same in both directions.
4. The values for the change in moisture content calculated from the theoretical relationship were somewhat higher than the experimental values. This may be due in part to the technique and in part to the assumptions incorporated in the theory. Conclusions (1) to (3), however, are predicted by the theoretical relationship, and therefore the principles underlying the Barkas theory appear to be correct.

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APPENDIX

Derivation of equation (3)

From the rules of partial derivatives, the following identity holds:

$$(\partial h/\partial \sigma)_m = -(\partial m/\partial \sigma)_h(\partial h/\partial m)_\sigma. \quad (1A)$$

Substituting equation (1A) into equation (2) in the text and rearranging,

$$(\partial m/\partial \sigma)_h = -(V/\nu)(1/x)(\partial x/\partial h)_\sigma. \quad (2A)$$

If the density, ρ, is defined as

$$\rho = W_m/(W_o V), \quad (3A)$$

where W_m and W_o are swollen and dry weights, respectively, then

$$V = (m + 1)/\rho. \quad (4A)$$

Substituting equation (4A) into equation (2A), and letting $h = h_o(h/h_o)$ results in equation (3) in the text.

Paired comparisons

The symbology for the 12 paired comparisons between slopes and levels of equation (5) in the text is as follows: The first letter (T or R) is for stress in the tangential or radial direction; the second letter (C or T) is for compression or tension; the third letter (C or E) is for calculated results (from equation [4]) or experimental results. The first group of four compares the calculated and experimental results.

TCC vs. TCE
TTC vs. TTE
RCC vs. RCE
RTC vs. RTE

The second group of four compares the results of compression stress to those of tension.

TCC vs. TTC
TCE vs. TTE
RCC vs. RTC
RCE vs. RTE

The final group of four compares the results of stress in the tangential direction to those in the radial direction.

TCC vs. RCC
TTC vs. RTC
TCE vs. RCE
TTE vs. RTE

Derivation of equation (6)

Barkas (1949) defines the differential swelling for a constant stress in the radial direction, for example, as

$$S_R = (V/R)(\partial R/\partial m)_{\sigma_R} = (TLR/W_oR)(\partial R/\partial m)_{\sigma_R} = (TL/W_o)(\partial R/\partial m)_{\sigma_R}, \qquad (5A)$$

where R, T, and L are the radial, tangential, and longitudinal dimensions. The term W_o is the mass of dry wood and is necessary since V was originally defined as the volume of moist wood per unit mass of dry wood. Equation (5A) states that the differential swelling, S_R, is due to a volume change caused by a change in the radial dimension at constant stress, and that T and L remain constant and do not contribute to this change in volume. In reality, T and L do change because of Poisson expansion or contraction. If δT and δL are the Poisson expansion or contraction, equation (5A) becomes

$$S_R = (T + \delta T) \times (L + \delta L)/W_o(\partial R/\partial m)_{\sigma_R}. \qquad (6A)$$

For a stress in the radial direction:

$$\begin{aligned} \mu_{RT} &= -(\epsilon_T/\epsilon_R) = -(\delta T/T)/(\delta R/R) \\ \mu_{RL} &= -(\epsilon_L/\epsilon_R) = -(\delta L/L)/(\delta R/R), \end{aligned} \qquad (7A)$$

where the μ's are the Poisson ratios and the ϵ's are the strains in the subscripted directions. From equation (7A):

$$\begin{aligned} \delta T &= -\mu_{RT}T(\delta R/R) \\ \delta L &= -\mu_{RL}L(\delta R/R), \end{aligned} \qquad (8A)$$

Substituting equation (8A) into equation (6A)

$$S_R = [1 - \mu_{RT}(\delta R/R)]\,[1 - \mu_{RL}(\delta R/R)] \times (TL/W_o)(\partial R/\partial m)_{\sigma_R}. \qquad (9A)$$

Following the same general derivation that led to equation (3), the corrected form, for a radial stress, is equation (6) in the text.