DETERMINING YOUNG'S MODULUS OF WOODEN MEMBERS WITH TENON AND MORTISE JOINT USING LONGITUDINAL VIBRATION†

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Abstract. The aim of this study was to examine the effect of tenon and mortise joints on Young's modulus of wooden members and propose a correcting method of Young's modulus. Young's moduli of the specimens with two additional concentrated masses (CMs) and those of the specimens with tenon and mortise joints were obtained using the longitudinal vibration test. The frequency equation for the longitudinal vibration of a specimen with two additional CMs was experimentally proved. The maximum deviation of 17% in Young's modulus was observed when the specimens with tenon and mortise joints were treated as rectangular bars. The mass ratio (mass of a tenon and a mortise/mass of the main body) and the volume ratio (volume of a tenon and a mortise/volume of the main body) could be used for the aforementioned frequency equation. Using this method, it is possible for one to accurately estimate Young's modulus of a wooden member with a tenon and a mortise on a construction site.

Keywords: Additional mass, frequency equation, longitudinal vibration, tenon and mortise joint.

INTRODUCTION

At the construction site in which wooden materials are used, wooden members processed in advance, such as fittings and junctions, are carried onto the site, inspected, and used in various construction activities. In October 2010, the Japanese government established the Act on the Promotion of the Utilization of Wood in Public Buildings. Consequently, it is expected that construction of nonresidential wooden buildings will be activated, and such wooden buildings will require strict on-site quality confirmation similar to other nonwooden structures in the future. In the inspection of wooden members on-site, the accurate measuring method of Young's modulus of the wooden member having a complicatedly processed shape has not been established. In addition to the wood lumber used in wooden construction, the beam for the timber guardrail also has the complicated shape: for example in the beam, a hole was drilled near both ends to attach the beam to the post, and a notch for alleviating the impact during a vehicle collision was formed at one end of the beam (Kubojima et al 2018a).

Wood lumber having a tenon or a mortise at its ends was investigated in this study as a basic case. The tenon and mortise joints have been used for thousands of years around the world, mainly when joining lumber to 90°. In this study, model specimens with the tenon and the mortise were

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used. We assumed that such wood lumber has an inhomogeneity at its ends.

The effects of the inhomogeneity within the wood on wood properties have been studied. Modal analysis, resonance frequency shifts, and frequency equation of a bar with defects have been investigated.

Modal analysis by transfer function was applied to nondestructive testing of wood. Using the shape of the flexural vibration wave and the curvature of the wave, localized defects in wood were determined (Yang et al 2001a), modulus of elasticity of sections of differing quality within a wooden beam was evaluated (Yang et al 2001), knots were detected (Yang et al 2002), and elastic modulus distribution in wood was estimated (Yang et al 2002). It is difficult to use this testing method on actual sites because hitting many points of a specimen is needed to obtain the shape of the vibration wave.

Resonance frequency shifts of a power spectrum due to defects such as holes and knots were investigated by a transfer matrix method and experimental analysis, and the positions of defects were identified (Sobue et al 2010; Roohnia et al 2011). This testing method is not suitable on actual sites because resonance frequencies of plural modes are necessary.

To investigate the influence of a defect on a flexural vibration test, a flexural vibration equation was solved using the Rayleigh method, and the effects of a knot in the wood and loss of a section were studied, resulting in a high correlation between dynamic Young's modulus and bending strength (Nakayama and Aoki 1967; Nakayama 1968; Nakayama and Oshiumi 1970; Nakayama and Oshiumi 1970a; Nakayama 1974; Nakayama and Yoshikai 1974; Nakayama 1974a; Nakayama 1974b; Nakayama 1975). Coefficient of correlations is 0.505 for *Quercus serrata* and 0.856 for *Castanopsis cuspidata* (Nakayama 1968). Because the apparent deflection in the bending vibration consists of shear and rotatory inertia as well as pure bending deflection, the longitudinal vibration is more appropriate than the flexural vibration on actual sites.

We studied Young's modulus of wood having the inhomogeneity of density and assumed that the inhomogeneity of density was a concentrated mass (CM) attached to the wood in those studies. The frequency equations for a bar with the CMs were derived for several end conditions, and a calculation method of Young's modulus of the wood with the inhomogeneity of density was proposed (Kubojima et al 2003; Kubojima et al 2005; Kubojima et al 2006; Kubojima et al 2014). The frequency equations can be used for the vibration method without weighing a specimen (Skrinar 2002; Türker and Bayraktar 2008; Kubojima and Sonoda 2015; Matsubara et al 2015; Kubojima et al 2016; Matsubara et al 2016; Sonoda et al 2016; Kubojima et al 2017a; Kubojima et al 2017b; Kubojima et al 2018; Kubojima et al 2019).

The objective of this study was to investigate the effect of a tenon and a mortise at the end of lumber on Young's modulus and to propose a correcting method of Young's modulus.

THEORY

A bar with a constant cross section S and length l is considered. The differential equation used for its longitudinal vibration is given as follows:

$$\frac{\partial^2 y}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 y}{\partial x^2}, \tag{1}$$

where x, y, t, E, and ρ are the distance along the bar, the longitudinal displacement, time, Young's modulus, and density, respectively.

Solving Eq (1), we get

$$y(x,t) = Y(x)(A\cos\omega t + B\sin\omega t), \tag{2}$$

where A and B are constants, ω (= $2\pi f$, f: resonance frequency) is the angular frequency, and

$$Y(x) = C\cos\frac{\omega}{v}x + D\sin\frac{\omega}{v}x, \tag{3}$$

where C and D are constants and v is velocity.

Young's modulus is expressed by

$$E = \rho v^2. \quad (4)$$

The CMs M_1 and M_2 are placed at (x_1, x_2) = (*al*, 0) and (x_2, x_3) = (*bl*, 0) (Fig 1).

Because the axial force does not exist at each extremity,

$$\begin{cases} x_1 = 0: & \dfrac{\partial y_1}{\partial x_1} = 0 \\ x_3 = cl: & \dfrac{\partial y_3}{\partial x_3} = 0 \end{cases}. \quad (5)$$

Because both parts of the bar are connected and the difference in the axial force in each bar is equal to the inertia force exerted by the CM at (x_1, x_2) = (*al*, 0) and (x_2, x_3) = (*bl*, 0),

$$\begin{cases} y_1 = y_2 \\ ES\dfrac{\partial y_2}{\partial x_2} - ES\dfrac{\partial y_1}{\partial x_1} = M_1\dfrac{\partial^2 y_1}{\partial t^2} = M_1\dfrac{\partial^2 y_2}{\partial t^2} \\ y_2 = y_3 \\ ES\dfrac{\partial y_3}{\partial x_3} - ES\dfrac{\partial y_2}{\partial x_2} = M_2\dfrac{\partial^2 y_2}{\partial t^2} = M_2\dfrac{\partial^2 y_3}{\partial t^2} \end{cases}. \quad (6)$$

From Eqs (2), (3), (5), and (6),

$$\begin{aligned} &\sin m_n + \mu_1 m_n \cos a m_n \cos(b+c)m_n \\ &+ \mu_2 m_n \cos(a+b)m_n \cos c m_n \\ &- \mu_1\mu_2 m_n^2 \cos a m_n \sin b m_n \cos c m_n = 0, \end{aligned} \quad (7)$$

where

$$m_n = \frac{\omega_n}{v} l, \quad (8)$$

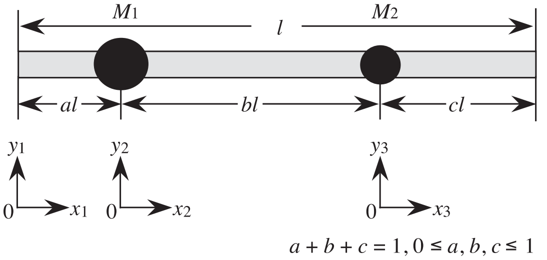

Figure 1. Beam with two additional concentrated masses.

and $\mu_1 = M_1/\rho Sl$ and $\mu_2 = M_2/\rho Sl$ are the ratios of the CMs to those of the bar. Subscript *n* is the resonance mode number. Eq (7) is the frequency equation for the free–free longitudinal vibration with two CMs.

1. If $\mu_1 = \mu_2 = 0$, then Eq (7) is as follows:

$$\sin m_n = 0. \quad (9)$$

2. If $\mu_1 = 0$ or $\mu_2 = 0$, Eq (7) is as follows:

$$\sin m_n + \mu_2 m_n \cos(a+b)m_n \cos c m_n = 0 \quad (10a)$$

$$\sin m_n + \mu_1 m_n \cos a m_n \cos(b+c)m_n = 0. \quad (10b)$$

These are the frequency equations for a bar that has free ends with a CM (Kubojima et al 2014).

3. If $a = c = 0$, Eq (7) is as follows:

$$\begin{aligned} &\sin m_n + (\mu_1 + \mu_2)m_n \cos m_n \\ &- \mu_1\mu_2 m_n^2 \sin m_n = 0. \end{aligned} \quad (11)$$

From Eqs (4) and (8),

$$E = \rho\left(\frac{2\pi f_n l}{m_n}\right)^2. \quad (12)$$

MATERIALS AND METHODS

Specimens

Sitka spruce (*Picea sitchensis* Carr.) was used as the specimen. Two kinds of specimens were made as follows: Group 1 was 1,000 mm longitudinal (L), 30 mm radial (R), and 10 mm tangential (T), and Group 2 was 600 mm (L), 30 mm (R), and 30 mm (T). The Group 1 specimens were used to experimentally confirm the frequency equation for the free–free longitudinal vibration with two CMs. The ends of Group 2 were processed into the models of the tenon and the mortise as illustrated in Fig 2. Hereafter, the specimen having the tenon and the mortise at the end is called the end-processed specimen in this study. There were four specimens in Group 1 (No. 1 - No. 4) and four in Group 2 for each type of end (No. 5 - No. 24). Considering the variation in the results of the vibration test, the number of specimens was four.

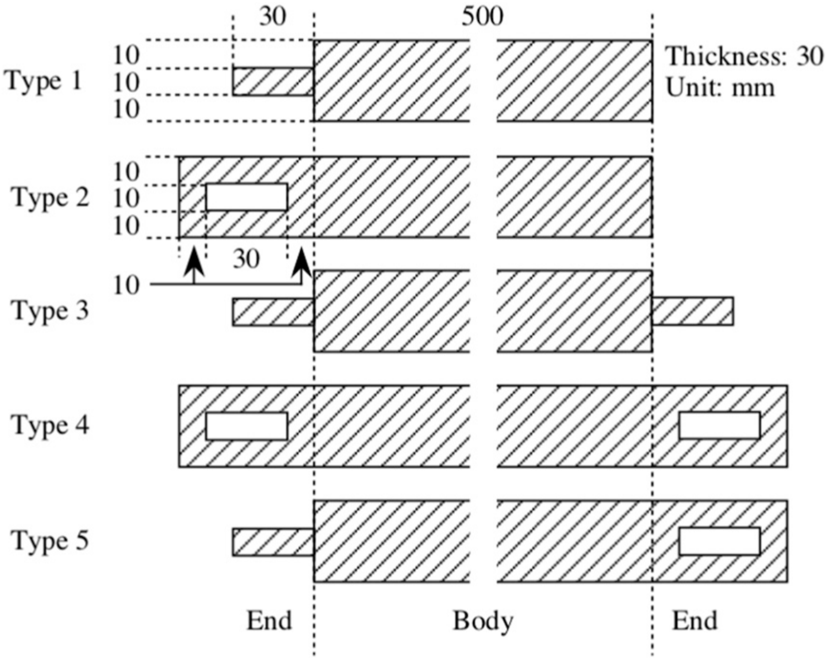

Figure 2. End-processed specimens.

The specimens were conditioned at 20°C in 65% RH until the weights were constant. All tests were conducted under the same conditions.

Longitudinal Vibration Test

To obtain Young's modulus, free–free longitudinal vibration tests were conducted by placing a specimen on a small sponge at the position of $x = l/2$. The vibration was initiated at one end in a longitudinal direction using a hammer. The motion of the first mode of longitudinal vibration of the specimen was detected by a microphone at the other end. The signal was processed through a fast-Fourier transform digital signal analyzer to yield high-resolution resonance frequencies. A diagram of the experimental setup is presented in Fig 3.

The vibration tests for Group 1 were conducted for the specimens with and without iron pieces 1 and 2. The dimensions of iron pieces 1 and 2 were 7 mm × 4 mm × 25 mm and 7 mm × 2 mm × 25 mm, respectively. The masses of iron pieces 1 and 2 were 5.03 g ($\mu_1 = 0.0344$) and 2.51 g ($\mu_2 = 0.0172$), respectively. The iron pieces 1 and 2 were bonded at $x = 0$ and l on the RT-plane and at $x = 0.1\,l$, $0.2\,l$, $0.3\,l$, $0.4\,l$, $0.5\,l$, $0.6\,l$, $0.7\,l$, $0.8\,l$, and $0.9\,l$ on the LR-plane with the two-sided adhesive tape. The vibration tests for Group 2 specimens were conducted on the specimen before and after their ends were cut.

RESULTS AND DISCUSSION

The average (standard deviation) density and Young's modulus were 477 (4) kg/m^3 and 14.10 (0.29) GPa for Group 1 (four specimens) and 429 (25) kg/m^3 and 11.93 (1.08) GPa for Group 2 (20 specimens), respectively. Values for Group 2 were obtained after cutting (length = 500 mm). Mathematica 10.4J software (Wolfram Research Co., Ltd.) was used with Eq (7) to calculate Young's modulus. Five times vibration tests were performed for each specimen, and the measured resonance frequency was repeatable.

Young's Modulus of the Specimen with Two Concentrated Masses

The results for the specimens with two CMs (Group 1) are presented in Tables 1 and 2.

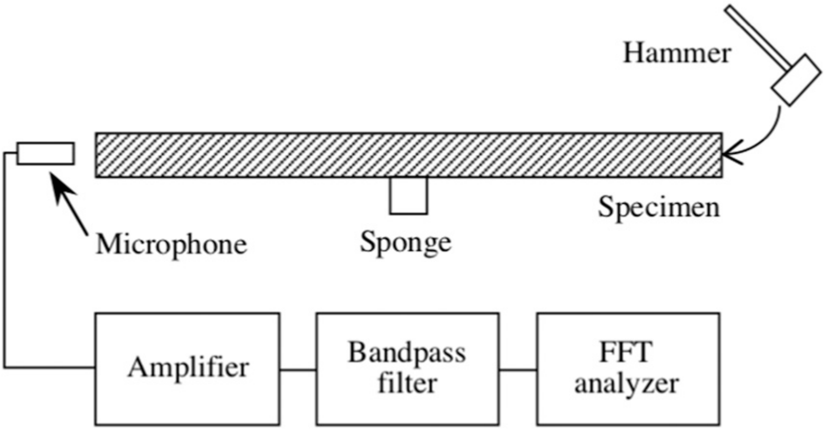

Figure 3. Schematic diagram of the experimental setup for the longitudinal vibration tests.

Young's modulus with two CMs (E_{CM}) was compared with that without the masses (E_0). An example of the positions of mass 1 and mass 2 is shown in Fig 4.

Results calculated for E_{CM} using $m_1 = \pi$ from Eq (9) for Eq (12) are presented in Table 1. The ratio E_{CM}/E_0 was nearly equal to 1 when a CM or two CMs were near the nodal position of the first mode of the longitudinal vibration ($x = 0.5l$), whereas the ratio decreased when they were near the antinodal position ($x = 0, l$). These tendencies were similar to the longitudinal and bending vibrations in the previous studies (Kubojima et al 2003; Kubojima et al 2005; Kubojima et al 2006; Kubojima et al 2014) using specimens with a CM.

The results of E_{CM} using m_1 from Eq (7) for Eq (12) are presented in Table 2. The average (standard deviation) of the results shown in Table 1 and those in Table 2 were 0.94 (0.026) and 0.99 (0.0028), respectively. From the t-test, there was a difference at 1% significant level. The ratio E_{CM}/E_0 approached 1 using m_1 from Eq (7) instead of $m_1 = \pi$ from Eq (9) for all cases. Hence, m_1 from Eq (7) should be used when two CMs are bonded on a specimen.

Young's Modulus of the End-Processed Specimen

The specimen length, density, and m_1 in Eq (12) are especially important in calculating Young's

Table 1. Ratio of Young's modulus of specimens with two CMs to that of specimens without CMs. m_1 for Eq (12) is π.

		Position of M_2									
		0.1 l	0.2 l	0.3 l	0.4 l	0.5 l	0.6 l	0.7 l	0.8 l	0.9 l	l
Position of M_1	0	0.90	0.91	0.92	0.93	0.93	0.93	0.92	0.91	0.90	0.90
	0.1 l	—	0.91	0.92	0.93	0.94	0.93	0.92	0.91	0.90	0.90
	0.2 l	—	—	0.94	0.95	0.95	0.95	0.94	0.93	0.92	0.92
	0.3 l	—	—	—	0.97	0.98	0.97	0.96	0.95	0.94	0.94
	0.4 l	—	—	—	—	0.99	0.99	0.98	0.97	0.96	0.96
	0.5 l	—	—	—	—	—	0.99	0.99	0.98	0.97	0.97
	0.6 l	—	—	—	—	—	—	0.98	0.97	0.96	0.96
	0.7 l	—	—	—	—	—	—	—	0.95	0.95	0.94
	0.8 l	—	—	—	—	—	—	—	—	0.92	0.92
	0.9 l	—	—	—	—	—	—	—	—	—	0.90

CMs, concentrated masses.
M_1 and M_2 are CMs attached to the specimens. Average of four specimens (No. 1 - No. 4).

Table 2. Ratio of Young's modulus of specimens with two CMs to that of specimens without the CMs. m_1 for Eq (12) is obtained using Eq (7).

		Position of M_2									
		0.1 *l*	0.2 *l*	0.3*l*	0.4 *l*	0.5 *l*	0.6 *l*	0.7 *l*	0.8 *l*	0.9 *l*	*l*
Position of M_1	0	0.99	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	1.00
	0.1 *l*	—	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
	0.2 *l*	—	—	0.99	0.99	1.00	0.99	0.99	0.99	1.00	0.99
	0.3 *l*	—	—	—	1.00	1.00	1.00	1.00	1.00	0.99	1.00
	0.4 *l*	—	—	—	—	1.00	1.00	1.00	1.00	1.00	1.00
	0.5 *l*	—	—	—	—	—	1.00	1.00	1.00	1.00	1.00
	0.6 *l*	—	—	—	—	—	—	1.00	1.00	1.00	1.00
	0.7 *l*	—	—	—	—	—	—	—	0.99	1.00	1.00
	0.8 *l*	—	—	—	—	—	—	—	—	0.99	0.99
	0.9 *l*	—	—	—	—	—	—	—	—	—	0.99

CMs, concentrated masses.
M_1 and M_2 are CMs bonded on the specimens. Average of four specimens (No. 1 - No. 4).

modulus of the end-processed specimen in this study. For calculation of Young's modulus of the specimen after cutting the ends, the specimen length is 500 mm of the main body, density is the measured value, and m_1 is π from Eq (9). Such Young's modulus is "true" Young's modulus and is described as E_0 as Group 1.

Young's modulus calculated regarding end-processed specimens as rectangular bars. In the inspection of members at an actual construction site, it is expected that the end-processed wood lumber will be treated as a rectangular bar. Thus, apparent Young's modulus when the end-processed specimen is regarded as a rectangular bar is examined.

The following parameters will be used for the calculation of Young's modulus. The lumber length will be a sum of the main body and the processed ends (Type 1: 530 mm, Type 2: 550 mm, Type 3: 560 mm, Type 4: 600 mm, and Type 5: 580 mm. Refer to Fig 2). The value of m_1 in Eq (12) will be π from Eq (9).

There are two possible specimen volumes required for calculating the density. One is the volume of the rectangular bar whose length is the sum of the aforementioned main body and the processed ends. Because this volume is larger than the true volume, the density calculated using this volume is smaller than the true density. The other can be calculated in advance in the material design stage. Because this volume is almost the same as the volume of the actual wood member, the density calculated from this volume and the measured mass can be said to be the "true density." In this study, the density of the rectangular bar after cutting with the length of 500 mm was used as the true density. Young's modulus using the smaller density and that using the true density are described as E_a and E_{a1} (a: apparent).

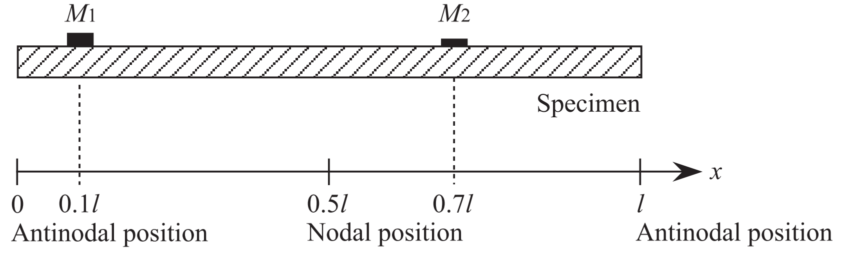

Figure 4. Example of the positions of the concentrated masses M_1 ($x = 0.1\ l$) and M_2 ($x = 0.7\ l$).

Young's modulus of the end-processed specimen is presented in Table 3. Considering the ratio E_a/E_0 and E_{a1}/E_0, the maximum deviation of 17% was observed and the deviation was largest in Type 3.

Estimation of Young's modulus of the end-processed specimen using the theory of the specimen with two additional masses. The deviation mentioned earlier was corrected by the following method. The processed ends correspond to the CM in the "THEORY" section. It is considered that the processed end is attached to the main body. Hence, "after cutting" and "before cutting" correspond to "without the CM" and "with the CM" in the "THEORY" section, respectively. The cut end completely adheres to the main body before cutting according to our previous study (Kubojima et al 2019). The values of μ_1 and μ_2 were obtained using the mass of the cut end and that of the specimen after cutting. The values of μ_1 and μ_2 were substituted into Eq (7) and m_1 was obtained. Substituting the true density, the obtained m_1, the specimen length of the main body (500 mm), and the resonance frequency before cutting into Eq (12), Young's modulus was calculated. Because Young's modulus obtained by this method corresponds to that with the CM, it is described as E_{CM} as Group 1.

The ratios of E_{CM}/E_0 were almost 1 as shown in Table 4 and Fig 5. Hence, this method was effective for calculating Young's modulus of the end-processed specimens.

Because end parts of wooden members cannot be cut on the construction sites, μ_1 and μ_2 from the volume ratio to be substituted for Eq (7) were discussed. Young's modulus using μ_1 and μ_2 from the volume ratio is described as E_{CMV}. The ratio E_{CMV}/E_0 was close to 1 for all specimen

Table 3. Results of the end-processed specimens.

		After cutting		Before cutting					
Type	Specimen	Density (kg/m^3)	E_0 (GPa)	Length (mm)	Density* (kg/m^3)	E_a (GPa)	E_{a1} (GPa)	E_a/E_0	E_{a1}/E_0
1	No. 5	440	12.90	530.0	424	13.47	14.00	1.04	1.08
	No. 6	464	13.46	530.5	447	13.95	14.48	1.04	1.08
	No. 7	446	11.17	530.0	429	11.58	12.03	1.04	1.08
	No. 8	443	12.72	530.0	427	13.20	13.70	1.04	1.08
	Average	448	12.56	530.1	432	13.05	13.55	1.04	1.08
2	No. 9	401	12.03	550.0	393	12.23	12.48	1.02	1.04
	No. 10	404	12.21	550.0	397	12.41	12.61	1.02	1.03
	No. 11	473	12.89	550.0	466	13.02	13.21	1.01	1.02
	No. 12	404	9.59	550.0	397	9.58	9.75	1.00	1.02
	Average	421	11.68	550.0	413	11.81	12.01	1.01	1.03
3	No. 13	407	12.39	560.5	378	13.37	14.38	1.08	1.16
	No. 14	411	12.80	560.0	382	13.86	14.92	1.08	1.17
	No. 15	457	10.29	561.0	424	11.06	11.92	1.08	1.16
	No. 16	407	10.48	560.0	379	11.27	12.12	1.08	1.16
	Average	421	11.49	560.4	391	12.39	13.34	1.08	1.16
4	No. 17	417	12.77	600.0	402	13.19	13.67	1.03	1.07
	No. 18	403	11.91	600.0	391	12.25	12.63	1.03	1.06
	No. 19	453	12.58	600.0	438	12.98	13.43	1.03	1.07
	No. 20	464	12.18	600.0	449	12.48	12.89	1.02	1.06
	Average	434	12.36	600.0	420	12.73	13.16	1.03	1.06
5	No. 21	436	12.85	580.5	413	13.60	14.36	1.06	1.12
	No. 22	407	12.46	580.0	386	13.03	13.72	1.05	1.10
	No. 23	444	10.67	580.5	421	11.26	11.87	1.06	1.11
	No. 24	401	10.26	580.5	380	10.74	11.33	1.05	1.10
	Average	422	11.56	580.4	400	12.16	12.82	1.05	1.11

* Density was calculated using the volume of a rectangular bar. E_0 is Young's modulus after cutting. E_a is Young's modulus when the end-processed specimen is treated as the rectangular bar using the smaller density. E_{a1} is Young's modulus when the end-processed specimen is treated as the rectangular bar using the true density.

Table 4. Corrected Young's modulus of the end-processed specimen.

			μ_1, μ_2: Mass ratio					μ_1, μ_2: Volume ratio				
Type	Specimen	E_0 (GPa)	μ_1	μ_2	m_1	E_{CM} (GPa)	E_{CM}/E_0	μ_1	μ_2	m_1	E_{CMV} (GPa)	E_{CMV}/E_0
1	No. 5	12.90	0.020	—	3.08014	12.96	1.00	0.02	—	3.08007	12.96	1.00
	No. 6	13.46	0.022	—	3.07427	13.43	1.00	0.02	—	3.08007	13.38	0.99
	No. 7	11.17	0.021	—	3.07757	11.15	1.00	0.02	—	3.08007	11.14	1.00
	No. 8	12.72	0.021	—	3.07605	12.72	1.00	0.02	—	3.08007	12.69	1.00
	Average	12.56	0.021	—	3.07701	12.57	1.00	0.02	—	3.08007	12.54	1.00
2	No. 9	12.03	0.078	—	2.91703	11.96	0.99	0.08	—	2.91266	12.00	1.00
	No. 10	12.21	0.083	—	2.90615	12.18	1.00	0.08	—	2.91266	12.12	0.99
	No. 11	12.89	0.084	—	2.90136	12.80	0.99	0.08	—	2.91266	12.70	0.99
	No. 12	9.59	0.081	—	2.90991	9.39	0.98	0.08	—	2.91266	9.37	0.98
	Average	11.68	0.082	—	2.90861	11.58	0.99	0.08	—	2.91266	11.55	0.99
3	No. 13	12.39	0.021	0.021	3.01581	12.42	1.00	0.02	0.02	3.02090	12.38	1.00
	No. 14	12.80	0.021	0.020	3.01917	12.88	1.01	0.02	0.02	3.02090	12.87	1.00
	No. 15	10.29	0.020	0.021	3.01849	10.26	1.00	0.02	0.02	3.02090	10.24	1.00
	No. 16	10.48	0.021	0.021	3.01646	10.48	1.00	0.02	0.02	3.02090	10.45	1.00
	Average	11.49	0.020	0.021	3.01748	11.51	1.00	0.02	0.02	3.02090	11.48	1.00
4	No. 17	12.77	0.077	0.081	2.71970	12.67	0.99	0.08	0.08	2.71399	12.72	1.00
	No. 18	11.91	0.082	0.082	2.70589	11.83	0.99	0.08	0.08	2.71399	11.76	0.99
	No. 19	12.58	0.080	0.080	2.71379	12.50	0.99	0.08	0.08	2.71399	12.50	0.99
	No. 20	12.18	0.078	0.084	2.70955	12.04	0.99	0.08	0.08	2.71399	12.00	0.98
	Average	12.36	0.079	0.082	2.71223	12.26	0.99	0.08	0.08	2.71399	12.24	0.99
5	No. 21	12.85	0.021	0.078	2.86180	12.84	1.00	0.02	0.08	2.85957	12.86	1.00
	No. 22	12.46	0.021	0.081	2.85403	12.35	0.99	0.02	0.08	2.85957	12.30	0.99
	No. 23	10.67	0.021	0.080	2.85760	10.65	1.00	0.02	0.08	2.85957	10.63	1.00
	No. 24	10.26	0.020	0.081	2.85761	10.16	0.99	0.02	0.08	2.85957	10.14	0.99
	Average	11.56	0.021	0.080	2.85776	11.50	1.00	0.02	0.08	2.85957	11.49	0.99

E_0 is the same as Table 3. E_{CM} and E_{CMV} are corrected Young's modulus.

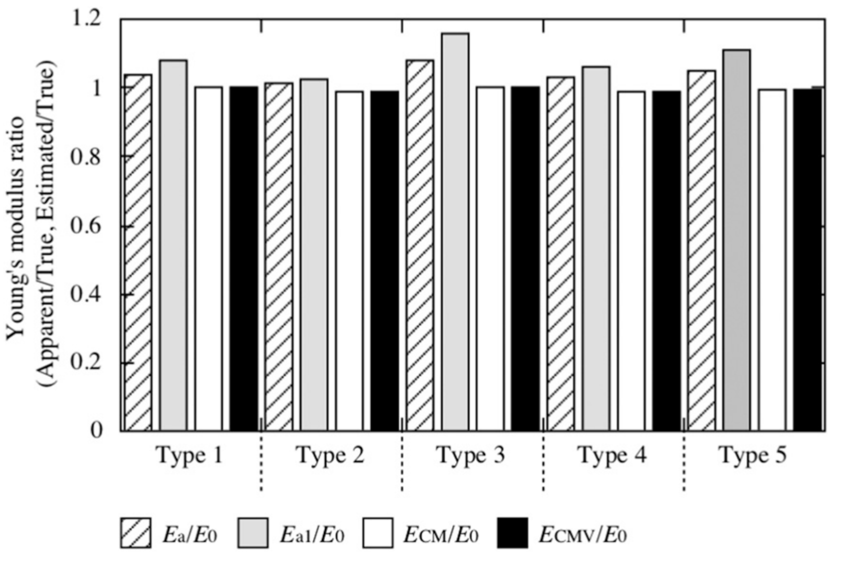

Figure 5. Accuracy of Young's modulus of the end-processed specimens. E_a and E_{a1}: Refer to Table 3. E_{CM} and E_{CMV}: Refer to Table 4.

types as shown in Table 4 and Fig 5. Hence, it is possible to estimate m_1 from the volume ratio (volume of processed parts/volume of main body) at the design stage of each kind of wooden member. Young's modulus of the end-processed wood member will be calculated using the estimated m_1 and the measured resonance frequency on the construction sites.

CONCLUSIONS

Young's moduli of the specimens with two additional CMs and those of end-processed specimen were obtained using the longitudinal vibration test as follows:

1. Young's modulus of the specimen with two additional CMs could be accurately obtained by using m_1 from Eq (7) instead of $m_1 = \pi$.
2. The maximum deviation of 17% in Young's modulus was observed when the end-processed specimens were treated as rectangular bars.
3. Young's modulus of the end-processed specimen could be accurately obtained using m_1 from Eq (7) instead of $m_1 = \pi$. The mass ratio (mass of processed part/mass of main body) could be used for μ_1 and μ_2 to be substituted for Eq (7).
4. Because the volume ratio (volume of processed ends/volume of main body) could be used as μ_1 and μ_2 for Eq (7), it is possible to simply estimate Young's modulus of a wooden member with processed ends on a construction site.

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