EMPIRICAL MODELS FOR PREDICTION COMPRESSION STRENGTH OF PAPERBOARD CARTON[1]

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Abstract. When designing packaging in the shape of a rectangular parallelepiped from various paperboard materials, it is important to determine their resistance to vertical compression force, which should be less than the maximum compression force. This is especially relevant when the products packed in these boxes are stacked during transport or storage. The developed empirical models make it possible to more optimally/more accurately determine the critical vertical compressive force of these packages. The purpose of this work is to create an semi-empirical model of the maximum compressive force of a paperboard box (carton) based on the corrected formulas of the maximum compressive force of the McKee corrugated cardboard box (taking into account the height) of the box and allowing to optimize its parameters. The accuracy of the developed semi-empirical models is presented by comparing the results of theoretical and experimental studies. It should be noted that the determination of the maximum compression force of the box is a contact problem of the nonlinear theory of elasticity and plasticity for structures whose elements are made of an anisotropic material. On this basis, semi-empirical models of three and one parameters were developed, which also estimated the values of experimental studies previously performed by other authors. One mathematical model also estimates the height of the box, which is not determined by the McKee formula. For the experiments, we used cartons of different geometric parameters and made from different types of paperboards. During the experiment, the boxes were compressed with vertical force until the packages collapsed. The results of the compared theoretical and experimental studies show the suitability of the proposed

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mathematical models for calculating the critical compressive force of packages, since the obtained mean absolute percentage error (MAPE) is within the acceptable limits. Taking into account the small discrepancy between the obtained experimental and theoretical research results, the proposed method for calculating the vertical maximum compressive force of the rectangular parallelepiped package is suitable for use. The methodology for calculating the carton compressive strength of such packages presented in this paper will be extended in the future for additional testing to verify the model with carton size and design variations.

Keywords: Paperboard, cartons, compression, strength, critical force, empirical model McKee.

INTRODUCTION

In these days the most important question in the packaging industry is—What should be the modern packaging? First of all, naturally, it should be environmentally friendly and completely harmless. Moreover, the packaging must be sufficiently strong, lightweight, reliable, and the manufacture of it should be cheap. All these criteria apply together to one type of material—paperboard. The wide range of applications, from the packaging of industrial goods to food, makes the future possible to continue using paperboard in the packaging industry.

Paperboard cartons are mainly used for light products. Paperboard cartons are used in food, cosmetics, clothing, and many other industries. This type of packaging may be glued or folded. Paperboard packaging gives wide advertising opportunities. The ability to apply any print design makes it one of the most commonly used marketing tools.

Packaging plays a pivotal role in the distribution and transportation of goods, which means that it must comply with all major requirements, both in terms of aesthetics and durability standards (RDC-Environment and Pira International 2003).

The most popular paperboard packaging type is cartons in the shape of a rectangular parallelepiped.

Models for the prediction of the maximum force the top-to-bottom compressive of folding cartons may be divided into analytical and numerical based on the finite element analysis (Beldie et al 2001; Garbowski and Przybyszewski 2015). In their turn, analytical models (mathematical formulae indicating relations between the value in question and parameters of the model) may be divided into empirical (Pyryev et al 2016), semi-empirical (McKee et al 1963; Coffin 2015) and "exact" within the scope of the assumptions (eg linearity of the model) made (Grangård and Kubát 1969; Pyryev et al 2019). "Exact" models for a maximum compression force does not require any experimental data. Models for different carton designs will also be different (Ristinmaa et al 2012). Models for the prediction of maximum compression force differ according to the material used to manufacture the boxes. More research has been done for corrugated cardboard and fewer for paperboards (Pyryev et al 2016, 2019; Kibirkštis et al 2007).

In 1963, McKee et al (1963) developed and published a mathematical equation to determine the compressive strength of a three-ply cardboard box, which is still in use today.

$$F_{\max} = aP_m^b(\sqrt{D_{CD}D_{MD}})^{1-b}P^{2b-1} \qquad (1)$$

Based on his research, McKee came to the conclusion that the dominant parameter affecting the properties of a box compressed from above and below by force F, as in the case of box storage, is the edge crush strength of corrugated cardboard in the cross direction (CD), which he defined as the parameter P_m and parameters describing the bending stiffness of a cardboard box in two directions D_{CD} and D_{MD}. McKee also used the parameter P, meaning the perimeter of the box, a and b constants defined experimentally. The ratio of height H to the circumference P must be $>$1:7 » 0.143. Let us assume that the direction of the force F coincides with the x-axis.

Mathematical models can take very divergent paths to try to achieve the goal. Little (1943) identified the main factors influencing box strength by analogy to column failure: box size, inherent material strength, and material stiffness.

One of the earliest practical formulations for estimating box compression strength (*BCT*) using these parameters was developed by Kellicutt and Landt (1958). Then, McKee et al published in 1963 likely the most popular industry method of box compression estimation and the one used among the most publicly available software programs.

Most packaging engineers are familiar with the McKee equation, typically in one of its numerical forms:

$$BCT = 2.028 \times ECT^{0.746} (\mathrm{RMSBending})^{0.254} \mathrm{Perimeter}^{0.492}, \tag{2}$$

$$BCT = 5.87 \times ECT(\mathrm{Caliper} \times \mathrm{Perimeter})^{1/2}. \tag{3}$$

The structure of these equations highlights the importance of different material parameters on box performance. The edge crush strength (*ECT*) of the combined board, has the largest role in estimating box strength. Measuring *ECT* attempts to quantify the inherent material strength of the complex corrugated board structure, and researchers (Frank 2014) have taken many approaches over the years to assess this material property of the combined board. Batelka and Smith (1993) expanded the original McKee box compression model (1) to include all box dimensions in their formula.

Two of the criteria proposed by Urbanik and Frank (2006) appear to lead to compression strength predictions in accordance with a large set of experimental results. The second approach considers that the buckling mode m, which yields the lowest critical buckling load for the side panel, has to be applied to both panels.

The carton board is an orthotropic material (Edholm 1998). This is because, during its manufacture, the majority of the fibers orient themselves in a direction, known as the machine direction (MD). The direction perpendicular to this is known as the CD. Furthermore, the properties of the board in the thickness direction (ZD) differ from those of MD and CD because of its laminar construction. A big effort of research has been taken to investigate the mechanical properties of the carton board (Sirkett et al 2006).

A simplified version of the McKee formula was fitted to the data set by Popil (2016) and the resulting equation improves the prediction average error.

Urbanik and Saliklis (2003) applied finite element analysis to observe the buckling phenomena in corrugated boxes.

Interesting studies carried out in the work by Gong et al (2020), where the effects of indentation shape on the compressive strength of corrugated cartons are studied by experiments and finite element analysis. A crippling analysis method has been utilized to estimate the compression strength of paperboard boxes in work (Linvill 2015). Two types of tests are required to characterize the crippling strength of material: compression tests of panels with one edge free and compression tests of panels with no edges.

The purpose of this work is to create an semi-empirical model of the maximum compressive force of a paperboard box (carton) based on the modified formulas (Batelka et al 1993) of the maximum compressive force of the McKee corrugated cardboard box, taking into account the height of the box and allowing to optimize of its parameters.

EXPERIMENTAL DATA AND MATERIALS

Six different types of paperboard packaging constructions were used for the creation of the engineering calculation procedure in relation to the maximum compression force. The paper proposes models to predict the top-to-bottom compressive strength of folding cartons. In the work, the experiment consisted of determining the compression data of 72 cartons ($i = 1, \ldots, 72$), (Fig 1[a]): six different geometrical parameter cartons and six different types of paperboard compressed in the CD, (Fig 1[b]), and six in the MD, (Fig 1[c]).

The number of repetitions of the tests for each of the 72 cartons is six. The geometrical parameters of the packaging are listed in the caption under Fig 2.

The simplified scheme for a compression stand and the view of packaging samples are both shown in Figs 1-2. Packaging of such sizes is

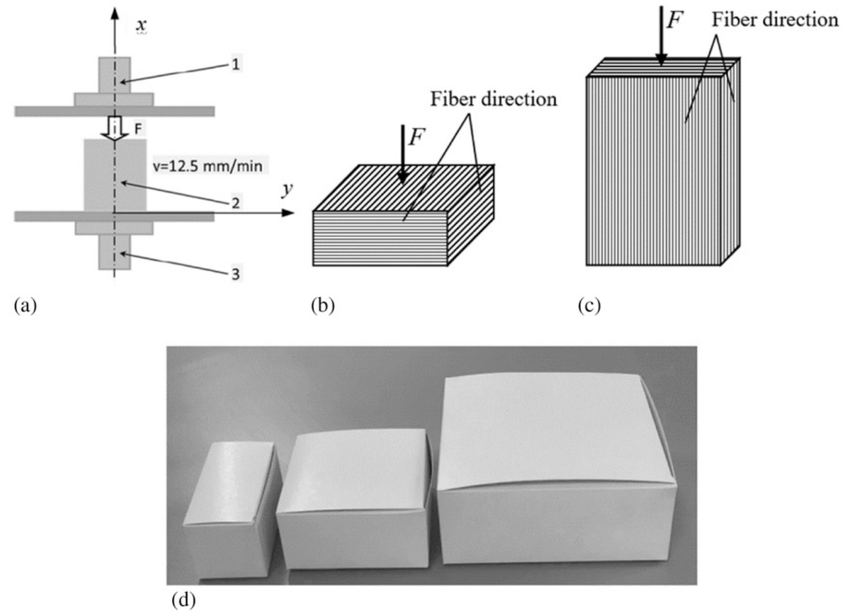

Figure 1. Compression testing scheme for a package under the action of vertical force F, N: (a) principal scheme, (b) compression testing scheme in cross direction (CD), (c) compression testing scheme in the machine direction (MD), (d) illustration the three different packages A60.20.00.03 classified according to ECMA considered in this study: 1—moving base support (v = 12.5 mm/min); 2—package under compression; 3—fixed base support.

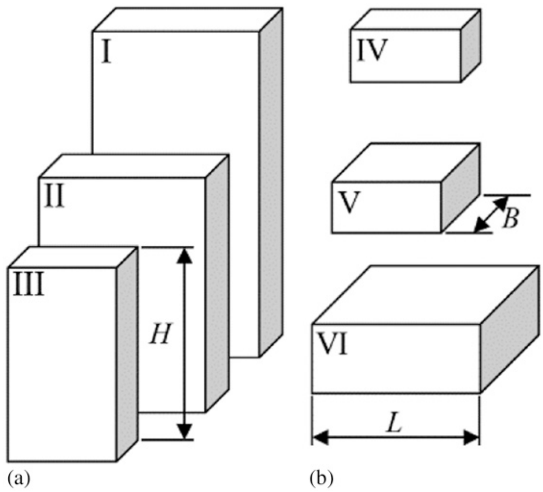

Figure 2. The packaging specimen's geometrical parameters: (a) carton size I: H = 230 mm, L = 118 mm, B = 48 mm; carton size II: H = 165 mm, L = 118 mm, B = 48 mm; carton size III: H = 137 mm, L = 77 mm, B = 37 mm; (c) carton size IV: H = 37 mm, L = 77 mm, B = 37 mm; carton size V: H = 37 mm, L = 77 mm, B = 77 mm; carton size VI: H = 48 mm, L = 118 mm, B = 118 mm.

widely used in Lithuania for packing products such as, for example, grain products (rice, buckwheat, etc.), which are prepacked into a separate carton for cooking (Kibirkštis et al 2007). The choice of this type of packaging specimen was determined by their wide-ranging usage for packing food products. For cartons with dimensions in Fig 2(a) (i = 1, . . ., 36), some of the experimental data carton strength and the short span compression strength, the bending stiffness of the paperboards was previously presented by the authors in earlier papers (Kibirkštis et al 2007; Pyryev et al 2016). Experimental data for paperboard package compression tests were obtained in standard atmosphere for conditioning and testing according to ISO 187:1990 requirements. The low cartons were also investigated for additional analysis (Fig 2[b]). Note that this paper presents new experimental data for 36 cartons (Fig 2[b]) (i = 37, . . ., 72) that have not been published before. The experiments for the low cartons in Fig 2(b) were carried out for the same type of paperboards as for the cartons in Fig 2(a).

When testing its compression strength, the carton is placed between two parallel rigid plates and is compressed at a constant rate of deformation of 12.5 mm/min in accordance with ISO 12048 recommendations. For the measurements were used: sensor—DBBMTOL-500 N, serial number AP34282. Load measurement accuracy: $\pm 0.5\%$ indicated load from 2% to 100% capacity, extended range. Position measurement accuracy $\pm 0.01\%$ of reading or 0.001 mm, whichever is greater. Speed accuracy $\pm 0.005\%$ of set speed. The maximum force of load that the sample can support is called the compression force F_{exp} (experimental value).

The main characteristics of the cartons are bending stiffness D and compressive strength SCT.

Figure 2 shows the sample cartons. For cartons I parameter $\lambda = P/H = 1.44$, for cartons II parameter $\lambda = 2.01$, for cartons III parameter $\lambda = 1.66$, for cartons IV parameter $\lambda = 6.17$, for cartons V parameter $\lambda = 8.33$, for cartons VI parameter $\lambda = 9.80$, where $P = 2\ (L + B)$ is the perimeter of rectangular plate $L \times B$.

With consideration to the potential conditions involved in storage, transportation, and maintenance, the packages were made of different paperboard types:

- Soft MC Mirabell paperboard (WLC) or (GD2)—Recycled coated white lined chipboard
- Kromopak paperboard (FBB) or (GC2)—Folding boxboard
- Korsnas Carry (SUB) or (GN4)—Solid unbleached board
- Korsnas Light (SUB) or (GN4)—Solid unbleached board

The boxes were made according to No A60.20. 00.03 PackDesign 2000 Standard Libraries for European Carton Makers Association (ECMA).

The technical characteristics provided by the manufacturers of these paperboards are listed in Table 1. MD—the direction in the board where the fibers are arranged in the direction of machine casting during the board manufacturing process. CD—the direction perpendicular to the MD. In the carton, it is most common to have the MD parallel to the vertical axis (for several reasons—primarily the reliability of opening on high-speed machinery), but to compare the experimental and theoretical results in both directions (MD) and (CD) need to be explored.

The findings from the proposed calculation procedures were later compared with the findings from experimental compression tests (see Table 2 (column 5), $i = 1, \ldots, 36$), which were analyzed in the paper (Kibirkštis et al 2007) for cartons from Fig 2(a).

Based on the work of Ristinmaa et al (2012) in this work, the proposed semi-empirical models have been tested on independent experimental data to predict the critical compressive force of a carton.

MODELING THE COMPRESSIVE STRENGTH

Formula McKee (1),[1] which takes into account the compressive force, is a two-parameter one. In addition, Eq 1 does not take into account the height of the box. It is anticipated that the height of the box may affect its strength. This assumption leads to a three-parameter formula. For the sake of simplicity, we will further reduce it to a one-parameter form, which will contribute to a simplified design of packages. For the above transformations, we will further use an empirical analysis of the obtained experimental data.

The Structural Formula of Maximum Compression Force

It should be noted that the determination of the maximum compression force of the carton is a contact problem of the nonlinear theory of elasticity and plasticity for a structure whose elements are made of an anisotropic material. The contact load on the side panels of the carton and the area of plasticity are unknown quantities. The solution to such a problem can be obtained only using numerical methods or a semiempirical approach.

A model can be devised based on the empirical results for structures that fail through combined compression and bending with the dimensionless

Table 1. A comparison of paperboard technical characteristics (Kibirkštis et al 2007).

	Type of paperboard	Grammage ISO 536 (g/m^2)	Thickness ISO 534 (μm)	D = Bending stiffness L&W[a], (5°), (mNm) ISO 5628		SCT = Compressive strength (kN/m) ISO 9895	
				MD[b]	*CD*[c]	*MD*[2]	*CD*[3]
1	MC Mirabell (WLC)	400	565	60.9	24.4	12.7	8.5
2	MC Mirabell (WLC)	320	435	31.8	13.3	9.8	7.4
3	Kromopak (FBB)	300	430	34.3	14.3	9.2	6.8
4	Kromopak (FBB)	275	395	29.0	12.0	8.6	6.2
5	Korsnas Carry (SUB)	400	585	113.0	55.3	11.2	8.4
6	Korsnas Light (SUB)	290	420	41.9	21.2	8.5	6.1

[a] L&W device-measured moment needed for bending the sample material to an angle of 5°.
[b] *MD*—machine direction.
[c] *CD*—cross machine direction.

form for the maximum force F_{max} (calculated value) as:

$$\frac{F_{max}}{F_{cr}} = a\left(\frac{SCT_x P}{F_{cr}}\right)^b \tag{4}$$

F_{max} = maximum compression force [N] (vertical maximum force in x directions); $P = 2(L+B)$ is the perimeter of the rectangular plate $L \times B$ [m]; H = height of the carton [m]; SCT_x = compressive strength in x directions of the board using a short-span compressive tester [N/m]; F_{cr} is the critical buckling load (coefficient of similarity) [N]; a, b = constant parameters defined on the basis of experimental data.

The choice of the power-law dependence of F_{max} on SCT_x and on the height of the carton H, on the one hand, is due to the analysis of the experimental data obtained, and on the other hand, the power-law dependence of the critical compression force of the corners on SCT_x by the formula presented in the article (Pyryev et al 2019). Therefore, the F_{max} becomes:

$$F_{max} = a(SCT_x)^b (F_{cr})^{1-b} P^b \tag{5}$$

We also assume that the critical compressive force F_{cr} is proportional to the sum of the box-critical compressive forces discussed in the article (Pyryev et al 2019) regarding four plates of the same height H:

$$F_{cr} \sim \frac{\sqrt{D_x D_y}}{P}\left(\frac{P}{H}\right)^d \tag{6}$$

where D_x, D_y = flexural rigidity in x and y directions [Nm], H = height of the carton [m], d = constant parameter, e.g. for low boxes H/L<<1, H/B<<1 (with all edges simply supported) parameter d = 2, for tall boxes H/L>>1, H/B >>1 parameter d = 0. This paper suggests building a theoretical model for a maximum compressive force of the carton as shown as follows:

$$F_{max} = a(SCT_x)^b \left(\sqrt{D_x D_y}\right)^{1-b} P^{2b-1}\left(\frac{P}{H}\right)^c \tag{7}$$

a, b, c = constant parameters defined on the basis of experimental data, c = d(1−b). A similar approach was used in the article (Coffin 2015) for corrugated box.

The expression (7) can be written as follows:

$$\tilde{y} = b_0 + b_1 x_1 + b_2 x_2 \tag{8}$$

where

$$\tilde{y} = \ln\frac{F_{max}P}{\sqrt{D_x D_y}},\ b_0 = \ln(a), b_1 = b, b_2 = c$$

$$x_1 = \ln\left(\frac{SCT_x P^2}{\sqrt{D_x D_y}}\right), x_2 = \ln\left(\frac{P}{H}\right)$$

Knowing the constant coefficients b_0, b_1, b_2 in Eq 8, you can write down a, b, c values in Eq 7:

$$a = e^{b_0}, b = b_1, c = b_2 \tag{9}$$

Table 2. Experimental data for paperboard package compression tests a prediction of the maximum compression force and their errors; for cases (17) $a = 15.2$, $b = 0.490$, and $c = 0.0329$; for case (19) $a = 15.8$, $b = 0.48$, and $c = 0.241$; for case (20) $a = 13.9$, $b = 0.45$, and $c = 0.247$; for cases (1) $a = 14.4$, $b = 0.499$, and $c = 0$; for cases (21) $a = 14.4$, $b = 0.5$, and $c = 0$.

Experiment number $i=1,\ldots 72$	The box design type Figure 2	Type of paperboard, No.	Load direction	F_{exp}, (N)[b]	F_{max}, (N) based on Equation (17)	(19)/(20)	(1)	(21)	100 ε_i, % based on equation (17)	(19)/(20)	(1)	(21)
1	I	1	MD	329 ± 11	305	295	316	318	7.29	10.4	4.10	3.25[a]
2	I	1	CD	235 ± 6	251	244	258	260	6.63	3.68[a]	9.88	10.8
3	I	2	MD	191 ± 5	195	188	202	204	2.07	1.81[a]	5.95	6.92
4	I	2	CD	164 ± 5	170	164	176	177	3.60	0.05[a]	7.25	8.21
5	II	1	MD	320 ± 6	308	320	316	318	3.64	0.15[a]	1.40	0.53
6	II	1	CD	253 ± 3	253	264	258	260	0.13[a]	4.33	2.06	2.92
7	II	2	MD	211 ± 2	197	203	202	204	6.59	3.71	4.09	3.21[a]
8	II	2	CD	167 ± 3	172	178	176	177	2.86[a]	6.44	5.33	6.27
9	III	1	MD	267 ± 12	309	311	316	318	15.7[a]	16.4	18.3	19.2
10	III	1	CD	230 ± 8	254	257	258	260	10.3[a]	11.6	12.4	13.2
11	III	2	MD	210 ± 7	197	198	203	204	5.99	5.87	3.57	2.75[a]
12	III	2	CD	161 ± 4	172	173	176	177	6.86[a]	7.42	9.33	10.2
13	I	3	MD	184 ± 3	196	189	203	205	6.68	2.82[a]	10.6	11.6
14	I	3	CD	165 ± 2	169	164	175	177	2.60	0.69[a]	6.06	6.99
15	I	4	MD	162 ± 3	174	167	181	182	7.40	3.37[a]	11.5	12.5
16	I	4	CD	132 ± 8	148	143	153	155	12.3	8.58[a]	16.2	17.2
17	II	3	MD	196 ± 3	198	205	203	205	1.25[a]	4.57	3.83	4.77
18	II	3	CD	184 ± 3	171	178	175	177	6.98	3.52[a]	4.89	4.06
19	II	4	MD	177 ± 2	176	181	181	182	0.62[a]	2.49	2.01	2.94
20	II	4	CD	150 ± 1	150	155	153	155	0.09[a]	3.51	2.24	3.14
21	III	3	MD	196 ± 2	199	199	204	205	1.42[a]	1.74	3.90	4.77
22	III	3	CD	167 ± 4	171	173	175	177	2.66[a]	3.42	4.87	5.71
23	III	4	MD	186 ± 4	176	177	181	182	5.27	5.11	2.86	2.04[a]
24	III	4	CD	142 ± 1	150	151	153	155	5.71[a]	6.35	8.07	8.95
25	I	5	MD	447 ± 2	414	405	425	428	7.42	9.43	5.00	4.24[a]
26	I	5	CD	366 ± 3	359	353	368	371	1.79	3.53	0.51[a]	1.28
27	II	5	MD	456 ± 3	418	439	425	428	8.25	3.82[a]	6.88	6.13
28	II	5	CD	370 ± 5	363	383	368	371	1.78	3.38	0.58	0.19[a]
29	III	5	MD	395 ± 6	419	427	425	428	6.09[a]	8.03	7.58	8.36
30	III	5	CD	360 ± 5	364	372	368	371	1.11[a]	3.37	2.26	2.97
31	I	6	MD	198 ± 7	220	213	227	229	11.0	7.53[a]	14.6	15.6
32	I	6	CD	195 ± 6	187	182	192	194	4.2	6.46	1.35	0.53[a]
33	II	6	MD	246 ± 9	222	231	227	229	9.69	6.24[a]	7.73	6.92
34	II	6	CD	224 ± 8	189	197	192	194	15.7	12.0[a]	14.1	13.4
35	III	6	MD	263 ± 7	223	224	227	229	15.4	14.7	13.6	12.9[a]
36	III	6	CD	219 ± 9	191	192	193	194	12.6	12.5	12.1	11.4[a]
37	IV	1	MD	272 ± 7	322	309	316	318	18.6	13.7[a]	16.1	17.0
38	IV	1	CD	239 ± 8	265	258	258	260	10.9	7.78[a]	8.12	8.95
39	V	1	MD	312 ± 7	324	324	316	318	3.73	3.88	1.14[a]	2.02
40	V	1	CD	268 ± 6	266	270	258	260	0.79	0.76[a]	3.64	2.84
41	VI	1	MD	324 ± 5	323	325	315	318	0.46	0.29[a]	2.69	1.76
42	VI	1	CD	238 ± 5	265	271	258	260	11.3	13.7	8.42[a]	9.41
43	IV	2	MD	217 ± 4	206	195	203	204	5.02[a]	10.1	6.68	5.89
44	IV	2	CD	168 ± 5	180	172	176	177	6.92	2.21[a]	4.78	5.63
45	V	2	MD	215 ± 6	207	205	202	204	3.79[a]	4.88	5.86	5.01
46	V	2	CD	172 ± 7	180	180	176	177	4.81	4.65	2.28[a]	3.18
47	VI	2	MD	198 ± 8	206	205	202	204	4.11	3.55	2.13[a]	3.14

(*continued*)

Table 2. Experimental data for paperboard package compression tests a prediction of the maximum compression force and their errors; for cases (17) $a = 15.2$, $b = 0.490$, and $c = 0.0329$; for case (19) $a = 15.8$, $b = 0.48$, and $c = 0.241$; for case (20) $a = 13.9$, $b = 0.45$, and $c = 0.247$; for cases (1) $a = 14.4$, $b = 0.499$, and $c = 0$; for cases (21) $a = 14.4$, $b = 0.5$, and $c = 0$. (cont.)

Experiment number i=1,… 72	The box design type Figure 2	Type of paperboard, No.	Load direction	F_{exp}, (N)[b]	F_{max}, (N) based on Equation (17)	(19)/(20)	(1)	(21)	100 ε_i, % based on equation (17)	(19)/(20)	(1)	(21)
48	VI	2	CD	167 ± 9	180	180	176	177	7.58	8.05	5.25[a]	6.27
49	IV	3	MD	185 ± 4	208	197	204	205	12.2	6.70[a]	10.1	11.0
50	IV	3	CD	153 ± 8	179	172	175	177	17.0	12.4[a]	14.5	15.4
51	V	3	MD	207 ± 1	208	207	204	205	0.62	0.04[a]	1.68	0.80
52	V	3	CD	197 ± 2	180	180	175	177	8.82	8.45[a]	11.2	10.4
53	VI	3	MD	231 ± 9	208	207	203	205	10.2[a]	10.2	12.0	11.1
54	VI	3	CD	218 ± 9	179	181	175	177	17.9	17.1[a]	19.8	19.0
55	IV	4	MD	174 ± 5	184	174	181	182	5.72	0.19[a]	3.84	4.72
56	IV	4	CD	148 ± 7	157	150	153	155	5.90	1.51[a]	3.69	4.53
57	V	4	MD	192 ± 1	185	183	181	182	3.85[a]	4.82	5.95	5.10
58	V	4	CD	161 ± 2	157	157	153	155	2.30[a]	2.19	4.74	3.91
59	VI	4	MD	171 ± 1	184	183	180	182	7.59	7.13	5.51[a]	6.55
60	VI	4	CD	145 ± 2	157	158	153	155	8.10	8.88	5.69[a]	6.70
61	IV	5	MD	428 ± 7	438	432	425	428	2.22	0.94	0.71	0.01[a]
62	IV	5	CD	397 ± 5	380	379	368	371	4.27[a]	4.52	7.27	6.63
63	V	5	MD	437 ± 8	439	453	425	428	0.48[a]	3.63	2.81	2.05
64	V	5	CD	352 ± 7	381	397	368	371	8.36	12.9	4.52[a]	5.31
65	VI	5	MD	483 ± 9	438	454	424	428	9.40	6.00[a]	12.1	11.4
66	VI	5	CD	402 ± 5	380	398	368	371	5.45	0.91[a]	8.56	7.79
67	IV	6	MD	253 ± 4	232	224	227	229	8.17[a]	11.5	10.2	9.50
68	IV	6	CD	218 ± 5	197	193	193	194	9.40[a]	11.9	11.7	11.0
69	V	6	MD	245 ± 5	233	235	227	229	4.83	4.21	7.34	6.54
70	V	6	CD	221 ± 8	198	202	192	194	10.3	8.68[a]	13.0	12.2
71	VI	6	MD	219 ± 7	232	235	227	229	6.10	7.43	3.58[a]	4.55
72	VI	6	CD	211 ± 7	198	202	192	194	6.39	4.11[a]	8.90	8.07
mean [%]									6.57	5.82[a]/6.38	6.99	7.11

CD, cross machine direction; MD, machine direction.

[a] Denotes the lowest value in the row.

[b] $F_{exp} \pm \sigma$—measured values from carton compression test with standard deviation.

Calculation of Coefficients of Multiple Linear Regression

Let us present the measurement data and the coefficients of the model in a matrix form:

$$\mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}, \mathbf{X} = \begin{bmatrix} 1 & x_{1,1} & x_{1,2} \\ \vdots & \vdots & \vdots \\ 1 & x_{n,1} & x_{n,2} \end{bmatrix}, \mathbf{b} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix},$$
$$\mathbf{e} = \begin{bmatrix} e_1 \\ \vdots \\ e_n \end{bmatrix}, \tilde{\mathbf{y}} = \mathbf{Xb}, n = 72, \quad (10)$$

where $\mathbf{y}$ is the measurement vector-column for measuring the compression force, $y_i = \ln(F^i_{exp} P^i / \sqrt{D^i_x D^i_y})$ (observed values of the dependent variable); $\mathbf{X}$—dimension matrix $n \times (m+1)$, m = 2, in which the i-th row $i = 1, 2, \ldots, n$ represents the i-th observation of the vector of independent variable values x_1, x_2 values corresponding to the variables at given free term b_0; $\mathbf{b}$—vector-column of dimension m+1 parameters of multiple regression equation; $\mathbf{e}$—vector-column of dimension n of deviations $e_i = y_i - \tilde{y}_i$ where y_i depends on $\tilde{\mathbf{y}}_i$ obtained from the regression equation:

$$\tilde{y}_i = b_0 + b_1 x_{i,1} + b_2 x_{i,2}, i = 1, 2, ..., n, \tilde{\mathbf{y}} = \mathbf{Xb} \quad (11)$$

The matrix form of the relation is:

$$\mathbf{e} = \mathbf{y} - \mathbf{X}\mathbf{b} \tag{12}$$

According to the least squares method:

$$\sum_{i=1}^{n} e_i^2 = \mathbf{e}^T\mathbf{e} = (\mathbf{y} - \mathbf{X}\mathbf{b})^T(\mathbf{y} - \mathbf{X}\mathbf{b}) \to min, \tag{13}$$

where $\mathbf{e}^T = (e_1, \ldots, e_n)$, ie the superscript T means a transpose matrix. It may be shown that the previous condition is fulfilled if the vector-column of coefficient **b** can be obtained by the following formula:

$$\mathbf{b} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y} \tag{14}$$

where $\mathbf{X}^T$ is a matrix transposed to matrix **X**, and $(\mathbf{X}^T\,\mathbf{X})^{-1}$ is a matrix inverse to $(\mathbf{X}^T\,\mathbf{X})$. The relation is valid for equations of regression with a random number m of explanatory variables.

The model of multiple regression is evaluated by using the determination coefficient R^2, R is the multiple coefficient of correlation between the dependent variable and the explanatory parameters:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \tilde{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{15}$$

where the average value of the dependent variable

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \tag{16}$$

The model is based on experiments with different mechanical and geometrical dimensions of packages (72 different cartons). The model will also be valid for cartons with other parameters, which are within the parameter range studied in the present paper.

RESULTS AND DISCUSSION

To calculate the maximum compression force, Eq 7, it is necessary to know the geometrical P, H, and physical SCT_x, D_x, D_y parameters of the sidewalls of the carton. Manufacturers produce a paperboard with the bending stiffness levels shown in Table 1. As mentioned previously, the index x corresponds to the direction of the acting compression force. Direction y is perpendicular to the direction of the acting compression force. Parameters D_x, D_y correspond to S_{DIN}^x, S_{DIN}^y.

The experimental findings and the parameters of the cartons under testing are presented in Table 2. The experimentally obtained values for the maximum force of compression F_{exp} are shown in column 5 of Table 2. For example, for the first experiment (i = 1) obtained: mean carton compression strength, 329 N, maximum, 344 N, minimum, 315 N, standard deviation, 11 N, coefficient of variation, 3.29%. The experimental data contains measurement uncertainty and individual replicate variability. The samples will not be perfect or consistent (folding/gluing, etc.), the board will have local variability (±5% in thickness, approximately = ±15% in stiffness).

The analysis of the data presented in Table 2 allows us to determine the range of nondimensional parameters: $SCT_x/SCT_y \in [1.0; 2.08]$; D_x/D_y Î [0.52; 9.42]; $B/H \in [0.2; 2.5]$; $L/H \in [0.51; 8{,}6]$; $\lambda = P/H \in [1.44; 9.80]$; ln $[F_{\mathrm{exp}}\,P\,(D_xD_y)^{-0.5}] \in [6.95; 8.50]$; $\ln[SCT{\cdot}P^2(D_xD_y)^{-0.5}] \in [8.62; 11.6]$.

In our case, $n = 72$. Having the findings of the experiment, we can evaluate the coefficients of linear regression Eq 11 by using the least squares method: $b_0 = 2.722$, $b_1 = 0.490$, $b_2 = 0.0329$.

The multiple coefficient of correlation (Eq 15) between the dependent variable and the explanatory parameters is equal to $R = 0.975$.

In accord with Eq 9, the following constant values are found: $a = 15.2$, $b = 0.490$, $c = 0.0329$.

Finally, the following mathematical model was developed upon the basis of the experimental findings:

$$F_{\max} = 15.2 \cdot SCT_x^{0.49}\left(\sqrt{D_xD_y}\right)^{0.51} P^{-0.020}(P/H)^{0.0329}$$

$$R^2 = 0.950. \tag{17}$$

By entering the data from Table 2 into Eq 17, we calculated the values of the critical compression force (Table 2, column 6).

The average deviation of the calculated values $F_{\max}^i$ from the experimental data $F_{\exp}^i$, $i = 1, \ldots, 72$, as determined by the following formula:

$$MAPE = \frac{100}{n}\sum_{i=1}^{n} \varepsilon_i, \varepsilon_i = \frac{|F_{\exp}^i - F_{\max}^i|}{F_{\exp}^i} \quad (18)$$

turned out to be *MAPE* (mean absolute percentage error) = 6.57%.

A comparison of the predicted forces (Table 2, column 6) and the experimental failure forces (Table 2, column 5) is shown in Fig 3, revealing a close correlation. The line $F_{\max} = F_{\exp}$ represents the calculated maximum forces (Eq 17), and the lines indicated +20% and −20% show the region with the absolute value of the relative error $\varepsilon_i < 0.2$ and includes no <80% of the obtained values.

As can be seen from the work (Pyryev et al 2019), the critical value of the parameter λ can be $\lambda^* = 4(D_y/D_x)^{1/4}$. For six types of paperboards (Table 1), the parameter $\lambda^* \in [3.18; 5.03]$. The parameter λ for the cartons shown in Fig 2(b) is <5.03. We find semi-empirical formulas for the critical compressive strength of the carton based

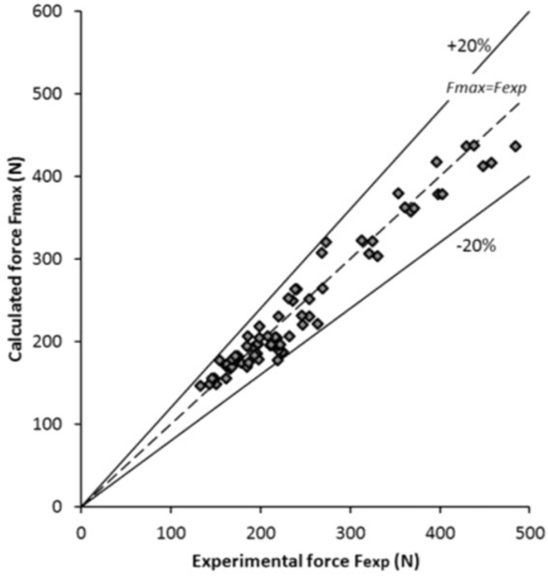

Figure 3. A prediction of the maximum compression force $F_{\max}$ (Eq 17) for the packaging compared with experimental data $F_{\exp}$.

on the results for cartons from Fig 2(a) ($\lambda \leqslant \lambda^*$) and for cartons from Fig 2(b) ($\lambda \geqslant \lambda^*$).

$$F_{\max} = 15.8 \cdot SCT_x^{0.48}\left(\sqrt{D_x D_y}\right)^{0.52} P^{-0.04}(P/H)^{0.241}$$
$$\lambda<\lambda^*, R^2 = 0.954, MAPE = 5.82\%. \quad (19)$$

$$F_{\max} = 13.9 \cdot SCT_x^{0.45}\left(\sqrt{D_x D_y}\right)^{0.55} P^{-0.1}(P/H)^{0.247}$$
$$\lambda>\lambda^*, R^2 = 0.954, MAPE = 6.38\%. \quad (20)$$

A prediction of the maximum compression force $F_{\max}$ is shown in column 7 of Table 2 according to Eqs 19 and 20 for $\lambda \leqslant \lambda^*$ (experiments 1-36) and $\lambda \geqslant \lambda^*$ (experiments 37-72), respectively. The absolute value of the relative errors $100{\cdot}\varepsilon_i$ are shown in column 11 of Table 2.

The small value of the parameter c in Eq 17 allows the empirical formula to be written in the form(1), where $a = 14.4$, $b = 0.499$, $R^2 = 0.946$, $MAPE = 6.99\%$.

A prediction of the maximum compression force $F_{\max}$ according to McKee's Eq 1 is shown in column 8 of Table 2. Errors for force $F_{\max}$ are shown in column 12 of Table 2.

Given that the empirical formula $b \approx 0.5$ can be found in the following form:

$$F_{\max} = a \cdot SCT_x^{0.5}(\sqrt{D_x D_y})^{0.5} \quad (21)$$

where the parameter a is calculated from the experimental data:

$$a = \frac{\sum_{i=1}^{72} F_{\exp}^i \left(SCT_x^i \sqrt{D_x^i D_y^i}\right)^{0.5}}{\sum_{i=1}^{72} SCT_x^i \sqrt{D_x^i D_y^i}} \quad (22)$$

According to the data (Table 2), the parameter a in Eq 22 is $a = 14.4$, $R^2 = 0.947$, $MAPE = 7.11\%$. Error's $100{\cdot}_i$ (%) for force $F_{\max}$ are shown in column 12 of Table 2.

Figure 4 shows the *MAPE* (%) of the calculated values $F_{\max}^i$ from the experimental data $F_{\exp}^i$, $i = 1, \ldots, 72$ based on Eqs 17, 19, 20, 1, and 21, as well as the maximum absolute value of the relative error $100{\cdot}\varepsilon_i$. Based on Eqs 17, 1, and 21, the maximum values are calculated by the formula $\max_{i=1,\ldots72}\{100\cdot{}_i\}$, based on Eq 19—by the formula $\max_{i=1,\ldots36}\{100\cdot{}_i\}$, and based on Eq 20—according to the formula $\max_{i=37,\ldots72}\{100\cdot{}_i\}$.

This work presented 72 experiments and the empirical formulas obtained on the basis of them, which show that the accuracy of the prediction, although slightly, increases when the height of the carton is taken into account. McKee Eq 1 according to Fig 4 gives the mean error MAPE = 6.99%, and Eqs 17, 19, and 20, which estimate the height of the carton, give lower values MAPE = 6.57%, MAPE = 5.81%, and MAPE = 6.39%, respectively.

Note that the analysis of the obtained results $F_{\max}$ based on Eqs 17, 19, 20, 21, and 1 and their errors shows that the obtained model based on McKee's Eq 1 gives the best prediction (asterisk for the lowest error) for only 10 experiments: 26, 39, 42, 46, 47, 48, 59, 60, 64, and 71. Only experiment 26 (I 6 CD) was carried out for a tall drawer (drawer design type I) and with the maximum board thickness (type 6 carton), load direction CD. The remaining nine experiments were carried out on low cartons V and VI.

Simplified model (21) gives the best estimate of the maximum compressive force compared with Eqs 17, 19, 20, and 1 for 10 experiments: 1, 7, 11, 23, 25, 28, 32, 35, 36, and 61. Experiment 61 (IV 5 MD) was carried out on a low carton and with the largest cardboard thickness, and the remaining nine experiments were carried out with high cartons I, II, and III. Only for the two experiments 35 (III 6 MD) and 36 (III 6 CD), the relative errors ($100\cdot\varepsilon_i$) using Eq 21 are more than 10%, and for the rest of the experiments the relative errors using Eq 21 are <5%.

The testing of the proposed semi-empirical prediction models for the maximum compressive force of a cardboard box was carried out on the experimental results of Ristinmaa et al.[8] Experimental results are presented for three different materials and for four different box sizes and for two different types A1111 (i = 1, . . . 16) and A6020 (i = 17, . . .20) according to the ECMA classification (Table 3, columns 1-3). The bending resistance values BR_{MD}, BR_{CD} (mN), (ISO 2493) and SCT_{MD}, and SCT_{CD} (mN/m) are presented in Table 3, column 3.

A comparison of the predicted forces (Table 3, column 5) and the experimental failure forces (Table 3, column 4) is shown in Fig 5, revealing a close correlation.

Moreover, a comparison of the predicted loads (Table 3, columns 5-7) and the experimental failure loads (Table 3, column 4) is shown in Table 3,

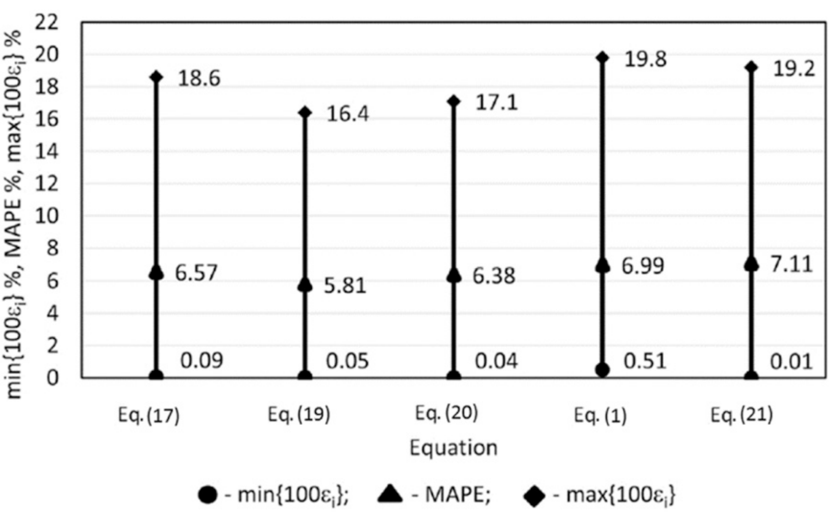

Figure 4. $\min\{100\cdot{}_i\}$ (%), *MAPE* (%), and $\max\{100\cdot{}_i\}$ (%) based on Eqs 17, 19, 20, 1, and 21.

Table 3. Experimental data of paperboard package compression tests (Ristinmaa et al 2012) a prediction of the maximum compression force and their errors.

		Dimensions, mm		Technical characteristics of paperboard (Ristinmaa et al 2012)					F_{max} (N)			100 ε_i, %		
		$L = 150$ mm		*BR*, (mN); *SCT*, (kN/m)					Based on Equation			Based on Equation		
Experiment number	Material	*H*	*B*	BR_x	BR_y	SCT_x	SCT_y	F_{exp} (N) (Ristinmaa et al 2012)	(17)	(1)	(21)	(17)	(1)	(21)
1	Mtrl-4	200	40	165	382	4.70	6.80	162	160	164	162	1.23	1.23	0.00
2	Mtrl-4	200	40	382	165	6.80	4.70	169	191	197	195	13.0	16.6	15.4
3	Mtrl-5	200	40	317	682	6.60	8.30	238	258	264	262	8.40	10.9	10.1
4	Mtrl-5	200	40	682	317	8.30	6.60	285	289	296	293	1.40	3.86	2.81
5	Mtrl-6	50	50	293	400	6.07	7.02	241	222	217	215	7.88	9.96	10.8
6	Mtrl-6	50	50	400	293	7.02	6.07	240	238	234	231	0.83	2.50	3.75
7	Mtrl-7	100	50	879	1340	7.81	8.74	409	452	439	435	10.5	7.33	6.36
8	Mtrl-7	100	50	1340	879	8.74	7.81	450	478	464	460	6.22	3.11	2.22
9	Mtrl-6	100	50	293	400	6.07	7.02	257	217	217	215	15.6	15.6	16.3
10	Mtrl-6	100	50	400	293	7.02	6.07	257	233	234	231	9.34	8.95	10.1
11	Mtrl-7	100	50	879	1340	7.81	8.74	422	442	439	435	4.74	4.03	3.08
12	Mtrl-7	100	50	1340	879	8.74	7.81	467	467	464	460	0.00	0.64	1.50
13	Mtrl-6	250	50	293	400	6.07	7.02	247	210	217	215	15.0	12.1	13.0
14	Mtrl-6	250	50	400	293	7.02	6.07	260	226	234	231	13.1	10.0	11.1
15	Mtrl-7	250	50	879	1340	7.81	8.74	441	429	439	434	2.72	0.45	1.59
16	Mtrl-7	250	50	1340	879	8.74	7.81	475	453	464	460	4.63	2.32	3.16
17	Mtrl-4	200	40	382	165	6.80	4.70	181	191	197	195	5.52	8.84	7.73
18	Mtrl-4	200	40	165	382	4.70	6.80	148	160	164	162	8.11	10.8	9.46
19	Mtrl-5	200	40	682	317	8.30	6.60	318	289	296	293	9.12	6.92	7.86
20	Mtrl-5	200	40	317	682	6.60	8.30	238	258	264	262	8.40	10.9	10.1
mean [%]												7.29	7.35	7.32

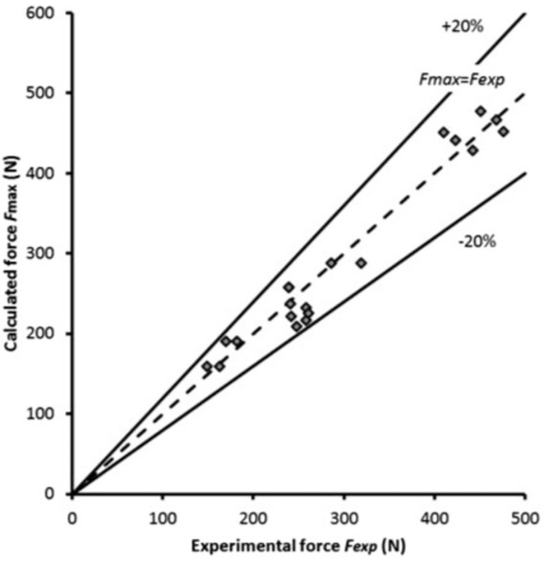

Figure 5. A prediction of the maximum compression force $F_{\max}$ (Eq 17) for the packaging compared with experimental data $F_{\exp}$ (Ristinmaa et al 2012).

which reveals a close correlation; the average error is 7.29% (based on Eq 17), 7.35% (based on Eq 1), 7.32% (based on Eq 21), respectively (Table 3, columns 8-10). In the absence of values of bending stiffness, it was taken the dependence $\sqrt{D_{MD}D_{CD}} = 0.103\sqrt{BR_{MD}BR_{CD}}$.

Note that different models give similar results. This means that common elements (or highly correlated elements) dominate their results. The dominant factor leading to this model similarity is the combination of compressive and flexural stiffnesses $SCT_x^{0.5}S^{0.5}$, where S = geometric mean of MD and CD bending stiffness $\sqrt{D_{MD}D_{CD}}$. Such a combination of stiffness is found in works of Grangård and Kubát (1969) and Pyryev et al (2019).

CONCLUSIONS

1. Formula McKee (1) has two parameters. Developed three-parameter theoretical models (17), (19), (20), and one-parameter model (21) of the maximum compression force were proposed and analyzed, describing the compression of paperboard packaging.
2. Table 3 shows, it does not any of the equations give significantly improved results over McKee (1). One would expect that with more parameters one would get a better fit, but it is not as if the fit got extremely better.
3. For the experimental data presented in the paper, we get $MAPE = 6.57\%$ (model (17)).
4. The comparison between theoretical (17) and experimental testing has shown sufficient accuracy in terms of the results.
5. For cartons with the parameter $\lambda \leqslant \lambda^*$, it is better to use Eq 19, $MAPE = 5.82\%$ for predicting the compressive strength, and for $\lambda \geqslant \lambda^*$ Eq 20, $MAPE = 6.38\%$.
6. The semi-empirical Eq 21 for predicting the compression force of the carton $MAPE = 7.11\%$ is noteworthy. The formula has a simple shape and does not depend on the geometric parameters of the carton, including the height of the carton H. As a first approximation, new Equation (24) is to be used.
7. Based on McKee's Eq 1 and experimental data, a model was obtained for which $MAPE = 6.99\%$.
8. Analyzing the obtained models, we can state that the semi-empirical model (19) for $\lambda \leqslant \lambda^*$, and (20) for $\lambda \geqslant \lambda^*$ have the best $MAPE$ value; then the semi-empirical model (17), then the semi-empirical model (1), the worst $MAPE$ value is model (21). However, the latter model allows you to perform calculations on a calculator (Fig 4).
9. The proposed models (17), (1), and (21) have been successfully tested on 20 independent experimental data (Ristinmaa et al 2012) the average error is 7.29% (based on Eq 17), 7.35% (based on Eq 1), 7.32% (based on Eq 21).
10. The considered theoretical model (17) allows us to predict the height of the carton if the expected load F_{cr} is set on it.

$$H = \left(\frac{15.2 \cdot SCT_x^{0.49} \left(\sqrt{D_x D_y} \right)^{0.51} P^{0.0129}}{F_{\max}} \right)^{30.4} \tag{23}$$

11. If the geometric parameters and the expected force $F_{\max}$ of the carton are

indicated, we can choose cardboard with a predicted compressive strength SCT_x according to (21).

$$SCT_x = \frac{F_{\max}^2}{14.39^2 \sqrt{D_x D_y}} \tag{24}$$

12. The developed simplified semi-empirical models allow to optimize the design of rectangular parallelepiped packaging with sufficient accuracy.

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