YOUNG'S MODULUS PARALLEL TO THE GRAIN IN WOOD AS A FUNCTION OF STRAIN RATE, STRESS LEVEL AND MODE OF LOADING[1]

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ABSTRACT

Moduli of elasticity were determined parallel to the grain in wood specimens using dynamic flexural tests, static tensile tests, and static compression tests. In the static tests the specimens were repetitively loaded at different strain rates below their proportional limits. Stress rate and strain rate data were taken at discrete stress levels. Species used were red oak, Douglas-fir, and western hemlock.

Stress rate plotted as a linear function of strain rate with zero intercept. The slopes of the plots were Young's moduli. For the majority of specimens, there were no significant differences between Young's moduli in tension and Young's moduli in compression. There were also no indications that Young's moduli were either a function of stress level or strain rate. Dynamic flexural moduli uncorrected for shear deflection averaged 3% less than static tensile moduli.

Additional keywords: *Tsuga heterophylla*, *Quercus* sp., *Pseudotsuga menziesii*, dynamic tests, static tests, bending tests, compression tests, tension tests.

INTRODUCTION

Several characteristics of Young's moduli parallel to the grain in wood are investigated in this paper. These concern the equality of Young's moduli in tension and compression, the variability of Young's moduli with strain rate, and the variability of Young's moduli with stress level. The testing technique and data analysis are based on stress rate and strain rate data collected at discrete stress levels in repetitive loadings of test samples.

The variation of Young's moduli with rate of loading is conceded to be small. James (1968) did not find any significant differences between Young's moduli in bending that were determined at rates of loading differing by a factor of 10,000. However, King (1957) observed small differences in moduli measured at speeds from 0.008 inches per min to 0.035 inches per min in tension specimens. More recently, Okuyama et al. (1970) noticed slight increases in Young's moduli as strain rates ranged from 0.05% to 10% per min in compression loading.

It is generally believed that measured Young's moduli decrease with increase in stress level even below the proportional limit because of the hypothesis that stress-strain curves are not straight lines. The reasoning for this theory comes from the fact that when a constant load is maintained on a wood member, the member often continues to deflect or creep. When the load is increased, the total strain observed is the strain due to the change in stress and the strain due to creep during the time required for the load change. King (1961) found measurable creep occurring in tensile specimens at stresses as low as 5% of their ultimate tensile strength parallel to the grain for some species and at least as low as 15% for all species tested. The creep rate increased with increase in stress. Consequently, he concluded that the stress-strain relationship was not linear below the proportional limit. However, since creep strain is relatively small at loadings below this limit, nonlinearity of the stress-strain curves and therefore changes in Young's moduli are not very apparent.

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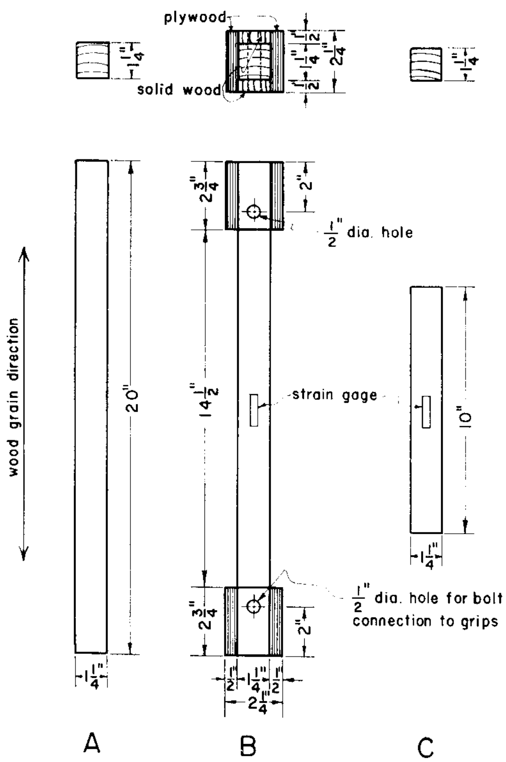

FIG. 1. Specimen geometries: A—beam for dynamic flexural vibration, B—converted to member for tensile testing by addition of end reinforcement, C—cut to shorter length for compression tests.

Ethington and Youngs (1965) found no difference between instantaneous elastic compliances (instantaneous strain as a function of stress) measured in tension and compression perpendicular to the grain. However, flow in compression was greater than flow in tension. Carefully matched samples of red oak were used in their work.

One of the more popular models for the fitting of stress-strain data for viscoelastic materials is a Maxwell Model, which consists of a spring and a dashpot in series. Collection of data in the form of stress and strain at constant strain rates is very desirable for this type of analysis. Special control procedures are required to achieve constant strain rates with most testing machines. Okuyama and Asano (1970) and Okuyama et al. (1970) collected stress-strain data at controlled strain rates for wood and fitted them to Maxwell Models. Similar analysis of data with Maxwell Models is given for rubber testing by Smith and Dickie (1970). Sliker's (1972) determination of Young's moduli from stress rate and strain rate data may also conform to a Maxwell Model.

PROCEDURE

Wood members were each tested in dynamic flexure, static tension parallel to the grain, and static compression parallel to the grain. Test specimens are shown in Fig. 1. Members 1.25 inches by 1.25 inches by 20 inches parallel to the grain were first cut as at *A*. After dynamic testing in bending, end grips were added for tensile testing as in sketch *B*. When the tensile testing was completed, the central 10 inches of the specimens were cut out as at *C* for compression loading. Baldwin type A-3-S6 bonded wire

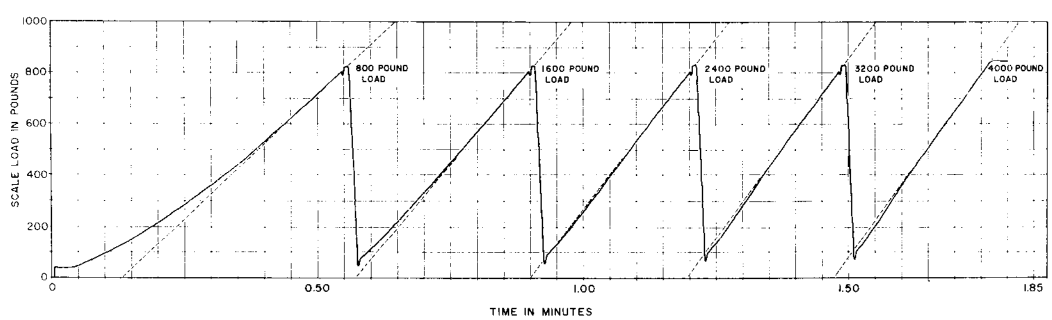

FIG. 2. Example of load versus time data (solid sloping lines) as drawn by strip chart recorder for a single load sequence at one testing machine crosshead speed. Dashed lines are those used to determine stress rates at indicated load levels.

TABLE 1. *Stress rates and strain rates measured at seven load levels in tension parallel to the grain for western hemlock sample H-4.*

TEST SERIES NUMBER 1	NOMINAL CROSSHEAD RATE IN INCHES PER MINUTE 2	LOAD LEVEL IN POUNDS 3	STRESS RATE IN PSI PER MINUTE 4	STRAIN RATE IN MICROSTRAIN PER MINUTE 5	YOUNG'S MODULUS IN POUNDS PER SQUARE INCH x 10^{-6} (Col. 4 ÷ Col. 5) 6
T-1	0.02	800	495	287	1.725
		1600	509	328	1.552
		2400	566	361	1.568
		3200	660	391	1.688
T-2	0.05	800	1014	651	1.558
		1600	1546	864	1.789
		2400	1621	971	1.669
		3200	1592	988	1.611
T-3	0.10	800	2400	1452	1.653
		1600	3035	1860	1.632
		2400	3303	1943	1.700
		3200	3386	1998	1.695
T-4	0.20	1600	6118	3488	1.754
		2400	6458	3769	1.713
		3200	6971	4164	1.674
		4000	6949	4105	1.693
		4800	6934	4256	1.629
T-5	0.02	800	531	311	1.707
		1600	644	341	1.889
		2400	641	400	1.603
		3200	691	419	1.649
		4000	759	434	1.749
		4800	720	447	1.611
T-6	0.05	800	1322	825	1.602
		1600	1545	976	1.583
		3200	1630	966	1.687
		4000	1661	1032	1.609
		4800	1771	1039	1.705
T-7	0.10	800	2284	1380	1.655
		2400	3287	1947	1.688
		3200	3397	2047	1.660
		4000	3467	2073	1.672
		4800	3497	2058	1.699
		5600	3344	2051	1.630
T-8	0.50	4000	17151	10446	1.642

resistance strain gages (13/16 inch by 9/64 inch grid; thin paper backing) were attached with Duco cement for measuring tension and compression strain: one gage was centered on a longitudinal tangential face of each specimen; a second gage was placed in the same location on the opposite face. By connecting the two gages in a Wheatstone bridge circuit so that like signals were added and unlike canceled, strains from bending of the test sample were suppressed while the average axial strain was being recorded.

Initially, there were five specimens of western hemlock (*Tsuga heterophylla* (Raf.) Sarg.), five of red oak (*Quercus* sp.), and

TABLE 2. *Stress rates and strain rates measured at five lead levels in compression parallel to the grain for western hemlock sample H-4*

TEST SERIES NUMBER 1	NOMINAL CROSSHEAD RATE IN INCHES PER MINUTE 2	LOAD LEVEL IN POUNDS 3	STRESS RATE IN PSI PER MINUTE 4	STRAIN RATE IN MICROSTRAIN PER MINUTE 5	YOUNG'S MODULUS IN POUNDS PER SQUARE INCH x 10^{-6} (Col. 4 ÷ Col. 5) 6
		800	1155	800	1.444
		1600	1542	946	1.630
C-1	0.02	2400	1691	1021	1.656
		3200	1740	1063	1.637
		4000	1777	1074	1.655
		800	2997	1862	1.610
		1600	3762	2283	1.648
C-2	0.05	2400	4080	2448	1.667
		3200	4206	2499	1.683
		4000	4219	2565	1.645
		1600	7741	4595	1.685
C-3	0.10	2400	8313	5001	1.662
		3200	8519	5106	1.668
		4000	9020	5384	1.675
C-4	0.20	4000	17400	10700	1.626

four of Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco). However, one of the Douglas-fir samples failed during tensile testing because of spiral grain that had not been previously detected; this specimen was eliminated from the analysis. Moisture content conditioning and testing were done in a room kept at 68 F and 65% relative humidity. The oak had an equilibrium moisture content close to 11%; the equilibrium moisture contents of the other two species were from 12½% to 13% in this atmosphere.

Testing for dynamic modulus in bending was accomplished as described in a publication by Kitazawa (1952). The 20-inch-long wood members were simply supported as beams 4.5 inches from either end and vibrated to determine their resonant frequencies. Coil magnets on rigid stands interacted with staples driven into the bottom surface at the beam ends to provide driving force and signal pickup.

Procedures for static testing were similar for the compression and tensile loading except for the direction of stress application. Tensile specimens were successively loaded at five different rates of machine crosshead movement (0.02, 0.05, 0.10, 0.20, 0.50 inches per min) up to a maximum load of 5600 pounds. Compression specimens were loaded at four different rates (0.02, 0.05, 0.10, 0.20 inches per min) up to a load of 4000 pounds. Successive loadings of a given specimen were 48 hr or more apart. (See Tables 1 and 2 for typical load sequences at the different crosshead rates. Order of testing was as in order of listing in tables.)

During a particular test sequence, load versus time and strain versus time were being recorded on strip chart recorders. Simultaneous marking pips were placed on each chart for every load increment of 800 pounds. In addition, after passing each 800 pounds load increment, the existing load and the existing strain were suppressed on the chart scales in order to maintain greater chart accuracy. (See Fig. 2 for an example of a typical load chart.) Slopes of the charts in terms of microstrain per minute and pounds per minute (converted to stress per minute) were measured at each of the desired load increments. A few readings were discarded because of electronic noise in the strain measuring circuits.

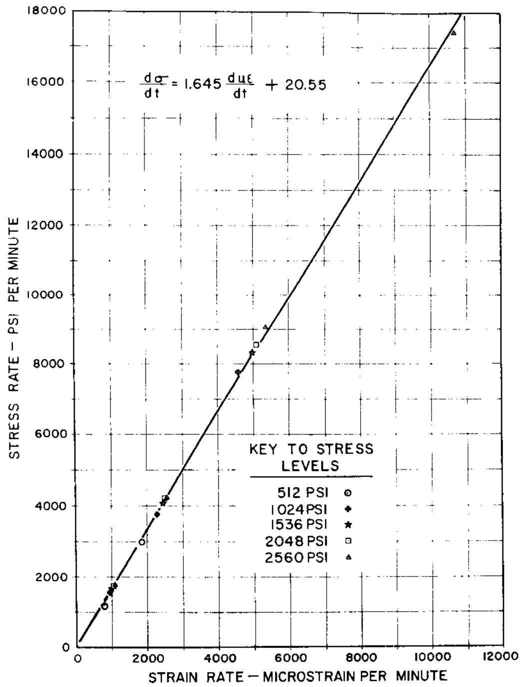

FIG. 3. Stress rate plotted as a function of strain rate for specimen H-4 in compression parallel to the grain.

RESULTS AND DISCUSSION

Stress rate $\frac{d\sigma}{dt}$ (psi per min) can be plotted as a linear function of strain rate $\frac{d\mu\epsilon}{dt}$ (microstrain per min) for a given specimen over the range of data taken. A graph of the two variables is shown in Fig. 3 for western hemlock specimen H-4 in compression. Linear regression analysis yields the equation:

$$\frac{d\sigma}{dt} = a \frac{d\mu\epsilon}{dt} + b \qquad (1)$$

where a and b are the regression constants. Coefficients of determination for equations of this type, which were calculated for each test specimen, were 0.998 in two cases; otherwise they were 0.999 or 1.000.

If an analogy is made to a Maxwell Model, Equation 1 becomes:

$$\frac{d\sigma}{dt} = E \frac{d\mu\epsilon}{dt} - E \frac{\sigma}{\eta} \qquad (2)$$

where E is Young's modulus, σ is stress and η is the coefficient of viscosity. A limit value for Young's modulus can be found equal to the slope of the straight line plot by making the strain rate infinitely large:

$$\text{as } \frac{d\mu\epsilon}{dt} \rightarrow \infty \quad E \rightarrow \frac{d\sigma/dt}{d\mu\epsilon/dt} \qquad (3)$$

Young's modulus at lower strain rates would be:

$$E = \frac{\frac{d\sigma}{dt}}{\left(\frac{d\mu\epsilon}{dt} - \frac{\sigma}{\eta}\right)} \qquad (4)$$

Therefore, strain-rate dependency of Young's modulus would be indicated if the last term of Equations 1 and 2 were other than zero. For compression sample H-4, the equation determined from the statistical routine was:

$$\frac{d\sigma}{dt} = 1.645 \frac{d\mu\epsilon}{dt} + 20.55 \qquad (5)$$

The constant term 20.55 might be assumed to be the value of the stress rate when the strain rate was zero. However, the standard error of this particular constant term is 44.58, and according to the t test, the constant term is not significantly different from zero at the 0.1 level of probability. The same situation occurred for the linear regression analyses of all the other compression samples and for all but four of the tension samples. In the case of the tension samples, the intercept for one was significant at the 0.1 level of probability, for another at the 0.05 level, for a third at the 0.02 level, and for a fourth at the 0.01 level. The most logical statistical assumption is that the straight line relationships determined for the twenty-six test specimens pass through the origins of the graphs with possibly a few exceptions as mentioned and, therefore, the Young's moduli are equal to stress rate divided by strain rate for all values of strain rate.

Young's moduli determined in tension, in compression, and by dynamic free-free vi-

TABLE 3. *Comparison of Young's moduli determined from static tension testing, static compression testing, and dynamic flexure testing.*

SPECIMEN NUMBER *	OVEN-DRY SPECIFIC GRAVITY	STATIC YOUNG'S MODULI				COMPRESSION E ÷ TENSILE E Col. 5 ÷ Col. 3	DYNAMIC YOUNG'S MODULI IN FLEXURE PSI x 10^{-6}	DYNAMIC E ÷ TENSILE E Col. 8 ÷ Col. 3
		TENSION		COMPRESSION				
		E PSI x 10^{-6}	STANDARD ERROR x 10^{-6}	E PSI x 10^{-6}	STANDARD ERROR x 10^{-6}			
1	2	3	4	5	6	7	8	9
H-1	0.438	1.866	0.010	1.877	0.020	1.006	1.795	0.962
H-2	0.451	1.892	0.012	1.935	0.019	1.023	1.833	0.969
H-3	0.451	1.815	0.011	1.819	0.018	1.002	1.747	0.963
H-4	0.476	1.656	0.008	1.645	0.011	0.993	1.579	0.954
H-5	0.446	1.856	0.009	1.842	0.012	0.992	1.764	0.950
RO-1	0.688	2.515	0.014	2.474	0.030	0.984	2.397	0.953
RO-2	0.709	2.431	0.010	2.370	0.019	0.975	2.302	0.947
RO-3	0.683	2.228	0.014	2.331	0.013	1.046	2.248	1.009
RO-4	0.686	1.900	0.009	1.894	0.013	0.997	1.938	1.020
RO-5	0.667	2.102	0.014	2.120	0.022	1.009	2.158	1.027
DF-1	0.556	2.600	0.012	2.653	0.027	1.020	2.228	0.857
DF-2	0.464	1.991	0.016	2.099	0.012	1.054	1.936	0.972
DF-3	0.563	1.717	0.011	1.712	0.023	0.997	1.692	0.985

*H = hemlock; RO = red oak; DF = Douglas-fir brations in bending are compared in Table 3. Differences between the Young's moduli in tension and those in compression were less than 1% for seven test specimens, were between 1% and 2.5% for four others, and for the remaining two were 4.6% and 5.4%. The latter two differences, which were for specimens RO-3 and DF-2, were statistically significant at the 0.001 level of probability when analyzed by covariance with dummy variables as described by Freese (1964). In addition, the difference for RO-2 was significant at the 0.01 level of probability. The remaining differences were not significant at the 0.05 level of probability. No apparent reason was observed for the larger differences other than that they occurred in specimens with distinct change in properties between earlywood and latewood. For most of the specimens, it appears as though Young's modulus in compression equals Young's modulus in tension. It should be noted that stress was calculated on the basis of original cross-sectional dimensions and not with the dimensions of the stressed specimens.

Dynamic flexural Young's moduli without correction for shear deflection averaged 3% less than the static tensile moduli (Column 9 of Table 3). This compares closely with the results of Bell et al. (1954) where Young's moduli from dynamic flexural tests approximately equaled the Young's moduli from static compression parallel to the grain tests on Douglas-fir before an allowance had been made for shear deflection in bending. As can be seen in Table 3, the ratios of these two moduli were more consistent for the hemlock specimens than for the red oak or Douglas-fir. It might be expected that for these two species the effective moment of inertia of a beam would be affected more than in the hemlock by growth ring placement, growth ring width, and differences in properties of earlywood and latewood. It is also well to keep in mind that the standard errors associated with the strain gage determinations for Young's moduli do not take into account errors from improper gage placement, incorrect gage factor, etc.

There was no clear indication that Young's moduli varied with either strain rate or stress level over the range of the test data. In multiple correlation analysis with stress rate run as a function of strain rate and stress level, the stress level factor did not prove to be significant. A number of additional regressions were tried with Young's moduli versus various manipulations of stress and strain rate, such as stress, log of stress, strain rate, log of strain rate, reciprocal of strain rate, and log of reciprocal of strain rate. Young's moduli were formed for these analyses by dividing stress rates by strain rates. Occasionally, the mathematical statistics would indicate significance at the 0.01 level of probability for a few specimens examined with a particular set of variables. However, no consistent pattern emerged for the majority of samples for any system.

A potential complicating factor in observing strain rate phenomena was that the strain rate during the testing of a single specimen at a given machine crosshead speed was not a constant. This can be seen in Table 1 for specimen H-4 in tension and in Table 2 for the same specimen in compression. At the lower loads, rate of strain increased rapidly and then appeared to approach an equilibrium at the higher loads. Strain rates occurring at the higher stress levels were often 25% more than those occurring at 800 pounds. It was possible to take all the data from the 452 observations of tensile testing irrespective of species and to write a predictive equation of strain rate ($d\mu\epsilon/dt$) in microstrain per minute as a function of testing machine crosshead speed setting (cs) in inches per minute, load level (P) in pounds, and Young's modulus (E) in pounds per square inch:

$$\frac{d\mu\epsilon}{dt} = 29{,}648(cs) + 0.08911(P) - 0.006050(E)(cs) - 324.0 \quad (6)$$

The coefficient of multiple determination (R^2) for this equation is 0.989. Similarly from the 190 observations for compression testing, the equation determined by linear regression analysis is:

$$\frac{d\mu\epsilon}{dt} = 70{,}554(cs) + 0.1668(P) - 0.01185(E)(cs) - 566.2 \quad (7)$$

For this equation, the coefficient of multiple determination is 0.991. Not included in the equations is the effect of specimen dimensions on strain rate.

Strain rates under compression loading, which are easier to compare than those under tension loading, were approximately half or were less than half of those predicted from measured crosshead rates without load. For instance, the expected strain rate in a 10-inch-long specimen for a crosshead movement of 0.02 inches per min would be 2000 microstrain per min. Measured strain rates from Table 2 for this speed setting were between 800 and 1074 microstrain per min.

The test results do not agree completely with the literature reviewed with regard to the relationships between Young's moduli and stress level and strain rate. From several references (King 1957; Okuyama et al. 1970), it was to be expected that Young's moduli would decrease with stress level. Either this is not true or the test method was not sensitive enough to detect the changes. Variation in stress-strain relationships reported by King (1957) would be large enough to be detected if data were taken at stress levels where creep occurred. Data for this research were taken below and slightly above the lowest stress levels where King reported creep.

This report also disagrees with some others (King 1957; Okuyama et al. 1970) with regard to the effect of the time-dependency of Young's moduli. Again the changes are small and could be easily overlooked. It is important that observed strain rate effects are properties of the test material and not of the testing apparatus. Data taken over a wider range of rates than described here would be helpful.

CONCLUSIONS

For western hemlock, red oak, and Douglas-fir samples tested at a number of strain rates in compression parallel to the grain to a maximum stress of 2560 psi and in tension parallel to the grain to a maximum stress of 3580 psi at 68 F and 12% moisture content:

1. Stress rate could be plotted as a linear function of strain rate with a zero stress rate intercept. Relationships were independent of stress level.
2. The slope of the above-mentioned plot for a given specimen was its Young's modulus. The standard errors associated with the slopes were from 0.5% to 1.0% of the slope values.
3. No significant differences were found between Young's moduli from tension testing and Young's moduli from compression testing for the majority of test specimens.
4. The dynamic moduli measured in free-free vibration in flexure averaged 3% less than the tensile moduli from static testing when no correction was made for shear deflections in bending.
5. There was no apparent relationship between Young's modulus and stress level nor between Young's modulus and strain rate.
6. For a given crosshead speed setting on the testing machine, the strain rates observed in the test specimens were considerably less than those expected. Strain rates were functions of crosshead speed settings, load level, and interaction between crosshead speed setting and specimen stiffness.

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