SHEAR DEFLECTION OF COMPOSITE WOOD BEAMS

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ABSTRACT

Shear deflection of wood beams usually is not included in design calculations. Ignoring shear deflection could lead to significant errors in total beam deflection predictions, especially for composite wood beams that have less and/or lower quality material in the core (or web) as compared to the outer zones (flanges).

A generalized shear deflection equation was developed for layered composite beams. The model can accommodate variable numbers of laminations, nonprismatic shapes, and variable elastic properties between laminations and along the beam length. The model was validated using full-sized glued-laminated timber beam test data. Sensitivity analyses were conducted on numerical integration step sizes, ratio of modulus of elasticity to shear modulus, and span-to-depth ratio (L/d). One important finding was that the common engineering design practice of not including shear deflection for solid-sawn wood beams with L/d ratios of 15 to 25 could lead to significant errors for composite wood beams.

Keywords: Beams, composite, shear, wood.

INTRODUCTION

Deflection of beams is comprised of two parts: 1) bending, and 2) shear deflection. Shear deflection is not included in many structural engineering calculations. This is an acceptable practice for steel structures because the total deflection usually is dominated by the bending component, except for very short, deep beams. United States engineering practice accounts for shear deflection of wood members by lowering published design values of E (by about 3.4%) to account for the omission of shear in common bending deflection equations. The Commission of the European Communities, in Eurocode 5 (1987), approaches this problem in a more direct manner. The true bending E is published as the design E, and the code specifies that shear deflection must be explicitly included in the total deflection prediction.

Shear deflection is related to the shear modulus (G) of the beam. The E/G ratio for steel is assumed to be 2.6 in the elastic range (Salmon and Johnson 1990); however, the ratio of E/G for wood is generally assumed to range from 11 to 16 (USDA 1987). This large ratio for wood indicates that the shear component of the total deflection can be more significant for a wood beam than for a steel beam. Shear deflection is even more important for composite beams because the cores usually are made from lower quality material than the rest of the beam. Furthermore, this less stiff material is positioned at the point of maximum shear stress.

Common engineering practice for calculating deflection of wood beams is to use flexural equations derived for bending only, and since the design E is reduced to account for shear

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deflection, they give reasonably accurate predictions for span-to-depth (L/d) ratios ranging from 15 to 25. However, if the L/d ratio is less than 15, the predicted deflection will be significantly less than the actual deflection. Modified equations can be used (Hoyle and Woeste 1989) that take shear deflection into account; however, they are valid only for homogeneous materials.

Simple methods of predicting shear stress and deflection are needed for layered composite wood beams. The finite element approach is effective for predicting shear stresses and strains in beams; however, it is computationally intensive and requires detailed material property data for each element or cell. Transformed-section approaches have been developed for layered beams with homogeneous laminations; however, these methods do not fully account for lengthwise variability of the lamination elastic properties. A general, versatile method is needed to predict shear deflection in composite wood beams for the purposes of evaluating design procedures and assessing the need for research on localized elastic properties of wood (particularly shear modulus).

RESEARCH OBJECTIVES

1. Develop a model to predict shear stress and deflection in layered composite wood beams.
2. Experimentally validate the shear deflection model using test data from glued-laminated (glulam) timber beams.
3. Integrate the shear deflection model into an existing glulam beam model and perform parameter sensitivity analyses.

LITERATURE REVIEW

Models

Many mechanics of materials textbooks address deflection of homogeneous beams with rectangular-shaped cross sections. However, cross sections with irregular shapes or containing nonhomogeneous materials are often "beyond the scope of the textbook." The bases for many of the deflection equations for homogeneous materials are energy methods. Equation 1 (Boresi and Sidebottom 1985) represents the generalized form of the deflection equation due to flexure for homogeneous beams. The first term of Eq. 1 represents the bending component, and the second term represents the shear component.

$$\delta_i = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial F_i}\,dx + \int_0^L \frac{kV}{GA}\frac{\partial V}{\partial F_i}\,dx \quad (1)$$

where

δ_i = deflection at point i,
A = area of the cross section,
E = modulus of elasticity,
F_i = unit concentrated load at point i,
G = shear modulus,
I = moment of inertia,
k = form factor as a function of beam geometry,
M = bending moment as a function of x and
V = shear as a function of x.

Unique k factor values can be derived for different cross sections. The k factor equals 1.2 for rectangular cross sections, and it can be approximated as 1.0 for I-beams, provided the area of the I-beam web is used for A (Boresi and Sidebottom 1985). This general method for calculating deflection has a large range of applications; however, it is limited to homogeneous materials.

Wangaard (1964) studied the elastic deflection of small-scale composite beams. The wood cores of the beams were covered with fiberglass reinforced plastic faces on the outermost fibers about the axis of bending. He showed that by including the second term from Eq. 1, the accuracy of the deflection model increased. This method is limited to cross sections that are symmetric about the neutral axis, referred to as balanced layups, and does not account for the variability of elastic properties (E and G) along the length of the beam.

Biblis (1965) examined the deflection of small-scale solid wood beams of varying span-to-depth (L/d) ratios. An important finding was that the shear component could account for over 40% of the total deflection at an L/d ratio of 8 for Douglas-fir lumber. This is significant because Douglas-fir is commonly used to manufacture glulam beams.

Stieda (1967) presented a strain energy method to calculate shear deflection in plywood box beams and I-beams. He found that treating the web as a homogeneous section, (i.e., all laminations oriented in the same direction) was adequate for modeling the shear deflection. Orosz (1970) also used energy methods to calculate shear deflection of wood beams. He derived a form factor (k) for an I-beam with the flange and webs having different lumber properties. Orosz also assumed that the web of the beam was homogeneous. His method can be applied to glulam beams, but is limited to a cross section that is symmetric about the neutral axis, allows only two different lumber grades, and does not account for material property variability.

Hilson et al. (1988, 1990) and Pellicane and Hilson (1985) presented a method for calculating bending and shear deflection using a transformed-section approach. They developed a finite difference equation similar to Eq. 1; however, their equation is not exact for composite cross sections with varying E because it does not account for the changes in the shear stresses across the various laminations. They suggested using the transformed area of the cross section with the k factor equal to 1.2 for a rectangular cross section, but these two terms are based on the assumption of a rectangular cross section, not a transformed nonrectangular cross section.

Mansour and Gopu (1990) presented an exact method for predicting deflection of pitch-cambered glulam beams using Eq. 1 with a transformed-section analysis. Their derivation includes an equation to solve for the form factor k for unbalanced layered beams. Monte Carlo simulation was performed to randomly assign E values along the length and depth of the beam, and the E/G ratio was set to 16. Simulated beams then were analyzed using a finite element approach. They concluded that simple equations for homogeneous beams accurately predicted total deflection as long as the shear component was included.

Elastic properties

Gaining a better understanding of the mechanical properties of wood could influence designers to use this material for more complex structures. Many models can predict the theoretical stresses and strains of wood systems, but these models require input in the form of elastic constants that are not completely characterized for every wood species grouping.

Early work on the relationships of E and G was conducted by Doyle and Markwardt (1966, 1967). They tested full-sized southern pine dimension lumber for a variety of structural grades. They reported linear correlation coefficients between E and G ranging from −0.342 to +0.554, depending on the lumber grade. They stated that G appeared to be less affected by grade or quality of the material than E. Doyle (1968) tested another sample of No. 2 dense kiln-dried southern pine dimension lumber and again reported that G was not significantly correlated with flatwise E. These three studies formed the basis of the E/G ratio of 12 for southern pine lumber, published in the Wood Handbook (USDA 1987).

Palka and Barrett (1985) presented a report to the ASTM task group investigating the validity of Table 2 in ASTM D 2915-74. This testing consisted of two samples of Canadian spruce structural lumber specimens. They reported a wide range of values for the E/G ratios and that the average value was considerably larger than the reported value of E/G of 16. They concluded that E/G is dependent on the test method and lumber quality. The table that instigated this investigation has since been replaced by a footnote (ASTM 1991a) that states, "Limited data indicate that the E/G ratio for individual pieces of lumber can vary significantly from E/G = 16 depending on the number, size and location of knots present, the slope of grain in the piece and the span over which deflections are measured."

Bodig and Goodman (1973) used plate bending and plate twisting tests on small-scale clear wood specimens to predict the elastic properties. Power-type regression models were not statistically significant for G versus E. This study highlights the problem of trying to predict G as a function of E. However, they did find significant correlations between specific gravity and the elastic properties E and G.

Goodman and Bodig (1978) presented a review of literature and a commentary on the problem of modeling elastic behavior of wood. Many of the data used to characterize the elastic parameters of wood were collected from clear wood specimens. They observed that grain deviations around knots caused the principal axes to rotate, making modeling procedures very difficult. They theorized that the assumptions of orthotropic symmetry in the radial direction are most often the cause for deviation between theoretical and experimental measurements.

Bradtmueller et al. (1991) tested oriented strandboard (OSB) over quarter-point and five-point loading conditions to calculate G. Sensitivity analyses indicated that a small error in calculating E magnified the error in G. This is caused by E being considerably larger than G, resulting in ill-conditioned simultaneous deflection equations. They noted that experimental results of G were lower than expected and theorized that this was caused by the comparatively low shear stiffness in the core that corresponded to the point of highest shear stresses.

Chui (1991) used a vibration technique to simultaneously evaluate E and G. His findings revealed that the common assumption of E/G equal to 16 for Douglas-fir (USDA 1987) may not be valid. Chui's data indicated that E/G is a random variable, not a deterministic value; furthermore, it was suggested to use E/G of 20 for clear wood and 30 for lower quality lumber. This is meaningful since the beam combinations found in the American Institute of Timber Construction (AITC) 117—Manufacturing (1988) often specify lower quality lumber in the core, compared to the rest of the beam.

MODEL DEVELOPMENT

Calculating shear deflection is more complex for composite beams than for rectangular beams with homogenous properties. E and G vary within the cross section and along the length for multilayered beams and this variation compounds the difficulty of calculating shear deflection. Mansour and Gopu (1990) and Orosz (1970) presented methods to calculate k for composite beams using the traditional shear deflection equation (Eq. 1); however, a more general method is needed to facilitate studies of spatial variation of E and G.

Derivation

Critical evaluation of Eq. 1 reveals that only the second term needs to be modified for composite beams. Finding the shear stress distribution in a homogeneous beam is straightforward; however, as material properties vary, as in a composite beam, so do the shear stress distributions. After the shear stresses are derived for composite beams, such as glulam beams or I-beams, shear deflection can be found by applying the theory of complementary virtual work.

Figure 1(a) illustrates a simply supported composite beam that is stressed by arbitrary loads P, Q, and w. Figure 1(b) represents the cross section of this beam with width b and height h. The outer laminations of this composite beam have modulus of elasticity of E_1 and the inner laminations have modulus of elasticity of E_2. It is assumed that E_1 is greater than E_2, an assumption that generally would be true for glulam beams.

Composite beams often are analyzed using the transformed-section method, because the usual elastic beam formulas can be used with slight modification. Figure 1(c) illustrates the transformed cross section. This method transforms the composite cross section to a homogeneous material with a modulus of elasticity of E_*, where E_* is an arbitrary constant and the width of the i^{th} lamination is adjusted by the ratio of E_i to E_*. After transformation, the elastic flexural formulas then can be used with slight modification. The bending stress in

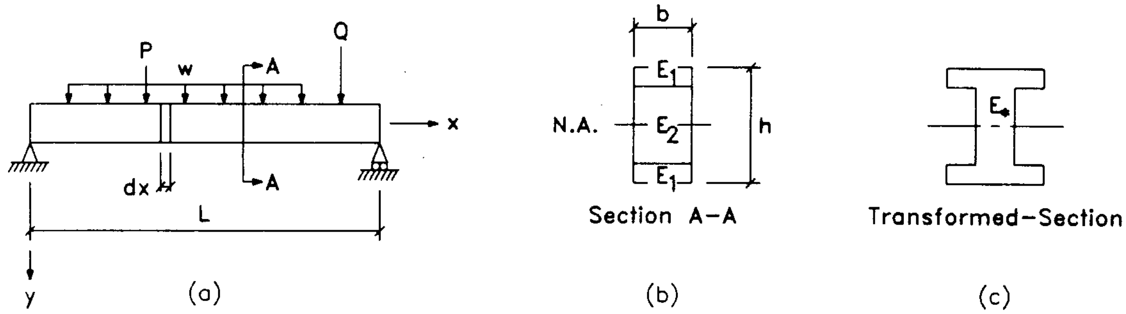

FIG. 1. Simple composite beam.

the cross section is represented by:

$$\sigma_{bi} = \frac{E_i}{E_*}\frac{My}{I_t} \qquad (2)$$

where

σ_{bi} = normal bending stress in the i^{th} lamination,
E_i = modulus of elasticity of the i^{th} lamination,
E_* = transformed modulus of elasticity,
I_t = moment of inertia of the transformed cross section,
M = bending moment applied at the cross section and
y = distance from the neutral axis to the point in question.

An element of the original simple composite beam, Fig. 1(a), is removed and examined in greater detail in Fig. 2(a). This cut has length dx and cross-sectional properties identical to the original beam shown in Fig. 1(b). The element is subjected to a moment M on the left side and an opposing moment on the right side M + dM. The bending stress is superimposed on Fig. 2(a). Note the discontinuity in the stress distribution that accompanies the different values of E_i, with the slope becoming steeper as E_i increases. Both of these properties are characterized by Eq. 2. The shaded area in Fig. 2(a) is now examined in greater detail in Fig. 2(b).

The bending stresses are resolved into resultant forces F_{L_i} and F_{R_i} in Fig. 2(b). The shear V_{OP} acting parallel to the line OP can be found using statics as follows:

$$V_{OP} = F_{R_1} + F_{R_2} - F_{L_1} - F_{L_2} \qquad (3)$$

where:

F_{R_i} = resultant bending force on the right side in the i^{th} lamination,
F_{L_i} = resultant bending force on the left side in the i^{th} lamination and
V_{OP} = shear force acting parallel to line OP.

The bending stress (Eq. 2) can be integrated over each area A_i, yielding the resultant component forces over their respective areas. These resultant forces then can be substituted into Eq. 3 to form Eq. 4. Figure 2(c) is a section removed from Fig. 1(a) illustrating the infinitesimal area dA_i. The area of integration A_i must be changed as the modulus of elasticity changes. After simplification, the equation can be expressed as follows:

$$V_{OP} = \int_{A_1} \frac{E_1}{E_*}\frac{dMy}{I_t}\, dA_1 + \int_{A_2} \frac{E_2}{E_*}\frac{dMy}{I_t}\, dA_2 \qquad (4)$$

where

A_i = area of integration of the i^{th} lamination.

The shear stress τ_{OP} can be found by dividing V_{OP} by the area that it acts over (dx b). The relationship can be further simplified by noting that the derivative of the moment, dM/dx, equals the shear force, V, and the area integral is the first moment of the area, Q. The resulting

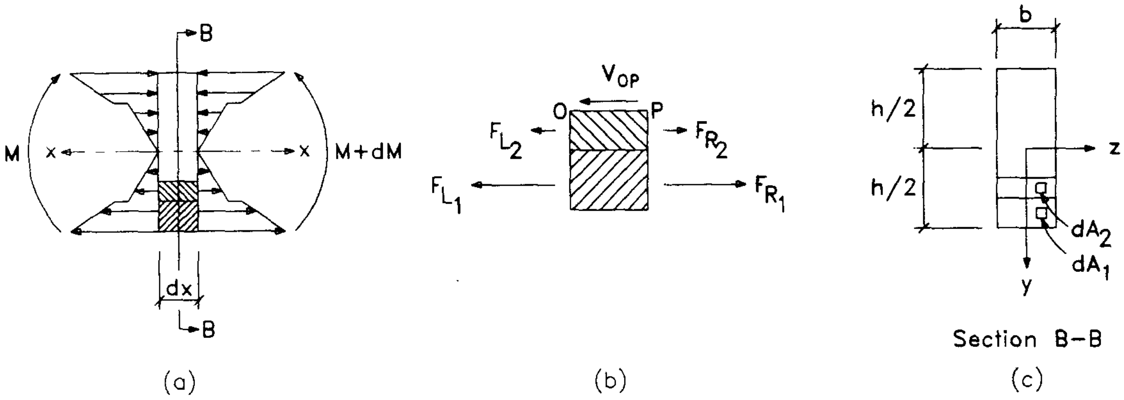

FIG. 2. Stresses, forces and moments on the composite cross section.

general form of the equation for shear stress at any point λ, along the depth of the beam is given by:

$$\tau_\lambda = \frac{V\sum_{k=1}^{\lambda} E_k Q_k}{E_* I_t b} \qquad (5)$$

where

τ_λ = shear stress at any point λ along the length and depth of the beam.
Q_i = first moment of the area of the i^{th} lamination.

After the general equation for shear stress is derived, the shear deflection can be characterized by using energy methods and Castigliano's theorem to yield:

$$\delta_{x,V} = \int_0^L \int_{-h/2}^{h/2} \frac{V}{(E_* I_t)^2} \frac{\left(\sum_{k=1}^{\lambda} E_k Q_k\right)^2}{bG} \frac{\partial V}{\partial F_x}\, dy\, dx \qquad (6)$$

where

$\delta_{x,V}$ = shear deflection at point x and
F_x = unit concentrated load at point x.

Equation 6 can be integrated numerically by expressing it in the form of Eq. 7. The integrals are replaced with summations and the partial derivative is replaced by v_i, which is the shear component of a unit load applied at midspan. The shear modulus, G_{ij}, is placed inside the summations so it can vary along the length and depth of the beam. E* is removed from inside the summations because it is constant across the beam length and depth. Additional details on the model derivation are given in Skaggs (1992).

$$\delta_{x,V} = \frac{1}{E_*^2}\left[\sum_{i=1}^{ncutx} \frac{V_i v_i}{I_{t_i}^2}\left[\sum_{j=1}^{ncuty} \frac{\left(\sum_{k=1}^{j} E_{ik} Q_k\right)^2}{b_j G_{ij}}\, \Delta y\right]\Delta x\right] \qquad (7)$$

where:

$\delta_{x,V}$ = shear deflection at point x,
Δx = width of intervals along the x-axis,
Δy = width of intervals along the y-axis,
b_j = actual width of the j^{th} lamination,
E_* = transformed modulus of elasticity,
E_{ik} = modulus of elasticity of the k^{th} lamination at i,
G_{ij} = shear modulus of the j^{th} lamination at i,
I_{t_i} = moment of inertia of the transformed cross section at i,
ncutx = number of intervals along the x-axis,
ncuty = number of intervals along the y-axis,
Q_k = first moment of the area of the k^{th} lamination,

ν_i = shear force at i due to unit load at x and

V_i = shear force at i.

Assumptions

1. The shear force on the beam acts parallel to the shear stresses.
2. The shear stresses act uniformly across the width of the beam.
3. The material is linearly elastic and is only subjected to small displacements.
4. Deformations are about the plane of bending (i.e., no lateral-torsional buckling).

Limitations

The shear stress formula (Eq. 5) is limited to beams that are deeper than they are wide. When beam width equals depth, true maximum shear stresses can be significantly larger (13% for a homogeneous beam) than what is predicted by Eq. 5 (Gere and Timoshenko 1984). This underprediction of shear stress also would cause an error in the amount of shear deflection predicted by Eq. 7 for a composite beam.

MODEL VERIFICATION AND VALIDATION

The shear deflection model (Eq. 7) was verified by comparing it to an exact theoretical solution for homogeneous beams. The shear deflection model was validated using test data on full-sized glulam beams and the constituent lumber. Lumber data were used as input to the deflection model, and by setting the predicted deflection equal to the value predicted by the bending deflection equation, glulam beam apparent E was calculated. The results were compared to actual glulam beam E's measured in the laboratory.

Experimental procedure

Hernandez's (1991) and Hernandez et al.'s (1992) work on a probabilistic glulam beam model (called PROLAM) was conducted concurrently with an extensive glulam beam test program undertaken by the American Institute of Timber Construction (AITC). Before the beams were fabricated, the laminating stock was run through a CLT stress-grading machine to obtain continuous E-profiles for each piece of lumber. These pieces then were stamped with an identification number so they could be identified in the glulam beam after fabrication.

A group of thirty 16-lamination 24F-V4 Douglas-fir glulam beams was tested. The 61-cm (24-in.)-deep beams were manufactured to a length of 12.2 m (40 ft) using nominal 5-cm by 15-cm (2-in. by 6-in.) Douglas-fir laminating lumber. After fabrication, the beams were planed to a final width of 13.0 cm (5.125 in.). The beams were destructively tested under symmetric two-point loading at a total span of 11.6 m (38 ft) and 2.4 m (8 ft) between the load points. The beams were restrained from buckling out of plane and the apparent E was measured for each beam.

The E profiles were averaged for each 61-cm (2-ft) lumber segment. Beam maps were constructed using the 61-cm (2-ft) average E_{CLT}. Cross-sectional profiles were then taken at 30.5-cm (1-ft) intervals, and the resulting values were recorded in data files. The dimension of the matrices were 39 by 16, representing 39 30.5-cm (1-ft) intervals and 16 laminations. One beam map could not be constructed due to a data collection problem; therefore, the final sample size was 29.

Adjustment of lumber E values

A FORTRAN program was written to transform the array of E_{CLT} values to the corresponding static bending modulus of elasticity (E_S) values using the following regression equation developed at the time the lumber was sampled:

$$E_S = 1.227\ E_{CLT} - 0.191 \tag{8}$$

where

E_S = static flatwise bending E and

E_{CLT} = raw CLT-E values averaged over a 61-cm (2-ft) segment.

The total deflections of the beams were calculated using the first term of Eq. 1 (numerical

TABLE 1. *Comparison of actual and predicted apparent beam E.*

Prediction method	Sample size	Apparent beam E			
		Average	Average error†	COV*	COV error†
		GPa (Mpsi)		%	
Actual test data	29	14.20 (2.06)		4.09	
Predicted (E/G = 16)	29	14.48 (2.10)	+1.64	3.62	−11.4
Predicted (E/G varies‡)	29	13.93 (2.02)	−1.98	3.52	−14.0

* Coefficient of variation.
† 100(Actual − Predicted)/Actual.
‡E/G = 30 for L3 and 20 for all other lumber grades.

integration) and Eq. 7. After the deflections were found, the apparent beam E's were calculated using the classic deflection (bending deflection only) equation for symmetric two-point loading.

Test results and discussion

Predictions of apparent E were made under two assumptions of E/G ratios: 1) E/G equal to 16 (USDA 1987), and 2) E/G equal to 30 for the lumber in the core (grade L3) and 20 for all other lumber grades (Chui 1991). Summary statistics for the actual test data and two prediction assumptions are given in Table 1. Average predicted beam E's bracketed the actual test value with less than 2% error. Predicted coefficients of variation (COV) were slightly less than the actual test value. This could be explained by the 4-ft span used within the CLT stress rating machine. This span would tend to smooth out some of the localized variability in lumber E.

Prediction errors for individual beams ranged from −3.1 to 10.2% for E/G equal to 16, and −6.4 to 6.2% when E/G varied (30 for L3 and 20 for other grades). Empirical cumulative distributions functions (CDFs) were plotted for the actual test data and the two prediction methods as shown in Fig. 3. Again the two predicted CDFs bracketed the actual CDF, indicating good agreement. The assumption of E/G varying from 20 to 30 gave the more conservative prediction of apparent beam E. Research is needed to better characterize G and its relationship to E as a function of lumber grade.

MODEL IMPLEMENTATION

Overview

The shear deflection model (Eq. 7) was implemented into a probabilistic glulam beam model developed by Hernandez et al. (1992), called PROLAM. PROLAM is a stochastic model that simulates glulam beam fabrication and generates random values of E and tensile strength (T) for each 61-cm (2-ft) lumber segment in the beam. These E and T values are spatially correlated along the lengths of the individual pieces of lumber. The simulated pieces of lumber are joined together with finger joints that also are assigned random E and T values. This model is used to predict statistical distributions of glulam beam strength and apparent E.

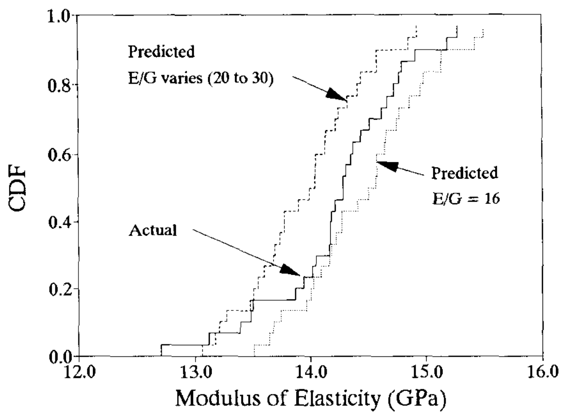

FIG. 3. Comparison of actual and predicted empirical cumulative distribution functions (sample size equals 29).

PROLAM uses a subroutine to perform the transformed-section analyses at specified increments, Δx, and then it repeats the analyses at each finger joint location in the tension zone of the beam. Due to the calculation requirements of the composite shear deflection model presented here, it was placed in a separate subroutine within PROLAM. Another addition to PROLAM was allowing the E/G ratio to vary for the different lumber grades across the cross section instead of being held constant.

Sensitivity analyses

Parameter sensitivity analyses were conducted using PROLAM with the composite shear deflection model. Parameters included numerical integration step size, E/G ratio, and span-to-depth ratio (L/d). The glulam beam type and size studied were the same as the validation beams previously discussed. All PROLAM input data were identical to those given by Hernandez et al. (1992).

Effect of numerical integration step size

The effect of numerical integration step size on predicted apparent beam E was studied over a wide range of values. Numbers of increments along the 11.6-m (38-ft) beam span were chosen to be 500, 250, 100, 50, 38, and 25 resulting in increment sizes of 2.32, 4.63, 11.58, 23.16, 30.48, and 46.33 cm (0.91, 1.82, 4.56, 9.12, 12.00, and 18.24 in.). A total of 1,000 beams were simulated with the same random number seed for the six computer runs (as a means of statistical blocking).

Since the accuracy of numerical integration increases as the number of increments becomes larger (for a fixed beam length), the scenario with 500 increments (ncutx equal to 500) was used as a benchmark for comparisons. The maximum prediction error observed was 0.56% for the case where Δx equalled 46.33 cm (18.24 in.). All other errors were less than one-fourth of 1%, indicating that E is fairly insensitive to the integration step size. It is recommended that Δx not exceed the lumber property cell size (cell size is 61 cm in PROLAM). An increment of Δx = 30.48 cm (12 in.) was selected for all other sensitivity analyses.

A similar sensitivity analysis was performed for the number of increments in the y-direction. The number of intervals, ncuty, was set at 160, 48, and 16 resulting in increment sizes of 0.38, 1.27, and 3.81 cm (0.15, 0.50, and 1.50 in.). There was no effect on the average apparent E for the 1,000 simulated beams from the number of increments in the y-direction; therefore, it is recommended to use ncuty equal to the number of laminations.

Effect of E/G ratio

The effect of the E/G ratio on predicted apparent beam E was studied next. Three cases were investigated: 1) E/G ratio equal to 16, 2) E/G ratio equal to 20 for all lumber grades except L3 which had an E/G ratio of 30 and 3) no shear deflection allowed. One thousand beams were simulated for each case using the same initial random seed value.

Predicted probability density functions (pdf) of apparent beam E are shown in Fig. 4. As expected, the case with no shear gave the lowest deflection predictions, resulting in the highest apparent E. The average apparent E for this case was 17.13 GPa (2.484 Mpsi). By adding shear deflection, total predicted deflections increased, resulting in lower values of apparent E (Fig. 4). Average apparent E values for Cases 1 and 2 were 16.09 and 15.40 GPa (2.333 and 2.234 Mpsi), respectively. In all cases, the coefficients of variation of apparent E were 4.0%.

The ratios of apparent E with and without shear deflection included were 0.94 (Case 1) and 0.90 (Case 2). These values are similar to the empirically derived 0.95 shear correction factor found in ASTM D3737 (ASTM 1991b).

Effect of L/d ratio

The effect of span-to-depth ratio (L/d) was studied using PROLAM for three different loading cases: 1) two-point loading (with distance between load points equal to 20% of total span), 2) uniform loading, and 3) single-point loading at the midspan. These simulations were performed with an E/G ratio of 16 and then

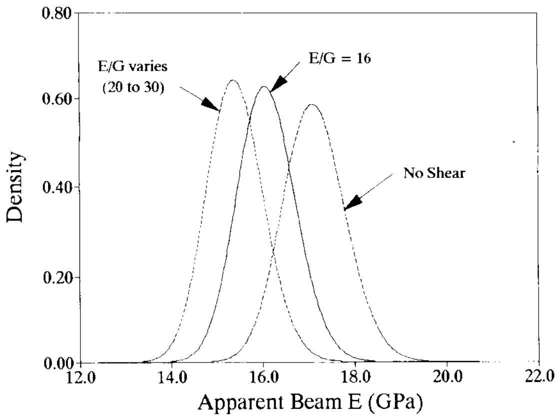

FIG. 4. Predicted probability density functions of apparent beam E for three E/G assumptions.

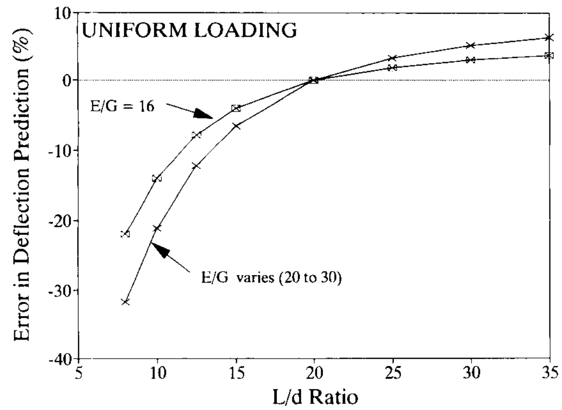

FIG. 6. Error in predicted deflections for glulam beam with uniform loading.

repeated with an E/G ratio of 20 for all lumber grades except L3 which had E/G of 30. One thousand beams were simulated for each case using the same initial random seed value.

The average apparent E predicted by PROLAM at an L/d ratio of 20 was assumed to be the design value of E. This design E was substituted into common beam deflection equations that do not include shear to predict deflections for the three different loading cases and the two E/G assumptions over a range of L/d ratios. Corresponding "true" deflections were predicted using PROLAM over the same range of L/d ratios.

Figures 5, 6, and 7 illustrate the errors in the predicted deflections for the various L/d ratios, loading conditions, and E/G ratios. All three graphs depict similar trends. For relatively long spans (L/d greater than 20), the simple bending equations slightly overpredicted deflections that could be considered conservative errors. Conversely, for wood beams with L/d ratios less than 20, the simple bending equations significantly underpredicted deflection, resulting in unconservative errors. Beam deflection equations (without shear) are usually considered sufficiently accurate for solid-sawn wood in the L/d range of 15 to 25 (Hoyle and Woeste

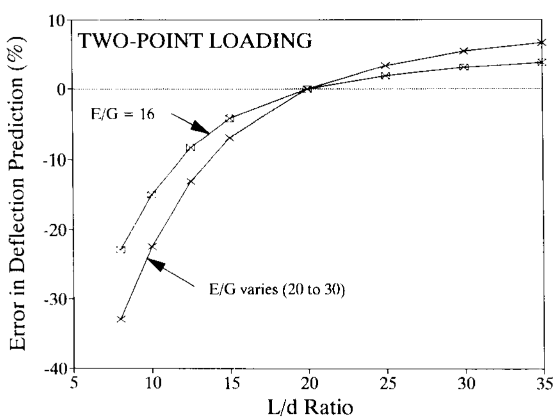

FIG. 5. Error in predicted deflections for glulam beam with two-point loading.

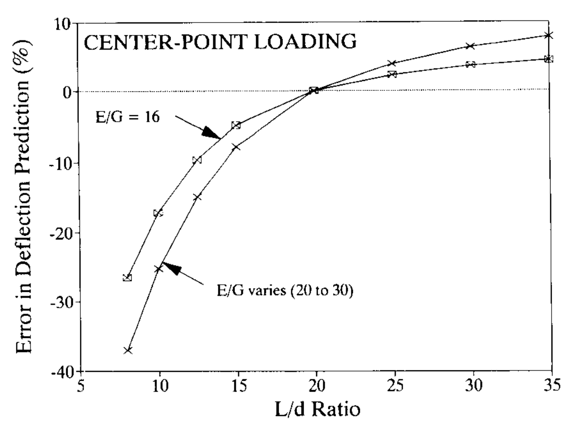

FIG. 7. Error in predicted deflections for glulam beam with center-point loading.

1989). However, the results shown in Figs. 5, 6, and 7 for a composite beam indicate errors of approximately 10% for L/d of 15, which may not be acceptable.

SUMMARY AND CONCLUSIONS

Beam deflection is comprised of two components: 1) bending, and 2) shear deflection. Shear deflection is not included in design calculations for many structural materials. However, shear deflection for wood beams can exceed the bending deflection under certain situations due to the relatively low shear modulus of wood, and it should be considered. This problem is reasonably straightforward for solid-sawn wood beams; however, it becomes more complex for composite beams such as glued-laminated timber beams and wood I-beams.

An equation was developed for predicting shear deflection in composite beams using the transformed-section method of analysis. This equation is similar to other published deflection equations (based on energy methods); however, it was derived to be more general to facilitate studies of localized variability of shear modulus and modulus of elasticity. The composite shear deflection model was validated using data from full-sized glued-laminated beam tests. The difference between average actual and predicted beam E was less than 2%. During development of the shear deflection model, an intermediate step was the development of an equation that characterized the shear stress distribution for composite beams. A possible application of the shear stress equation would be to use it in a probabilistic model to predict shear strength of composite beams.

The composite shear deflection model was incorporated into an existing glulam beam model and sensitivity studies were performed. It was found that the number of increments along the length and depth for the numerical integration had little effect on the apparent beam E predictions. It is recommended to use values of $\Delta x = 30.48$ cm (12 in.) and Δy equal to the lamination thickness. The ratio of modulus of elasticity to shear modulus (E/G) had a significant effect on predictions of beam deflection. Conflicting data on G appear in the literature, suggesting the need for additional research on G and its relationship with E, especially as a function of lumber quality. As expected, span-to-depth ratio (L/d) had the most significant effect on beam deflection. The common engineering design practice of not including shear deflection for solid-sawn wood beams with L/d ratios between 15 to 25 could lead to significant errors for composite wood beams. An unconservative error of approximately 10% was observed for a glulam beam at an L/d ratio of 15.

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