STOCHASTIC MODEL FOR LOCALIZED TENSILE STRENGTH AND MODULUS OF ELASTICITY IN LUMBER

Steven E. Taylor

Assistant Professor
Department of Agricultural Engineering
Auburn University, AL 36849-5417

and

Donald A. Bender

Associate Professor
Department of Agricultural Engineering
Texas A&M University
College Station, TX 77843-2117

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ABSTRACT

Localized modulus of elasticity (MOE) and tensile strength (T) were modeled for two visual grades of Douglas-fir laminating lumber. These material property models will be used as input to other structural analysis models that predict the strength and reliability of glued-laminated beams. Tensile strength and MOE are important material properties since most glued-laminated (glulam) timber beam failures initiate in the tension zone. Localized MOE and T exhibited significant within-piece variability as well as between-piece variability. These localized properties were also spatially correlated. A method that uses a transformation of the multivariate normal distribution was developed to simulate these localized properties for lumber up to 8-ft long. This method preserved the probability distributions of localized MOE and T as well as the spatial correlations between the localized property values. Procedures were described for expanding the model to simulate boards of any length. The method was also used to simulate long-span tensile strength. Mean simulated tensile strengths compared favorably with test results. Test results also confirmed a reduction in tensile strength as test span increased.

Keywords: Modulus of elasticity, tensile strength, laminating lumber, spatial variability, stochastic.

INTRODUCTION

The design of engineered wood structures, such as trusses and glued-laminated beams, is governed by the mechanical properties of their individual components. Tensile strength (T) of the lower truss chord or tension lamination can be the critical design parameter in long-span trusses or glued-laminated timber beams, respectively. Deflection of the truss or beam, which is a function of modulus of elasticity (MOE), can also be the critical design criterion in many applications.

If the MOE and T of the constituent lumber are known with certainty, the structural design process is straightforward. Unfortunately, the biological nature of wood causes wide variations in these two lumber properties. Variability in MOE and T exists between each piece of lumber and within each piece of lumber. This wide variation in strength and stiffness characteristics complicates the design process and leads to uncertainty in the performance of the total wood structure.

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Reliability-based design techniques allow the designer to account for this variability in structural performance. This design method requires accurate statistical characterization of the strength of the structural member. Unfortunately, the time and expense required to destructively test a sufficient sample of trusses or beams would be prohibitive. One solution to this testing dilemma is to use computer simulation. Monte Carlo simulation has been used to predict reliability of many engineered wood structures, such as glued-laminated timber beams (Foschi and Barrett 1980; Glos and Michel 1983; Bender et al. 1985; Ehlbeck et al. 1985a, b; Schaffer et al. 1986). Although there are differences in the structural analysis techniques employed in these models, one common attribute of these analyses is that they require accurate information on localized strength properties of the individual components (or elements) of the structural member.

Most current models of lumber material properties are based on data that were collected from long-span tests; i.e., they account only for variability between different pieces of lumber. These models do not account for the variation of material properties within a given piece of lumber. However, this within-piece variability information is critical for structural analysis techniques that require localized properties of individual elements, such as the finite element method.

Within-piece, or spatial, variability of MOE and T was recently investigated by Kline et al. (1986) and Showalter et al. (1987) for two grades and two sizes of southern pine lumber. Further research is needed to model spatial variability of localized MOE and T for other important species and grades of lumber that are used in engineered wood components, such as glued-laminated timber beams.

The objectives of this research were:

1. To characterize localized MOE and T properties in two visual grades of Douglas-fir laminating lumber.
2. To develop a stochastic model for simulating localized MOE and T. This model can be used as input to structural analysis models that predict the reliability of glued-laminated beams.

BACKGROUND

Since most failures of glulam beams initiate in the tension zone, special grades of lumber have been developed for use as tension laminations in these beams (AITC 1987). Several studies (Marx and Evans 1986, 1988; Wolfe and Moody 1981) have investigated characteristics of MOE and T for high quality southern pine and Douglas-fir laminating lumber suitable for use as tension laminations. As in many other investigations, these studies of lumber strength properties emphasized long-span values of tensile strength and MOE.

Strength reducing characteristics in lumber, such as knots and grain deviations, can cause variation in strength and stiffness within pieces of lumber. A few studies have addressed the problem of lengthwise variability in the mechanical properties of lumber. However, much of this research emphasized relating lumber strength to the minimum stiffness within the piece of lumber and therefore was performed to develop more improved methods of lumber grading (Corder 1965; Kass 1975; Bechtel 1985; Foschi 1987). These studies were primarily concerned with predicting long-span strength and did not address techniques for modeling localized stiffness and strength.

Kline et al. (1986) investigated the lengthwise variability of MOE along 30-inch segments of southern pine lumber. They found that 30-inch segment values of MOE within each piece of lumber were serially correlated. They developed a second-order Markov, or second-order autoregressive, model to simulate the lengthwise variability of MOE.

Showalter et al. (1987) expanded the work of Kline by modeling spatial variability of T as well as MOE for two grades and two sizes of southern pine lumber. Showalter combined the Markov MOE model presented by Kline with a regression approach developed by Woeste et al. (1979) that related MOE to T. Within the regression approach, Showalter modeled the regression residuals as a parallel Markov process.

A new approach for simulating long-span correlated lumber properties was developed by Taylor and Bender (1988). This approach used a transformation of the multivariate normal distribution and could be extended to as many variables, or properties, as desired. This method exactly preserves the probability distributions of each random variable and closely approximates the correlation between the variables. Taylor and Bender (1989) demonstrated how the method could be used to simulate multiple correlated lumber properties by simulating four spatially correlated values of localized MOE. This paper will build on the description of modeling MOE and extend the procedure to simulate both localized MOE and tensile strength.

EXPERIMENTAL PROCEDURE

Two high-quality, visual grades of Douglas-fir laminating lumber were studied: 302-24 and L1. Lumber was sampled from five laminators. Each laminator supplied 100 randomly selected 14-ft 2 by 6's from each of the two grades as they were graded by the laminating plant grader. This sampling scheme was intended to produce a representative sample of the type of lumber being used in the manufacture of glulam timber beams. The 302-24 and L1 groups of lumber, when combined, contained 500 and 502 boards, respectively.

All testing procedures followed ASTM Standards D198 and D4761 (ASTM 1989a, b) and are summarized as follows:

1. The lumber was conditioned to a moisture content of approximately 12%, (dry-basis). Length, width, depth, moisture content, and weight were then recorded for each board. Location and size of knots were also recorded for each board.
2. Localized MOE was measured on 4 contiguous 2-ft segments within each board using flatwise bending over a 6-ft span with third-point loading (the maximum applied load was 400 pounds). The influence of shear was minimized by measuring deflection relative to the load head, over the center 2-ft portion of the total 6-ft span.
3. The lumber specimens of each grade were then randomly divided into 3 groups with sample sizes of 300 boards, 100 boards, and 100 boards, respectively.
   a) The group with 300 boards was destructively tested in tension parallel-to-grain at two locations within each board using a 2-ft test span. These data were used to develop stochastic models of MOE and T.

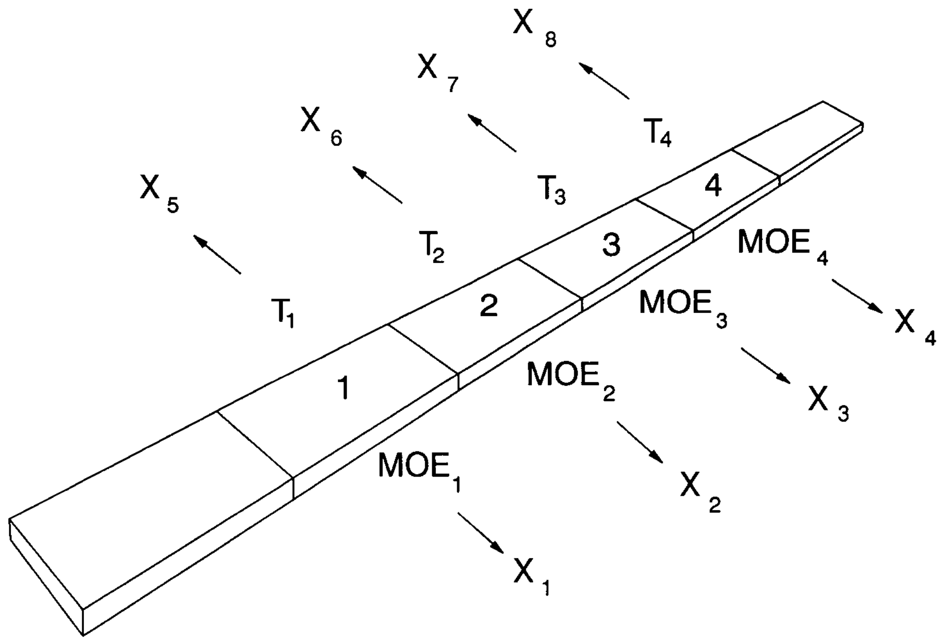

FIG. 1. Sketch of 14-ft 2 × 6 shown with numbering scheme for 2-ft segments. Also shown is the graphical illustration of the transformation from segment MOE and T values to the multivariate vector $\underline{X} = (x_1, x_2, \ldots, x_8)^T$.

b) The remaining two groups (of 100 boards each) were destructively tested in tension at test spans of 5-ft and 7-ft, respectively. These data were used to validate the stochastic MOE and T models.

The rate of loading for all of the tensile tests was approximately 4000 psi/minute, resulting in failure times from 1 to 2 minutes.

The 24-inch element length was chosen because of testing limitations related to the flatwise-bending and tensile strength measurements. The numbering scheme for the 24-inch segments is shown in Fig. 1. It is desirable to obtain tensile strength measurements for each of the four segments; however, this was impossible since space had to be allowed for the grips of the tension machine. Furthermore, there was a possibility that specimens could split during a test and damage adjacent segments. Because of these constraints, the lumber was cut in half, and tensile strengths were obtained for Segments 1 and 4.

LABORATORY TEST RESULTS

Summary statistics for localized MOE and T are listed in Table 1 for each grade and supplier. Multivariate analyses of variance (MANOVA) were performed on the localized MOE and T data to determine if there were significant differences in the properties of the lumber from the different laminators. The MANOVA tests indicated significant differences (at a significance level of 0.01) in the properties of lumber from the various laminators. Hence, it may be advisable to use varying statistical characteristics of lumber to simulate mill variation when modeling glulam beam performance. Therefore lumber property statistics are reported

TABLE 1. *Test results for 2-ft segment modulus of elasticity (MOE) and tensile strength parallel-to-grain (T).*

Grade and supplier		Modulus of elasticity				Tensile strength				
		Sample size	Mean (10^6 psi)	Standard deviation (10^6 psi)	COV (%)	Sample size	Mean (psi)	Standard deviation (psi)	COV (%)	5% exclusion limit* (psi)
302-24	A	240	2.609	0.332	12.72	117	10,303	3,051	29.61	5,294
302-24	B	240	2.658	0.395	14.86	120	10,396	2,993	28.79	5,277
302-24	C	240	2.221	0.463	20.85	120	10,513	2,437	23.18	7,055
302-24	D	240	2.712	0.415	15.30	119	9,266	2,779	29.99	5,223
302-24	E	240	2.427	0.367	15.10	119	8,937	2,595	29.04	4,762
302-24	All	1,200	2.526	0.435	17.22	595	9,884	2,846	28.79	5,257
L1	A	244	2.306	0.377	16.34	122	7,338	2,722	37.09	3,553
L1	B	240	2.427	0.388	16.00	120	7,968	2,456	30.82	4,577
L1	C	240	2.198	0.339	15.41	120	7,378	2,249	30.48	4,261
L1	D	240	2.288	0.412	18.02	119	6,851	2,320	33.86	3,708
L1	E	240	2.132	0.355	16.41	120	6,492	2,297	35.38	3,342
L1	All	1,204	2.270	0.388	17.08	601	7,206	2,460	34.14	3,645

* Non-parametric fifth percentile at 50% confidence.

in this paper for each supplier. However, to illustrate the simulation model, the data from all suppliers were combined into one group.

Serial and cross-correlations in localized material properties were estimated from the test data for each of the laminators and each of the two grades as shown in Appendix 1. The lag-k correlation, ρ_k, is the correlation between an observation from one segment and an observation from k previous segments. For example, the lag-2 MOE serial correlation refers to the correlation between the MOE's of Segments 1 and 3 and Segments 2 and 4. In general, the estimated lag-k serial correlation, $\hat{\rho}_k$, for a data set x(1), . . . x(n) is given by:

$$\hat{\rho}_k = \frac{\sum_{i=1}^{n-k} [x(i) - \bar{x}][x(i+k) - \bar{x}]}{\sum_{i=1}^{n} [x(i) - \bar{x}]^2} \tag{1}$$

where x(i) is the i_{th} observation of X and $\bar{x}$ is the sample mean. Similarly, the estimated lag-k cross-correlation for a data set x(1), . . . , x(n) and y(1), . . . , y(n) is given by:

$$\hat{\rho}_k = \frac{\sum_{i=1}^{n-k} [x(i) - \bar{x}][y(i+k) - \bar{y}]}{\sqrt{\sum_{i=1}^{n} [x(i) - \bar{x}]^2 \sum_{i=1}^{n} [y(i) - \bar{y}]^2}} \tag{2}$$

The lag-1, lag-2, and lag-3 serial correlations in localized MOE were estimated directly from the data since there were four observations of 2-ft MOE per board. Since there were only two observations of 2-ft T per board and since they were separated by two segments, only the lag-3 serial correlation in T was calculated directly from the test data. Procedures for estimating the lag-1 and lag-2 serial correlation coefficients in T will be discussed in a succeeding section. Cross-correlation between localized MOE and T was calculated for lag-0, lag-1, lag-2, and lag-3. Additional details on estimation of the correlation coefficients can be found in Taylor (1988).

Significant serial and cross-correlation existed for localized MOE and T. For example, the lag-1, lag-2, and lag-3 MOE serial correlation coefficients for the 302-24 lumber from all suppliers were 0.9581, 0.9277, and 0.8902, respectively. The lag-3 T serial correlation coefficient was 0.5136. These high serial correlations can probably be attributed to the high quality of this laminating grade. The lag-0, lag-1, lag-2, and lag-3 MOE-T cross-correlation coefficients for the 302-24 lumber were 0.4015, 0.3603, 0.3407, and 0.3322, respectively. Overall, the lag-1 MOE serial correlation coefficients ranged from 0.8956 to 0.9581 for the various groups of lumber. Lag-3 T serial correlation coefficients ranged from 0.6637 to 0.8661 and lag-0 MOE-T cross-correlation coefficients ranged from 0.4015 to 0.7677. The serial correlation coefficients for MOE and T were similar to those reported by Kline et al. (1986) and Showalter et al. (1987). The lag-0 cross-correlation between MOE and T was similar to results reported by other researchers (Doyle and Markwardt 1967) for similar quality lumber.

SIMULATION OF LOCALIZED MOE AND T

A multivariate statistical approach developed by Taylor and Bender (1989) was used to model the localized MOE and T data. The following text describes how the multivariate method is applied to an 8-ft-long board comprised of four 2-ft segments. The four 2-ft segment MOE's shown in Fig. 1, denoted by X_1 through X_4, and the four 2-ft segment T's, denoted by X_5 through X_8, are treated as 8 correlated random variables. The procedure can be used to generate random vectors $\underset{\sim}{X}$ containing observations of 8 correlated variables $(x_1, x_2, \ldots, x_8)^T$. Hence, each random vector can be thought of as a single board with 4 observations of segment MOE's and 4 observations of segment T's.

The procedure for modeling the correlated variables begins by estimating the parameters for the best fitting probability distributions for each variable, X_1, X_2, . . . , X_8. These distribution functions, $Fx_1(x)$, $Fx_2(x)$, . . . ,$Fx_8(x)$ need not be normal. This step only involves fitting a distribution to the segment MOE's and the segment T's. It is assumed that the MOE's and T's of the different segments are identically distributed; therefore, one MOE and one T distribution is used to represent the properties of each segment. Appropriate goodness-of-fit tests should be performed on the hypothesized distributions.

The second step in modeling the variables is to estimate the correlation matrix, $\underset{\sim}{\Sigma}$, for the variables X_1, X_2, . . . , X_8. This step involves calculating the lag-0, lag-1, lag-2 and lag-3 serial correlation for the segment MOE's and the segment T's. It also involves calculating the lag-0, lag-1, lag-2 and lag-3 cross-correlation between segment MOE's and segment T's. The calculation of most of these correlation coefficients is straightforward. However, since it is not possible to determine the tensile strengths of Segments 2 and 3, or X_6 and X_7, the lag-1 and lag-2 serial correlations for tensile strength could not be directly estimated from experimental data; only the lag-3 serial correlation could be estimated from the test data.

Showalter et al. (1987) used the assumption that tensile strength can be approximated by a first-order autoregressive process [AR(1)], also referred to as a first-order Markov process. The autocorrelation function for the AR(1) model is:

$$\rho_k = \rho_1{}^k \qquad \text{for } k > 0. \tag{3}$$

Equation (3) can be used to estimate the lag-1 and lag-2 serial correlation coefficients, ρ_1 and ρ_2, in tensile strength given the lag-3 serial correlation, ρ_3, by:

$$\rho_1 = \rho_3{}^{1/3} \tag{4}$$

$$\rho_2 = \rho_3{}^{2/3} \tag{5}$$

All of the correlation coefficients can now be substituted into the 8 × 8 correlation matrix, Σ. The simulation procedure can proceed once the probability distributions have been selected and the correlation matrix has been determined.

Simulating the correlated variables begins by generating vectors from the multivariate normal distribution using the localized MOE and T correlation matrix, Σ. Although these vectors of lumber properties will have the correct spatial correlation structure, they will all be normally distributed.

A transformation is used to convert the normally distributed observations to observations with the best fitting univariate distribution. The transformation begins by evaluating the standard normal cumulative distribution function for each component of each vector resulting in vectors of observations distributed uniformly between 0 and 1, i.e., U(0, 1). These uniformly distributed vectors are then substituted into the appropriate inverse cumulative distribution function (either for MOE or for T) to obtain vectors of correlated observations from the correct univariate probability distributions.

The transformation between the normal and non-normal distributions is demonstrated graphically in Fig. 2. Software is widely available for numerically inverting distribution functions that cannot be expressed in closed form. Since all cumulative distribution functions are uniformly distributed between 0 and 1, the transformation exactly preserves each of the univariate distributions. This nonlinear transformation results in simulated data with a correlation matrix that closely approximates the original matrix. Experience with numerous sets of lumber properties data indicates the the transformation has no significant effect on the original correlation matrix.

SIMULATION RESULTS AND DISCUSSION

Modeling localized MOE and T

All of the 302-24 data were combined into one group to illustrate the simulation method. The three-parameter Weibull distribution provided the best fit for the segment MOE and T data from the 302-24 lumber. Figures 3 and 4 contain frequency histograms of the original data with overlays of the fitted probability density functions for MOE and T. Parameter estimates for the Weibull distributions are listed on the figures. Chi-square goodness-of-fit tests were performed on the fitted distributions. These distributions could not be rejected at the 0.10 significance level. Distribution parameters for the lumber from each laminator and each grade are listed in Appendix 2.

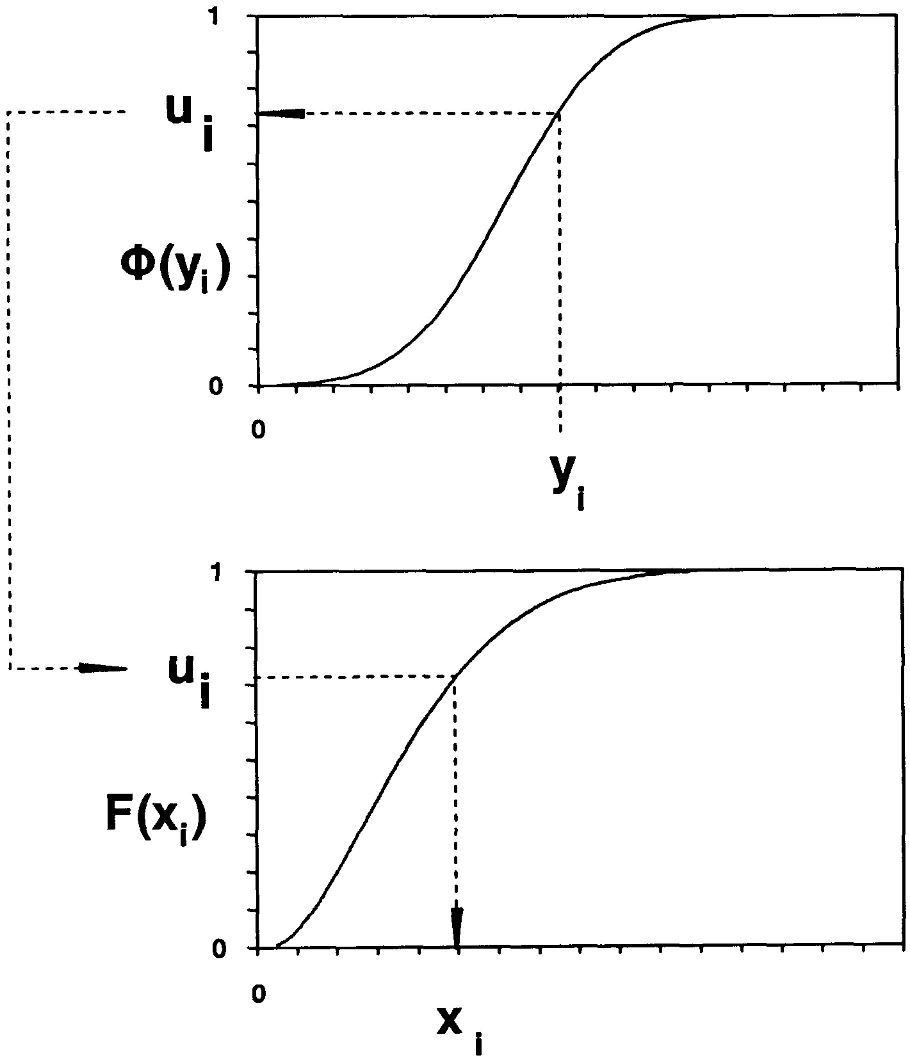

FIG. 2. Illustration of the transformation from the normal distribution, $\Phi(y)$, to the non-normal distribution, $F(x)$.

The serial and cross-correlation coefficients listed in Appendix 1 were used to fill in the correlation matrix for input to the simulation procedure. All correlation coefficients were estimated from the test data except the lag-1 and lag-2 serial correlation coefficients for tensile strength. These coefficients were estimated using the assumption of a first-order autoregressive model discussed earlier. The complete set of original correlation coefficients is listed in Table 2.

The algorithm presented earlier was used to simulate segment values of MOE and T for the 302-24 lumber. One thousand multivariate normal vectors of length 8 were generated using the correlation matrix estimated from the test data. Then the normal cumulative distribution function was evaluated for the 8 components of each vector. This step resulted in vectors containing 8 correlated U(0, 1)

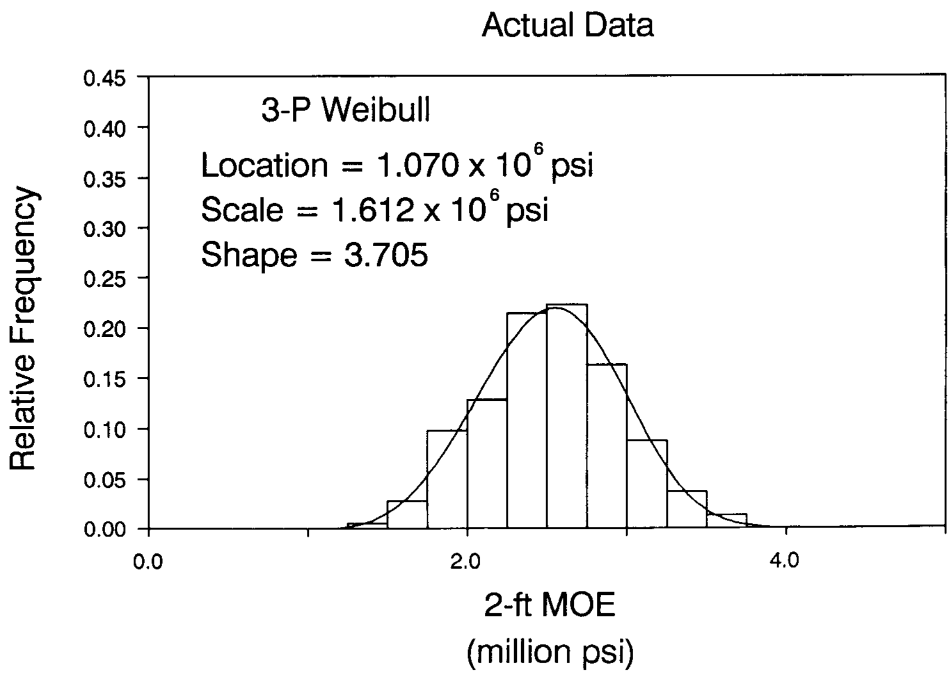

FIG. 3. Frequency histogram with best fitting probability density function overlaid for 2-ft segment MOE. The lumber group contained 302-24 lumber from all suppliers combined.

observations. These U(0, 1) observations were substituted into the appropriate inverse cumulative distribution functions for MOE and T. The result of this step was a set of 1,000 vectors containing 8 correlated components representing 1,000 imaginary 8-ft long boards with 4 segment MOE's and 4 T's. This entire procedure was replicated 10 times so confidence intervals could be constructed for the simulated correlation values.

Validation of the simulation procedure

Probability distributions.—As previously mentioned, the multivariate approach is formulated to exactly preserve the distributions of each random variable. As an additional check, histograms of the simulated segment MOE's and T's with overlays of the original probability density functions are shown in Figs. 5 and 6 for the MOE and T of the 302-24 lumber, respectively. Kolmogorov-Smirnov (KS) goodness-of-fit tests were used to test the hypotheses that the simulated data could have been a sample from the given probability distributions. Ninety-five percent confidence intervals were calculated for the values of the KS test statistics after the 10 replications. The confidence interval for the KS statistic for MOE was [0.01944, 0.03496]. The confidence interval for the KS statistic for T was [0.02394, 0.03124]. The critical KS statistic for $n = 1{,}000$, for $\alpha = 0.15$, and for the case where all parameters are known is 0.03585. Therefore, since the upper bounds of both confidence intervals were below the critical value, both hypotheses could not be rejected at the 0.15 level of significance (it is easier to reject the null hypothesis as the sample size and the level of significance increase).These figures

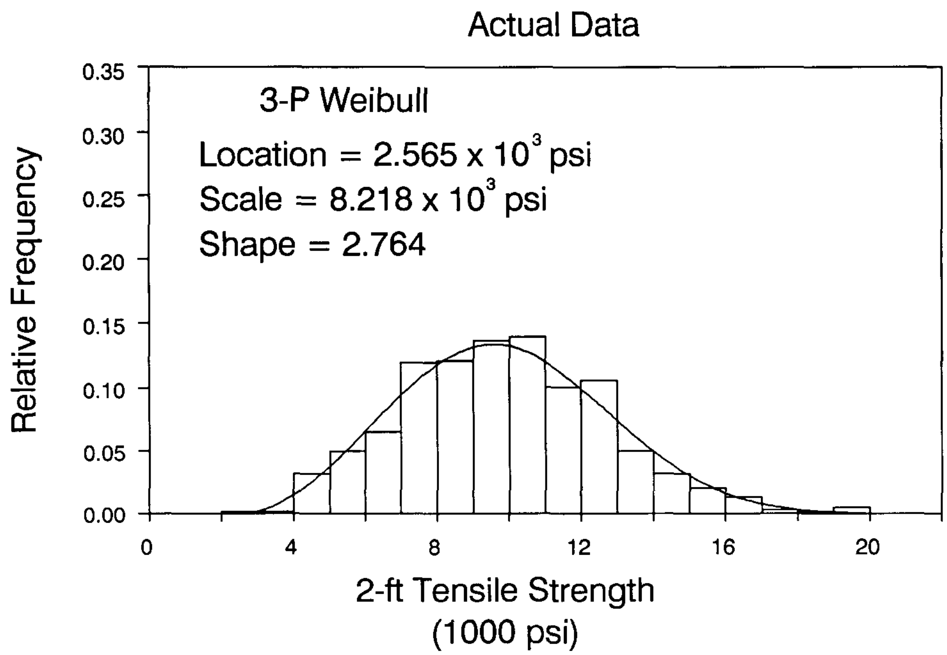

FIG. 4. Frequency histogram with best fitting probability density function overlaid for 2-ft segment tensile strength. The lumber group contained 302-24 lumber from all suppliers combined.

and hypothesis tests are confirmation that the random number generator functioned properly and that the algorithm exactly preserved the marginal distributions.

Correlation matrices.—The second step in the validation process was to determine if the correlation matrix was preserved by the simulation procedure. Sample

TABLE 2. *Serial and cross-correlation coefficients from actual data, average correlation coefficients from simulated data, and 99% confidence intervals for the simulated correlation coefficients. The lumber group contained all of the 302-24 lumber. The maximum difference between the simulated correlation coefficients and those from actual data was 1.8%. This difference is not practically significant.*

Correlation		Correlation from actual data	Mean correlation from simulated data	Simulated correlation confidence limits	
				Lower	Upper
Serial MOE	lag-0	1.0000	1.0000	1.0000	1.0000
Serial MOE	lag-1	0.9581	0.9580	0.9549	0.9611
Serial MOE	lag-2	0.9277	0.9260	0.9210	0.9310
Serial MOE	lag-3	0.8902	0.8894	0.8825	0.8963
Cross MOE-T	lag-0	0.4015	0.4045	0.3658	0.4432
Cross MOE-T	lag-1	0.3603	0.3647	0.3309	0.3985
Cross MOE-T	lag-2	0.3407	0.3430	0.3047	0.3813
Cross MOE-T	lag-3	0.3322	0.3383	0.3034	0.3731
Serial T	lag-0	1.0000	1.0000	1.0000	1.0000
Serial T	lag-1	0.8008	0.8020	0.7865	0.8174
Serial T	lag-2	0.6413	0.6394	0.6175	0.6613
Serial T	lag-3	0.5136	0.5105	0.4935	0.5275

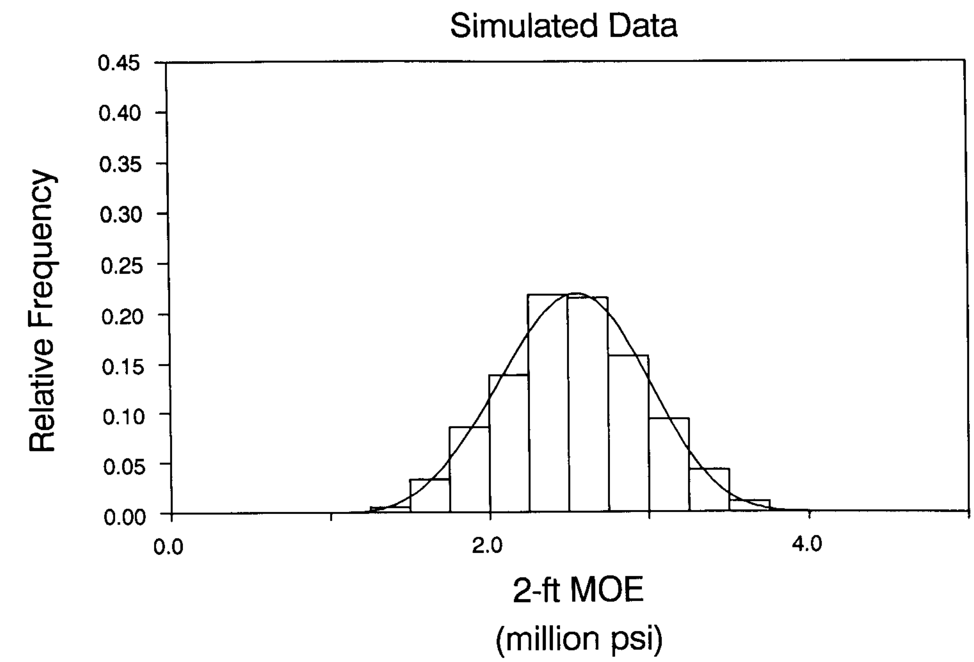

FIG. 5. Frequency histograms of simulated 2-ft segment MOE for the 302-24 lumber. The original probability density function is overlaid. The close agreement between the simulated histogram and the original density function provides visual confirmation that the distribution was preserved.

correlation matrices were calculated for each of the 10 replications of the simulation procedure. Table 2 contains original correlation coefficients, mean simulated correlation coefficients, and 99% confidence intervals for the simulated correlation coefficients for the 302-24 lumber. There is a close correspondence between the original correlation coefficients and the mean simulated correlation coefficients, with a maximum difference of 1.8%.

Independent validation of the model.—The simulation model has been shown to accurately model the probability distributions of localized MOE and T as well as the correlation matrix of these localized properties. However, a more fundamental test of the model involves validating the original assumptions on the spatial variation of MOE and T. For example, one question to ask is: was the use of the autocorrelation function from the first-order autoregressive model [Eqs. (4) and (5)] a valid method of estimating the lag-1 and lag-2 serial correlations in localized tensile strength? If the model accurately simulated localized tensile strength, then it should have been possible to simulate long-span tensile strength as well. These simulated long-span tensile strengths could then be compared to the long-span tensile test data to validate the original assumptions of the model.

Long-span tensile strength over a given span was defined as the minimum 2-ft tensile strength within that span. Simulation results from the model were used to calculate long-span tensile strength for spans of 2 ft, 4 ft, 6 ft, and 8 ft. The mean simulated long-span tensile strengths for the 302-24 lumber are plotted versus length in Fig. 7. Ninety-nine percent confidence intervals for the mean

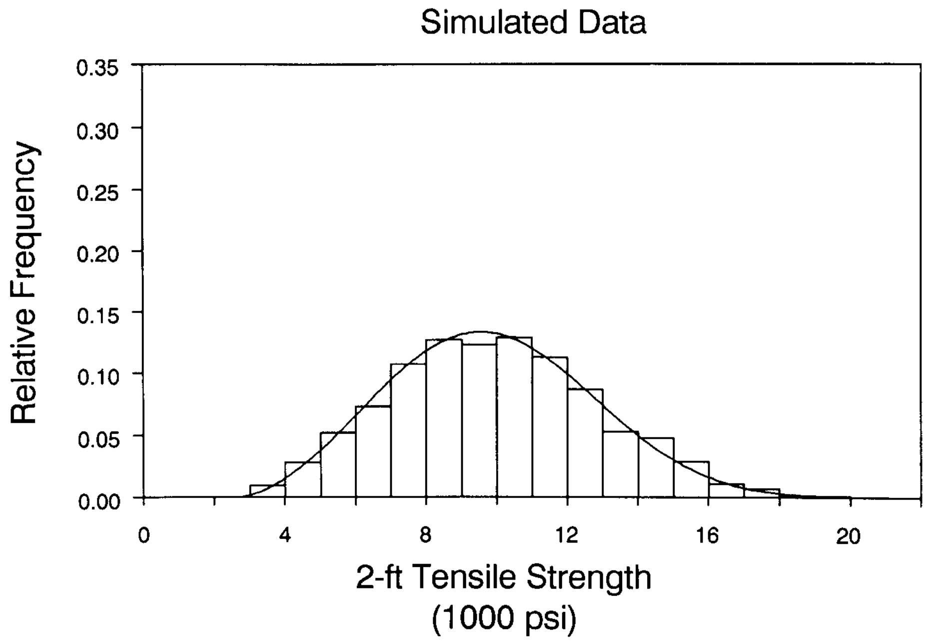

FIG. 6. Frequency histograms of simulated 2-ft segment tensile strength for the 302-24 lumber. The original probability density function is overlaid. The close agreement between the simulated histogram and the original density function provides visual confirmation that the distribution was preserved.

tensile strength (from the test data) are also shown in this figure for the 2-ft, 5-ft, and the 7-ft tensile test spans.

Although all confidence intervals contain the curve of the simulated strength, the mean tensile strength of lumber tested at the 7-ft span is slightly lower than the curve of the simulated tensile strength. This apparent overprediction of 7-ft tensile strength by the simulation model may be due to an overestimation of the lag-1 and lag-2 serial correlation in tensile strength or it may be due to sampling error in the actual 7-ft span test data. Overall, it appears that the simulation model accurately predicted long-span tensile strength for this set of lumber data. This procedure was repeated for all 302-24 and L1 lumber groups (Taylor 1988). The results showed that there was close agreement between the simulated and actual tensile strengths. Table 3 summarizes the results of the long-span tensile tests. These tables also confirm the significant reduction in tensile strength with increasing lumber length reported by Showalter et al. (1987).

EXTENSION OF THE MULTIVARIATE MODEL TO SIMULATE LONGER LUMBER

Extension of algorithm

The localized MOE-T simulation model presented here was initially developed to simulate localized MOE and T for 8-ft long boards. However, Monte Carlo simulation models that determine glulam beam strength quite often need the properties of lumber that is longer than 8 ft. Therefore, the original localized

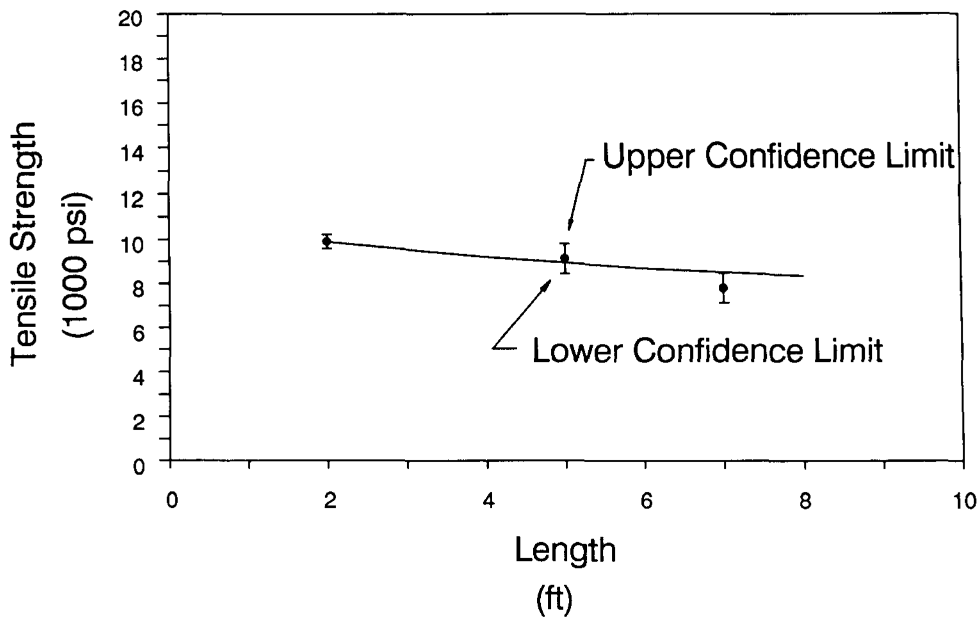

FIG. 7. Mean simulated long-span tensile strength versus lumber length for the 302-24 lumber. Ninety-nine percent confidence intervals for the actual mean tensile strength (from test data) are also shown.

MOE-T simulation procedure was embellished to simulate the localized properties of lumber that were up to 20-ft long (boards composed of ten 2-ft segments).

The embellishment of the model required the modification of the algorithm to generate vectors of length 20 i.e., $\underset{\sim}{X} = (x_1, x_2, \ldots, x_{20})^T$, instead of length 8. The new model treats the MOE's of Segments 1 through 10 as X_1 through X_{10} and the T's of Segments 1 through 10 as X_{11} through X_{20}. The initial step in modeling

TABLE 3. *Tensile strength parallel-to-grain test results for the lumber tested at spans of 5 ft and 7 ft.*

Grade and supplier		5-ft test span				7-ft test span			
		Sample size	Mean (psi)	Standard deviation (psi)	COV (%)	Sample size	Mean (psi)	Standard deviation (psi)	COV (%)
302-24	A	20	9,884	2,843	28.76	20	8,391	2,369	28.23
302-24	B	20	9,584	2,603	27.16	20	8,920	2,639	29.59
302-24	C	20	9,249	1,721	18.61	20	8,295	2,499	30.13
302-24	D	20	9,113	2,343	25.71	20	7,403	1,826	24.67
302-24	E	20	7,675	2,521	32.85	20	6,002	2,120	35.32
302-24	All	100	9,101	2,506	27.54	100	7,802	2,484	31.84
L1	A	20	5,439	1,595	29.33	20	5,115	2,099	41.04
L1	B	20	6,712	2,386	35.55	20	7,062	2,429	34.40
L1	C	20	5,684	1,553	27.32	20	5,916	1,690	28.57
L1	D	20	6,116	2,874	46.99	20	5,665	1,282	22.63
L1	E	20	5,204	1,281	24.62	20	4,550	1,610	35.38
L1	All	100	5,831	2,057	35.28	100	5,662	2,014	35.57

the probability distributions of localized MOE and T remains the same as in the original model. Distributions only need to be fit to localized MOE (which will be used for X_1 through X_{10}) and to localized T (which will be used for X_{11} through X_{20}).

The second step in modeling the variables, constructing the correlation matrix, required estimates of the lag-1 through lag-9 MOE serial correlations, the lag-1 through lag-9 T serial correlations, and the lag-0 through lag-9 MOE-T cross-correlations. However, no test data were available to directly estimate the lag-4 through lag-9 correlations. One solution to this problem is to assume that these higher-lag correlations were equal to 0. However, this assumption is not very realistic since the previously discussed lag-0 through lag-4 correlations were so high.

Another potential solution to the problem of estimating these higher-lag serial correlations in MOE and T is to estimate them using the autocorrelation functions for various order autoregressive models. The autocorrelation function for an AR(p) model requires knowledge of at least the lag-1 through the lag-p correlations. Therefore, since only the lag-1 through lag-3 correlations of MOE were known, an AR(3) model is the highest-order model possible to use for estimating the higher-lag correlations. The autocorrelation function for an AR(3) model is written as:

$$\rho_k = \beta_1\rho_{k-1} + \beta_2\rho_{k-2} + \beta_3\rho_{k-3} \qquad \text{for } k > 0 \tag{6}$$

where:

$$\beta_1 = \frac{\dfrac{\rho_1(1-\rho_2)}{(1-\rho_1{}^2)} - \dfrac{\rho_1\rho_2(\rho_1{}^2-\rho_2)}{(1-\rho_1{}^2)^2} + \dfrac{\rho_3(\rho_1{}^2-\rho_2)}{(1-\rho_1{}^2)^2}}{1 - \left[\dfrac{(\rho_1{}^2-\rho_2)}{(1-\rho_1{}^2)}\right]^2} \tag{7}$$

$$\beta_2 = \rho_2 - \rho_1 \left[\beta_1 + \left(\frac{\beta_1(\rho_1{}^2-\rho_2) - \rho_1\rho_2 + \rho_3}{(1-\rho_1{}^2)}\right)\right] \tag{8}$$

$$\beta_3 = \rho_3 - \beta_1\rho_2 - \beta_2\rho_1 \tag{9}$$

The higher-lag MOE correlations, which were estimated using the autocorrelation function for an AR(3) model and the original serial correlation coefficients from the test data, are listed in Table 4.

The higher-lag tensile strength serial correlations were estimated from the lag-3 T serial correlation by using the assumption of an AR(1) model as listed in Eqs. (4) and (5). These higher-lag serial correlation coefficients, ρ_k, were calculated by:

$$\rho_k = \rho_3^{\frac{k}{3}} \tag{10}$$

where ρ_3 is the lag-3 T serial correlation. The estimated higher-lag tensile strength correlation coefficients are also listed in Table 4.

The higher-lag MOE-T cross-correlation coefficients were estimated by a relationship presented by Matalas (1967) that preserved the lag-0 cross-correlation

TABLE 4. *Higher-lag correlation coefficients estimated from actual test data, mean correlation coefficients from simulated data, and 99% confidence intervals for the simulated correlation coefficients. The lumber group contained all of the 302-24 lumber. The maximum difference between the simulated correlation coefficients and those from the actual data was 5.7% (lag-9 T serial correlation). This difference is not practically significant.*

	Correlation from actual data	Mean correlation from simulated data	Simulated correlation confidence limits	
			Lower	Upper
Serial MOE lag-0	1.0000	1.0000	1.0000	1.0000
Serial MOE lag-1	0.9581	0.9579	0.9556	0.9602
Serial MOE lag-2	0.9277	0.9250	0.9205	0.9296
Serial MOE lag-3	0.8902	0.8870	0.8806	0.8934
Serial MOE lag-4	0.8559	0.8532	0.8446	0.8619
Serial MOE lag-5	0.8220	0.8209	0.8103	0.8314
Serial MOE lag-6	0.7897	0.7906	0.7787	0.8026
Serial MOE lag-7	0.7586	0.7556	0.7438	0.7675
Serial MOE lag-8	0.7288	0.7270	0.7120	0.7420
Serial MOE lag-9	0.7001	0.6995	0.6796	0.7194
Cross MOE-T lag-0	0.4015	0.4044	0.3807	0.4282
Cross MOE-T lag-1	0.3847	0.3881	0.3633	0.4130
Cross MOE-T lag-2	0.3724	0.3831	0.3535	0.4128
Cross MOE-T lag-3	0.3574	0.3664	0.3411	0.3917
Cross MOE-T lag-4	0.3436	0.3508	0.3239	0.3778
Cross MOE-T lag-5	0.3300	0.3367	0.3101	0.3633
Cross MOE-T lag-6	0.3171	0.3223	0.2959	0.3487
Cross MOE-T lag-7	0.3046	0.3069	0.2828	0.3311
Cross MOE-T lag-8	0.2926	0.2950	0.2703	0.3196
Cross MOE-T lag-9	0.2811	0.2850	0.2655	0.3044
Serial T lag-0	1.0000	1.0000	1.0000	1.0000
Serial T lag-1	0.8008	0.8006	0.7921	0.8092
Serial T lag-2	0.6413	0.6411	0.6265	0.6556
Serial T lag-3	0.5136	0.5144	0.4937	0.5351
Serial T lag-4	0.4113	0.4101	0.3817	0.4386
Serial T lag-5	0.3294	0.3214	0.2923	0.3506
Serial T lag-6	0.2638	0.2469	0.2132	0.2807
Serial T lag-7	0.2113	0.2046	0.1683	0.2410
Serial T lag-8	0.1692	0.1686	0.1374	0.1998
Serial T lag-9	0.1355	0.1278	0.1023	0.1533

and the first-order serial correlation in MOE. These additional correlation coefficients were calculated by:

$$\rho_{\text{MOE-T}_k} = \rho_{\text{MOE}_k}\rho_{\text{MOE-T}_0} \tag{11}$$

where $\rho_{\text{MOE-T}_k}$ is the lag-k MOE-T cross-correlation, ρ_{MOE_k} is the lag-k MOE serial correlation, and $\rho_{\text{MOE-T}_0}$ is the lag-0 MOE-T cross-correlation. These estimated cross-correlations are also summarized in Table 4 for the 302-24 lumber data.

Validation of embellished simulation model

The ability of the model to preserve the original probability distributions of localized MOE and T has already been discussed. However, the modification of the correlation matrices made it necessary to confirm that the method preserved these larger correlation matrices. Therefore, the simulation procedure was repeated using the original localized MOE and T probability distributions and the new 20 × 20 correlation matrix. The procedure was carried out exactly as before, i.e., replicating the simulation procedure 10 times and simulating 1,000 boards per replication. Correlation matrices for the simulated MOE-T data were calculated after each replication of the simulation procedure. The original correlation coefficients, the mean of the simulated correlation coefficients, and 99% confidence intervals for the simulated correlation coefficients are presented in Table 4 for the 302-24 lumber. All the original correlation coefficients fell within their respective 99% confidence intervals developed from the simulated data.

Simulation results from this embellished model were also used to calculate long-span tensile strengths for spans up to 20 ft. Again, the mean simulated long-span tensile strengths fell within 99% confidence intervals for mean tensile strength from the 2-ft, 5-ft, and 7-ft test data. This comparison of simulated and actual mean long-span tensile strengths and correlation coefficients indicates that the embellished model is mathematically valid. However, additional test data are needed to determine the model's accuracy in predicting tensile strength at these longer test spans.

CONCLUSIONS

The goal of this study was to develop accurate stochastic models for simulating localized MOE and tensile strength in Douglas-fir laminating lumber. Accurate models of localized MOE and tensile strength are needed as input to Monte Carlo simulation models that predict the strength and stiffness of glued-laminated beams.

One thousand 14-ft-long Douglas-fir 2 by 6's from 2 high quality laminating grades (302-24 and L1) were tested to determine variation in localized MOE along the length of the lumber. Six hundred of these specimens were destructively tested in tension (at a 2-ft test span) in two locations along the board. The remaining specimens were destructively tested in tension at test spans of 5 ft and 7 ft. The lumber was sampled from five laminators.

An approach for simulating multiple correlated lumber properties developed by Taylor and Bender (1989) was extended here to model the spatially correlated values of localized MOE and T for 8-ft long lumber. The method was illustrated for the 302-24 lumber data. The model preserved the probability distributions of MOE and T, and the correlation of the localized properties. In addition, the model was used to simulate long-span tensile strength for spans up to 8 ft. Simulated mean long-span tensile strength compared favorably with actual long-span tensile test data at spans of 5 ft and 7 ft. The long-span tensile test results also confirmed earlier work by other researchers and showed a significant decrease in tensile strength as lumber length increased.

The original simulation algorithm was embellished to allow the simulation of localized MOE and T for lumber up to 20 ft in length. Autocorrelation functions for various order autoregressive models were used to allow the estimation of higher-lag serial and cross-correlation coefficients. The revised simulation procedure preserved the larger 20 × 20 correlation matrices and appeared to successfully predict long-span tensile strength.

Additional research is needed to develop similar information and models for other grades and species groupings of laminating lumber. Further modeling and testing are needed to determine if this simulation procedure successfully predicts long-span tensile strength over a wider range of test spans. These results can be used in reliability models of glulam timber beams, trusses, wood I-beams, and many other engineered wood structural components.

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APPENDIX 1. *Serial and cross-correlation coefficients estimated from the test data for each group of lumber.*

Correlation		302-24 A	302-24 B	302-24 C	302-24 D	302-24 E	All suppliers 302-24
Serial MOE	lag-0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Serial MOE	lag-1	0.9264	0.9577	0.9558	0.9366	0.9491	0.9581
Serial MOE	lag-2	0.8878	0.9058	0.9341	0.9125	0.9079	0.9277
Serial MOE	lag-3	0.8759	0.8448	0.9239	0.8974	0.8087	0.8902
Cross MOE-T	lag-0	0.4887	0.5498	0.4172	0.4245	0.5347	0.4015
Cross MOE-T	lag-1	0.4380	0.5270	0.4055	0.3976	0.4637	0.3603
Cross MOE-T	lag-2	0.4192	0.4988	0.3718	0.3874	0.4303	0.3407
Cross MOE-T	lag-3	0.4153	0.4839	0.3805	0.3810	0.3940	0.3322
Serial T	lag-0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Serial T	lag-1	0.6637	0.8176	0.8432	0.8327	0.7723	0.8008
Serial T	lag-2	0.4405	0.6685	0.7110	0.6933	0.5964	0.6413
Serial T	lag-3	0.2924	0.5465	0.5995	0.5773	0.4606	0.5136

Correlation		L1 A	L1 B	L1 C	L1 D	L1 E	All suppliers L1
Serial MOE	lag-0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Serial MOE	lag-1	0.8956	0.9173	0.9082	0.9109	0.9253	0.9196
Serial MOE	lag-2	0.8785	0.8408	0.8677	0.8633	0.8855	0.8752
Serial MOE	lag-3	0.8525	0.7779	0.8155	0.8403	0.8541	0.8323
Cross MOE-T	lag-0	0.7677	0.4294	0.4872	0.6408	0.5555	0.5894
Cross MOE-T	lag-1	0.7022	0.3720	0.4361	0.6076	0.4616	0.5395
Cross MOE-T	lag-2	0.6796	0.3084	0.4240	0.5821	0.4308	0.5097
Cross MOE-T	lag-3	0.6548	0.2885	0.4190	0.4932	0.4571	0.4803
Serial T	lag-0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Serial T	lag-1	0.8661	0.8252	0.7848	0.8041	0.7991	0.8303
Serial T	lag-2	0.7501	0.6809	0.6158	0.6466	0.6385	0.6894
Serial T	lag-3	0.6496	0.5619	0.4833	0.5199	0.5102	0.5724

Appendix 2. *Parameters for the best fitting probability density functions for each group of lumber.*

Summary of probability distributions selected for short-span MOE of the 302-24 lumber.

Supplier	N	Distribution type	Parameters: Location (10^6 psi)	Scale (10^6 psi)	Shape
A	240	2-P Lognormal	NA	1.069	0.129
B	240	2-P Lognormal	NA	0.966	0.149
C	240	2-P Lognormal	NA	0.778	0.202
D	240	3-P Weibull	1.520	1.332	3.188
E	240	3-P Weibull	0.000	2.579	8.106
All suppliers	1,200	3-P Weibull	1.070	1.612	3.705

Summary of probability distributions selected for short-span MOE of the L1 lumber.

Supplier	N	Distribution type	Parameters: Location (10^6 psi)	Scale (10^6 psi)	Shape
A	244	3-P Weibull	1.376	1.046	2.668
B	240	3-P Weibull	1.395	1.158	2.907
C	240	3-P Weibull	1.270	1.039	3.026
D	240	3-P Weibull	1.149	1.273	2.969
E	240	3-P Weibull	0.978	1.279	3.597
All suppliers	1,204	3-P Weibull	1.111	1.292	3.245

Summary of probability distributions selected for short-span T of the 302-24 lumber.

Supplier	N	Distribution type	Parameters: Location (10^3 psi)	Scale (10^3 psi)	Shape
A	117	3-P Weibull	3.684	7.457	2.276
B	120	3-P Weibull	2.803	8.534	2.774
C	120	3-P Weibull	1.552	9.865	3.980
D	119	3-P Weibull	2.946	7.128	2.442
E	119	3-P Weibull	3.317	6.347	2.334
All suppliers	595	3-P Weibull	2.565	8.218	2.764

Summary of probability distributions selected for short-span T of the L1 lumber.

Supplier	N	Distribution type	Parameters: Location (10^3 psi)	Scale (10^3 psi)	Shape
A	122	3-P Weibull	2.382	5.585	1.902
B	120	2-P Lognormal	NA	2.029	0.309
C	120	2-P Lognormal	NA	1.953	0.306
D	119	2-P Lognormal	NA	1.868	0.339
E	120	2-P Lognormal	NA	1.806	0.365
All suppliers	601	3-P Weibull	2.310	5.527	2.090