DISTRIBUTIONS OF MOE AND MOR IN EIGHT MILL-RUN LUMBER POPULATIONS (FOUR MILLS AT TWO TIMES)

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Abstract. To evaluate the reliability of lumber structures, good models for the strength and stiffness distributions of visual and machine stress–rated (MSR) grades of lumber are necessary. Verrill and coworkers established theoretically and empirically that the strength properties of visual and MSR grades of lumber are not distributed as 2-parameter Weibulls. Instead, strength properties of grades of lumber must have "pseudo-truncated" distributions. To properly implement the pseudo-truncation theory (to correctly estimate the MOR and MOE distributions of graded subpopulations), one must know the MOE and MOR distributions of full ("mill-run") lumber populations. Owens and coworkers investigated the mill-run distributions of MOE and MOR at each of four mills. They found that univariate mill-run MOE and MOR distributions are well-modeled by skew normal distributions or mixtures of normal distributions but not so well modeled by normal, lognormal, 2-parameter Weibull, or 3-parameter Weibull distributions. They noted that it was important to investigate whether these results were stable over time. In this article, to investigate stability over time, the authors extend the analyses of "summer" data sets performed by Owens et al to new mill-run "winter" data sets. The results show that normal, lognormal, 2-parameter Weibull, and 3-parameter Weibull distributions continue to perform relatively poorly, and that skew normal distributions and mixtures of normal distributions continue to perform relatively well.

Keywords: Full lumber population, mill-run, MOE, MOR, normal distribution, Weibull, pseudo-truncated, MSR lumber, MOE binned lumber, visually graded lumber, lumber property distribution, lumber reliability, skew normal, mixture of bivariate normals.

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INTRODUCTION

To evaluate the reliability of lumber structures, good models for the strength and stiffness distributions of visual and machine stress–rated (MSR) grades of lumber are necessary. Verrill et al (2012, 2013, 2014, 2015, 2020) established theoretically and empirically that the strength properties of visual and MSR grades of lumber are not distributed as 2-parameter Weibulls. Instead, strength properties of grades of lumber must have (at least to a first approximation) "pseudo-truncated" distributions.

"Pseudo-truncation" has a technical meaning. The concept, at least, of pseudo-truncation was recognized in an American Society of Civil Engineers (ASCE 1988) prestandard report. Section B3 of that report notes that "an improved strength distribution can be obtained by … thinning the lower tail by sorting on a correlated variable" (p. 152). For example, if the full ("mill-run") bivariate MOE-MOR distribution were a bivariate Gaussian (normal)–Weibull, then truncating or "binning" on the basis of MOE values (as in MSR lumber) would lead to a pseudo-truncated MOR distribution. That is, because MOE and MOR are not perfectly correlated, truncating based on lower and upper MOE limits does not lead to perfect truncation of the MOR distribution, but it does lead to an MOR distribution whose tails are thinned. For the case in which the mill-run joint MOE-MOR distribution is a bivariate Gaussian–Weibull, Verrill et al (2012, 2015) derived the exact form of this "pseudo-truncated" Weibull distribution (they obtained its probability density function.) They also showed that it *cannot* have tail behavior that matches that of a Weibull distribution.

To properly implement the pseudo-truncation theory of Verrill et al (2012, 2015), one must know the true mill-run MOE and MOR distributions. Verrill et al (2017) and Owens et al (2018, 2019) investigated the mill-run distributions of MOE and MOR at each of four mills in northern Mississippi. They found that univariate mill-run MOE and MOR distributions are well modeled by skew normal distributions or mixtures of normal distributions but not so well modeled (in general) by normal, lognormal, 2-parameter Weibull, or 3-parameter Weibull distributions. Owens et al (2019) noted that it was important to investigate whether these results are stable over time. In this article, the authors extend the Owens et al (2019) analyses of mill-run "summer" data sets to new mill-run "winter" data sets to determine if distributional forms vary over time.

MATERIALS AND METHODS

Sampling

Mill-run samples of eight-foot 2 × 4 southern yellow pine (*Pinus* spp.) lumber were supplied by four dimension sawmills in northern Mississippi. Mills 1-4 were characterized as a full-complement mill (processing logs of all sizes), a small log mill, a full-complement mill, and a large log mill, respectively (Table 1).

Each mill provided one "summer" sample and one "winter" sample for a total of eight samples or 1600 specimens. The purpose of sampling material produced in different seasons was not to draw general conclusions about the distributions of summer and winter lumber but rather to merely determine if differences in distributional forms could be observed at the same mill over time. Although some variation in mechanical properties undoubtedly occurs in mill-run lumber from week to week and even day to day because of variations in raw materials, it might be reasonable to assume that large variations are more likely to occur over a period of months than over a period of days. If the span of months is approximately 6, one might also expect influence from seasonal variables such as log availability and forest tract access. For these reasons, two samples of sawn material were obtained from each mill—one in the summer (June through July production) and one in the winter (December through January production). The sampling of material produced roughly half a year apart at the same mills seemed an expedient way to maximize observed differences.

Each sample consisted of 200 pieces of rough sawn, kiln-dried lumber. A kiln package was randomly chosen from weekly production, the

Table 1. Production profiles of each sawmill sampled.

	Typical log size	Primary lumber dimensions
Mill 1 (pilot mill)	Full range (small to large)	2 × 4 through 2 × 12
Mill 2	Small diameter	2 × 4
Mill 3	Full range (small to large)	2 × 4 through 2 × 12
Mill 4	Large diameter	Wide dimensions (few 2 × 4)

top course of lumber was removed, and the next 200 pieces were sampled sequentially. The net dimensions of each specimen after drying and planing were 1.5 × 3.5 × 96 inches (3.81 × 8.89 × 243.84 cm). Each mill-run sampling was made before grading and included all qualities that developed—even the lower quality pieces that would not make grade and might otherwise be discarded.

Testing

For each specimen, MOR and three measures of MOE were assessed. Dynamic MOE was measured by two nondestructive tests. Metriguard's E-computer (Model 340, hereafter "E-computer," Metriguard, Inc., Pullman, WA www.metriguard.com) estimated the MOE of each specimen by transverse vibration. The test pieces were supported at each end. After a slight tap was applied to the midspan, the frequency of oscillation was measured by a transducer at one end. The computer calculated the MOE as per the following formula (Ross 2015).

$$E = \frac{f^2 W S^3}{C I g},$$

where E = modulus of elasticity, S = span, W = weight of specimen, f = resonant frequency, I = moment of inertia, g = acceleration due to gravity, and C = constant.

Fiber-gen's Director HM200 (hereafter "Director," Fiber-gen Limited, Christchurch, New Zealand, www.fibre-gen.com) estimated MOE by measuring acoustic velocity. Each specimen was laid across two sawhorses. The device's sensor was held against one end of the test piece while an acoustic wave was initiated with a hammer. The sensor measured the acoustic velocity and calculated the MOE based on the following formula (Ross 2015):

Figure 1. Static bending test setup as per ASTM D198-15.

$$E = \rho V^2,$$

where E = modulus of elasticity, ρ = density of the specimen, and V = acoustic velocity.

MOR and static MOE were measured by a destructive third-point static bending test per ASTM D198-15 (ASTM 2015) (Figs 1 and 2). Before testing, the MC of each specimen was measured by a Wagner L 601-3 handheld moisture meter (Wagner Electronic Products Inc., Rogue River, OR, www.wagnermeters.com). The mean MC of the specimens was 13.3% (SD = 1.70). The span-to-depth ratio was 17:1. Specimens were oriented edgewise. Load head placement along the length of the 59.5-inch (151.13 cm) test span was determined randomly. An extensometer under the bottom edge of the midspan measured the deflection. Force was applied until rupture. The testing time was approximately 5 min. All MOE and MOR values were adjusted to a common MC of 15% per ASTM 1990-16 (ASTM 2016) before analysis.

There was one broken specimen in the Mill 2 summer sample and one in the Mill 4 winter

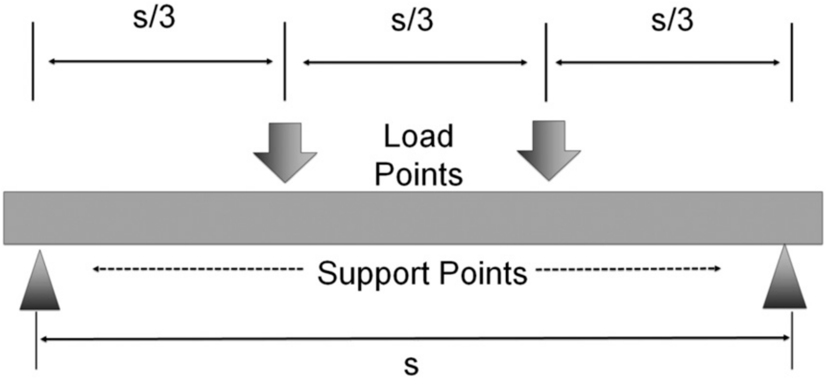

Figure 2. Third-point loading with a span-to-depth ratio of 17:1.

sample. These pieces could not be tested, so their records were removed listwise from the dataset reducing their N to 199 each. Also, there were two specimens from the Mill 1 winter sample, two specimens from the Mill 2 winter sample, and one specimen from the Mill 3 winter sample for which the Director device was unable to generate a reading even after multiple attempts. These missing data points were deleted on a pairwise basis.

Statistical Methods

Distributions were fit to each of the four data sets and evaluated for goodness-of-fit. Candidate distributions were selected based on previous research. Normal, lognormal, two-parameter Weibull, and three-parameter Weibull appear widely in the literature (eg Galligan et al 1986; Green and Evans 1987; Evans et al 1997; ASTM 2017a, 2017b). Skew normal and mixed normal distributions showed good fit in previous studies by the current authors (Verrill et al 2017; Owens et al 2018; Owens et al 2019). The probability density functions of the distributions are provided in the Appendix.

The normal and lognormal fits reported in this article were performed in the R programming environment (R Core Team 2018). The maximum likelihood fits for the other four distributions were performed via Fortran programs written by the authors (see http://www1.fpl.fs.fed.us/4mills.html for listings of these programs).

The R nortest package (Gross and Ligges 2015) was used to perform Cramér–von Mises and Anderson–Darling goodness-of-fit tests for normal and lognormal distributions. The R EWGoF package (Krit 2017) was used to perform Cramér–von Mises and Anderson–Darling goodness-of-fit tests for 2-parameter Weibull distributions. Because there were no readily available packages to perform goodness-of-fit tests for three-parameter Weibull, skew normal, and mixed normal distributions, the authors wrote Fortran programs that performed nonparametric bootstraps (a type of simulation) that yielded Cramér–von Mises p-values for these three distributions (see http://www1.fpl.fs.fed.us/4mills.html for listings of these programs). The specialized Shapiro–Wilk test of normality (Shapiro and Wilk 1965) often yields greater statistical power than the more general Cramér–von Mises and Anderson–Darling goodness-of-fit tests. Thus, the authors included Shapiro–Wilk (shapiro.test in R) results for normal and lognormal tests in Tables 2-5.

RESULTS

The results of the goodness-of-fit tests for mills 1-4 by season appear in Tables 2-5, respectively. Tables 2-5 are further summarized in Table 6. For each of the four properties (static MOE, E-computer E, Director E, and MOR), Table 6 presents the number of mills, by season, for which a distribution was rejected by a goodness-of-fit test at a 0.05 significance level. This number

Table 2. G. O. F. *p*-values for Mill 1 (full-complement mill).

		Distribution											
		Normal		Lognormal		2-par Weibull		3-par Weibull		Skew normal		Mixed normal	
Property	G. O. F. test	Sum	Win	Sum	Win	Sum	Win	Sum	Win	Sum	Win	Sum	Win
Static MOE	Shapiro–Wilk	**0.371**	**0.298**	<0.001	0.006	—	—	—	—	—	—	—	—
	Cramér–von Mises	**0.054**	**0.174**	0.040	0.044	0.002	0.027	—	—	—	—	—	—
	Anderson–Darling	**0.095**	**0.202**	0.015	0.022	0.006	0.016	—	—	—	—	—	—
	CVM simulation[a]	—	—	—	—	—	—	0.017	**0.229**	**0.129**	**0.207**	**0.104**	**0.069**
E-computer E	Shapiro–Wilk	**0.318**	**0.287**	<0.001	**0.081**	—	—	—	—	—	—	—	—
	Cramér–von Mises	**0.252**	**0.225**	0.018	**0.109**	0.006	0.049	—	—	—	—	—	—
	Anderson–Darling	**0.202**	**0.255**	0.011	**0.114**	0.003	0.018	—	—	—	—	—	—
	CVM simulation[a]	—	—	—	—	—	—	0.040	**0.523**	**0.481**	**0.143**	**0.498**	**0.879**
Director E	Shapiro–Wilk	**0.185**	0.041	<0.001	0.003	—	—	—	—	—	—	—	—
	Cramér–von Mises	0.011	0.037	0.007	**0.145**	<0.001	0.005	—	—	—	—	—	—
	Anderson–Darling	0.024	0.032	0.004	**0.130**	<0.001	<0.001	—	—	—	—	—	—
	CVM simulation[a]	—	—	—	—	—	—	0.001	0.029	0.020	**0.067**	**0.506**	**0.859**
MOR	Shapiro–Wilk	0.001	**0.258**	<0.001	<0.001	—	—	—	—	—	—	—	—
	Cramér–von Mises	<0.001	**0.512**	<0.001	<0.001	<0.001	**0.195**	—	—	—	—	—	—
	Anderson–Darling	<0.001	**0.375**	<0.001	<0.001	<0.001	**0.107**	—	—	—	—	—	—
	CVM simulation[a]	—	—	—	—	—	—	0.001	**0.163**	**0.277**	**0.343**	**0.584**	**0.650**

Sum, summer specimens; Win, winter specimens; G. O. F., goodness-of-fit; par, parameter; E-computer E, dynamic MOE as tested with E-computer device; Director E, dynamic MOE as tested with Director HM200 device; CVM, Cramér–von Mises; —, the test was not performed.

$N = 198$ for Director E (Winter). For all other data sets, $N = 200$. Bold values indicate that a test failed to reject a distribution at a 0.05 significance level.

[a] In cases where critical values for the Cramér–von Mises test were not available in D'Agostino and Stevens (1986), they were determined by simulation.

Table 3. G. O. F. p-values for Mill 2 (small log mill).

		Distribution											
		Normal		Lognormal		2-par Weibull		3-par Weibull		Skew normal		Mixed normal	
Property	G. O. F. test	Sum	Win	Sum	Win	Sum	Win	Sum	Win	Sum	Win	Sum	Win
Static MOE	Shapiro–Wilk	<0.001	<0.001	**0.350**	**0.666**	—	—	—	—	—	—	—	—
	Cramér–von Mises	<0.001	<0.001	**0.328**	**0.637**	<0.001	<0.001	—	—	—	—	—	—
	Anderson–Darling	<0.001	<0.001	**0.357**	**0.548**	<0.001	<0.001	—	—	—	—	—	—
	CVM simulation[a]	—	—	—	—	—	—	0.018	0.001	**0.167**	**0.478**	**0.920**	**0.053**
E-computer E	Shapiro–Wilk	<0.001	<0.001	**0.093**	**0.822**	—	—	—	—	—	—	—	—
	Cramér–von Mises	<0.001	<0.001	**0.249**	**0.476**	<0.001	<0.001	—	—	—	—	—	—
	Anderson–Darling	<0.001	<0.001	**0.196**	**0.518**	<0.001	<0.001	—	—	—	—	—	—
	CVM simulation[a]	—	—	—	—	—	—	0.030	0.014	**0.098**	**0.253**	**0.436**	**0.228**
Director E	Shapiro–Wilk	<0.001	<0.001	0.013	**0.260**	—	—	—	—	—	—	—	—
	Cramér–von Mises	<0.001	<0.001	0.043	**0.303**	<0.001	<0.001	—	—	—	—	—	—
	Anderson–Darling	<0.001	<0.001	0.028	**0.271**	<0.001	<0.001	—	—	—	—	—	—
	CVM simulation[a]	—	—	—	—	—	—	0.029	0.001	**0.087**	**0.132**	**0.245**	**0.439**
MOR	Shapiro–Wilk	<0.001	<0.001	**0.146**	<0.001	—	—	—	—	—	—	—	—
	Cramér–von Mises	<0.001	0.006	**0.310**	0.034	0.001	0.036	—	—	—	—	—	—
	Anderson–Darling	<0.001	<0.001	**0.273**	0.025	<0.001	0.004	—	—	—	—	—	—
	CVM simulation[a]	—	—	—	—	—	—	**0.108**	**0.145**	**0.567**	**0.492**	**0.598**	**0.095**

Sum, summer specimens; Win, winter specimens; G. O. F., goodness-of-fit; par, parameter; E-computer E, dynamic MOE as tested with E-computer device; Director E, dynamic MOE as tested with Director HM200 device; CVM, Cramér–von Mises; —, the test was not performed.

$N = 199$ for all summer data sets. $N = 198$ for Director E (Winter). For all other data sets, $N = 200$. Bold values indicate that a test failed to reject a distribution at a 0.05 significance level.

[a] In cases where critical values for the Cramér–von Mises test were not available in D'Agostino and Stevens (1986), they were determined by simulation.

Table 4. G. O. F. *p*-values for Mill 3 (full-complement mill).

		Distribution											
		Normal		Lognormal		2-par Weibull		3-par Weibull		Skew normal		Mixed normal	
Property	G.O.F. test	Sum	Win	Sum	Win	Sum	Win	Sum	Win	Sum	Win	Sum	Win
Static MOE	Shapiro–Wilk	<0.001	0.002	**0.392**	0.031	—	—	—	—	—	—	—	—
	Cramér–von Mises	<0.001	0.006	**0.365**	**0.662**	<0.001	0.006	—	—	—	—	—	—
	Anderson–Darling	<0.001	0.002	**0.463**	**0.389**	<0.001	0.002	—	—	—	—	—	—
	CVM simulation[a]	—	—	—	—	—	—	**0.080**	**0.363**	**0.181**	**0.655**	**0.088**	**0.169**
E-computer E	Shapiro–Wilk	<0.001	<0.001	**0.724**	**0.182**	—	—	—	—	—	—	—	—
	Cramér–von Mises	0.008	0.005	**0.997**	**0.587**	<0.001	0.003	—	—	—	—	—	—
	Anderson–Darling	0.002	<0.001	**0.976**	**0.507**	<0.001	<0.001	—	—	—	—	—	—
	CVM simulation[a]	—	—	—	—	—	—	**0.469**	**0.804**	**0.942**	**0.470**	**0.764**	**0.112**
Director E	Shapiro–Wilk	<0.001	<0.001	**0.597**	**0.499**	—	—	—	—	—	—	—	—
	Cramér–von Mises	<0.001	<0.001	**0.727**	**0.921**	<0.001	<0.001	—	—	—	—	—	—
	Anderson–Darling	<0.001	<0.001	**0.723**	**0.854**	<0.001	<0.001	—	—	—	—	—	—
	CVM simulation[a]	—	—	—	—	—	—	**0.111**	**0.434**	**0.563**	**0.777**	**0.404**	**0.370**
MOR	Shapiro–Wilk	**0.104**	0.005	<0.001	<0.001	—	—	—	—	—	—	—	—
	Cramér–von Mises	**0.737**	0.036	<0.001	<0.001	**0.784**	**0.815**	—	—	—	—	—	—
	Anderson–Darling	**0.423**	0.017	<0.001	<0.001	**0.770**	**0.828**	—	—	—	—	—	—
	CVM simulation[a]	—	—	—	—	—	—	**0.405**	**0.870**	**0.332**	**0.193**	**0.521**	**0.670**

Sum, summer specimens; Win, winter specimens; G.O.F., goodness-of-fit; par, parameter; E-computer E, dynamic MOE as tested with E-computer device; Director E, dynamic MOE as tested with Director HM200 device; CVM, Cramér–von Mises; —, the test was not performed.

$N = 199$ for Director E (Winter). For all other data sets, $N = 200$. Bold values indicate that a test failed to reject a distribution at a 0.05 significance level.

[a] In cases where critical values for the Cramér–von Mises test were not available in D'Agostino and Stevens (1986), they were determined by simulation.

Table 5. G. O. F. p-values for Mill 4 (large log mill).

		Distribution											
		Normal		Lognormal		2-par Weibull		3-par Weibull		Skew normal		Mixed normal	
Property	G.O.F. test	Sum	Win	Sum	Win	Sum	Win	Sum	Win	Sum	Win	Sum	Win
Static MOE	Shapiro–Wilk	0.024	**0.194**	**0.217**	0.004	—	—	—	—	—	—	—	—
	Cramér–von Mises	0.020	**0.757**	**0.340**	0.015	0.002	**0.060**	—	—	—	—	—	—
	Anderson–Darling	0.023	**0.636**	**0.419**	0.015	0.002	0.021	—	—	—	—	—	—
	CVM simulation[a]	—	—	—	—	—	—	**0.094**	**0.343**	**0.314**	**0.775**	**0.075**	**0.880**
E-computer E	Shapiro–Wilk	<0.001	0.006	**0.777**	**0.981**	—	—	—	—	—	—	—	—
	Cramér–von Mises	0.003	**0.087**	**0.855**	**0.794**	<0.001	0.001	—	—	—	—	—	—
	Anderson–Darling	0.001	**0.058**	**0.706**	**0.888**	<0.001	<0.001	—	—	—	—	—	—
	CVM simulation[a]	—	—	—	—	—	—	**0.272**	**0.693**	**0.766**	**0.731**	**0.306**	**0.395**
Director E	Shapiro–Wilk	<0.001	<0.001	**0.621**	**0.929**	—	—	—	—	—	—	—	—
	Cramér–von Mises	0.006	0.004	**0.551**	**0.907**	<0.001	<0.001	—	—	—	—	—	—
	Anderson–Darling	0.002	<0.001	**0.419**	**0.932**	<0.001	<0.001	—	—	—	—	—	—
	CVM simulation[a]	—	—	—	—	—	—	**0.382**	**0.388**	**0.339**	**0.562**	0.040	**0.223**
MOR	Shapiro–Wilk	**0.064**	0.027	<0.001	<0.001	—	—	—	—	—	—	—	—
	Cramér–von Mises	**0.197**	0.022	<0.001	<0.001	0.019	0.001	—	—	—	—	—	—
	Anderson–Darling	**0.144**	0.010	<0.001	<0.001	0.030	<0.001	—	—	—	—	—	—
	CVM simulation[a]	—	—	—	—	—	—	0.013	0.001	**0.096**	**0.238**	0.004	**0.348**

Sum, summer specimens; Win, winter specimens; G.O.F., goodness-of-fit; par, parameter; E-computer E, dynamic MOE as tested with E-computer device; Director E, dynamic MOE as tested with Director HM200 device; CVM, Cramér–von Mises; —, the test was not performed.

$N = 199$ for all winter data sets. For all summer data sets, $N = 200$. Bold values indicate that a test failed to reject a distribution at a 0.05 significance level.

[a] In cases where critical values for the Cramér–von Mises test were not available in D'Agostino and Stevens (1986), they were determined by simulation.

Table 6. Goodness-of-fit test summary score card.

Property	2-par Weibull		Normal		Lognormal		3-par Weibull		Mixed normal		Skew normal	
	Sum	Win	Sum	Win	Sum	Win	Sum	Win	Sum	Win	Sum	Win
Static MOE	4	4	3	2	1	3	2	1	0	0	0	0
Summer + winter	**8**		**5**		**4**		**3**		**0**		**0**	
E-computer E	4	4	3	3	1	0	2	1	0	0	0	0
Summer + winter	**8**		**6**		**1**		**3**		**0**		**0**	
Director E	4	4	4	4	2	1	2	2	1	0	1	0
Summer + winter	**8**		**8**		**3**		**4**		**1**		**1**	
MOR	3	2	2	3	3	4	2	1	1	0	0	0
Summer + winter	**5**		**5**		**7**		**3**		**1**		**0**	

par, parameter; E-computer E, dynamic MOE as tested with E-computer device; Director E, dynamic MOE as tested with Director HM200 device.
For each of the four properties (static MOE, E-computer E, Director E, and MOR), Table 6 presents the number of data sets (4 mills × 2 seasons) for which a distribution was *rejected* by a goodness-of-fit test at a 0.05 significance level. For each season, this number can range from 0 to 4. The centered number in bold is the subtotal of the summer plus the winter samplings for that property (0-8). Low numbers for a distribution suggest that it might be a good model for both stiffness and strength at multiple mills. Results of Table 6 indicate that skew normal and mixed normal models perform relatively well, and eg the 2-parameter Weibull model does not.

can range from 0 to 4 (in cases where multiple tests were performed, a rejection from one test was scored as a "full rejection" even if the other tests failed to reject. This seemed the most conservative and appropriate approach for assessing goodness-of-fit.) The centered number in bold is the subtotal of the summer plus the winter samplings for that property. This number can range from 0 to 8. Low numbers for a distribution suggest that it might be a good model for that property across multiple mills and/or seasons. Table 6 indicates that mixed normal and skew normal models perform relatively well, and eg the 2-parameter Weibull model does not, as illustrated in the plots in Fig 3.

Probability plots for 2-parameter Weibull distributions fit to MOR data appear in Fig 3. Histograms and probability plots for all 192 cases (4 mills × 2 seasons × 4 variables × 6 distributions) can be found at https://www1.fpl.fs.fed.us/4mills.plots.html.

DISCUSSION

As discussed in the Introduction section, the strength and stiffness distributions of visual grades of lumber and the strength distributions of MSR grades of lumber are pseudo-truncated versions of mill-run lumber strength and stiffness distributions. To estimate these pseudo-truncated distributions, it is necessary to start with the mill-run distributions.

Verrill et al (2017), Owens et al (2018, 2019), and the current study have focused on identifying reasonable models for mill-run strength and stiffness distributions. In these articles, they have established empirically that normal, lognormal, 2-parameter Weibull, and 3-parameter Weibull distributional forms do not generally perform as well as skew normal or mixed normal distributional forms.

It is important to note that even if distributional *forms* (eg skew normal and mixture of normals) are stable across mills and times, distributional *fits* (estimated parameter values) might not be. Anderson et al (2019) established that means and variances (as opposed to distributional forms) varied among the eight mill-run data sets discussed in the current article. This suggests that even though these data sets might share a distributional *form* (such as a skew normal or a mixed normal), their parametric *fits* might differ and thus their corresponding pseudo-truncated grade distributions and their associated reliability properties are likely to differ. The authors are currently investigating the extent to which distributional fits differ among the eight data sets.

CONCLUSION

Verrill et al (2017) and Owens et al (2018, 2019) investigated the mill-run distributions of MOE and MOR at four mills. They found that univariate mill-run MOE and MOR distributions are

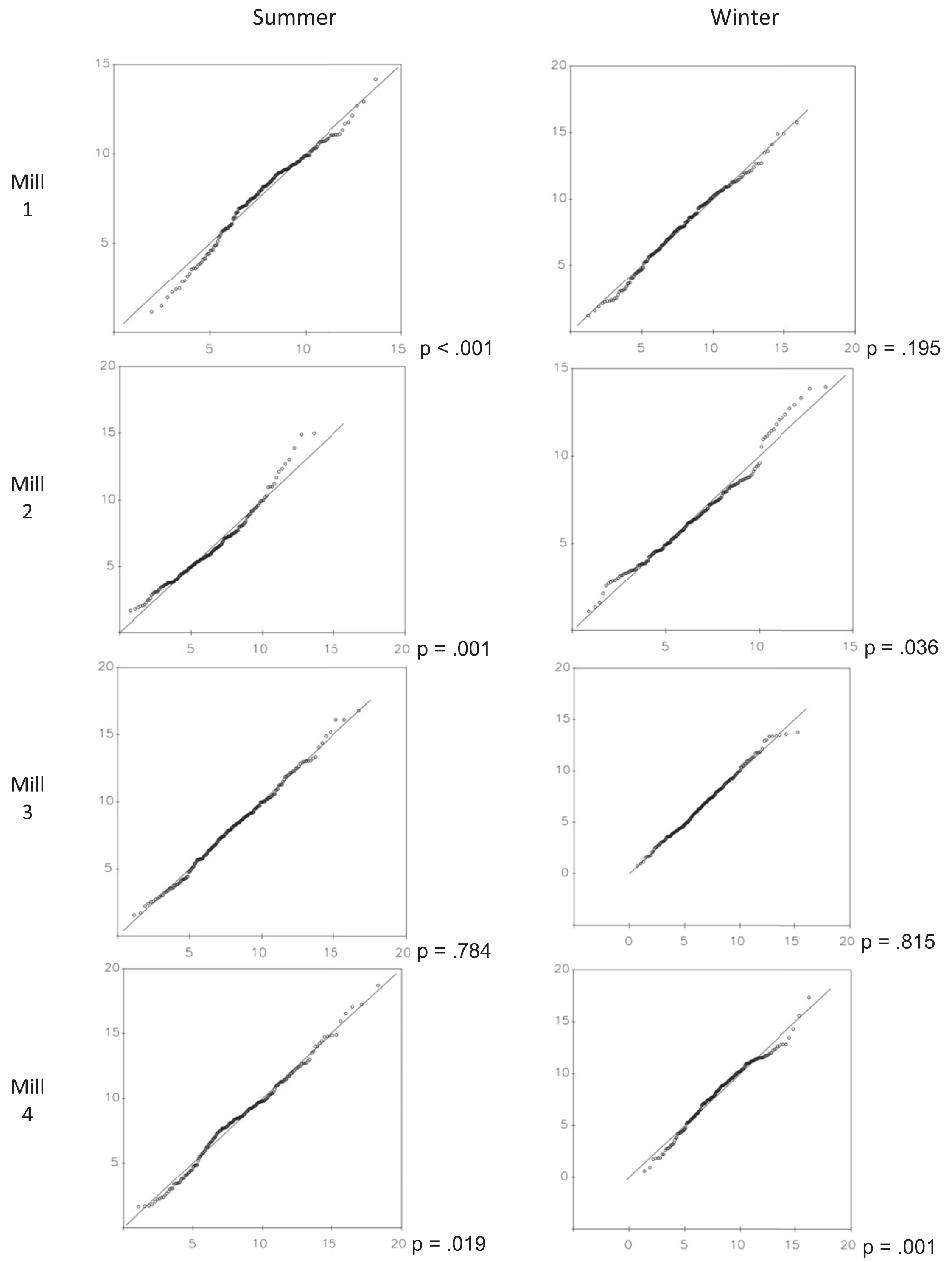

Figure 3. Probability plots for 2-parameter Weibull distributions fit to MOR data. For each plot, *X* and *Y* axes are "Ordered Expected Values" and "Ordered Observed Values," respectively. *p*-values are from Cramér–von Mises goodness-of-fit tests.

well-modeled by skew normal distributions or mixtures of normal distributions, but not so well-modeled (in general) by normal, lognormal, 2-parameter Weibull, or 3-parameter Weibull distributions.

Owens et al (2019) noted that it was important to investigate whether these results are stable over time. In this article, the authors have extended the analyses of "summer" data sets performed by Owens et al (2019) to new mill-run "winter" data sets. They have found that normal, lognormal, 2-parameter Weibull, and 3-parameter Weibull distributions continue to perform relatively poorly and that skew normal distributions and mixtures of normal distributions continue to perform relatively well.

Of course, it is possible that distributional forms (eg skew normal and mixture of normal) are stable across mills and times, whereas fits (estimated parameter values) are not. The authors are currently investigating the extent to which distributional fits differ among their eight data sets, and the corresponding consequences for reliability calculations.

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APPENDIX—PROBABILITY DENSITY FUNCTIONS

NORMAL DISTRIBUTION

The normal probability density function is given by

$$f(x;\ \mu,\sigma) = \frac{1}{\sqrt{2\pi}}\,\frac{1}{\sigma}\,\exp\left(-(x-\mu)^2\Big/2\sigma^2\right)$$

for $x \in (-\infty, \infty)$, where μ is the mean and σ is the SD. This distribution is denoted by the notation $N(\mu, \sigma^2)$.

LOGNORMAL DISTRIBUTION

The lognormal probability density function is given by

$$f(x;\ \mu,\sigma) = \frac{1}{\sqrt{2\pi}}\,\frac{1}{\sigma}\,\frac{1}{x}\,\exp\left(-(\log(x)-\mu)^2\Big/\left(2\sigma^2\right)\right)$$

for $x \in (0, \infty)$, where μ is the mean and σ is the SD of the log of the original data.

SKEW NORMAL

The skew normal distribution has probability density function

$$f(x;\xi,\omega,\alpha) = \frac{2}{\omega}\times\phi\left(\frac{x-\xi}{\omega}\right)\times\Phi\left(\alpha\left(\frac{x-\xi}{\omega}\right)\right)$$

for $x\in(-\infty,\infty)$ where ϕ denotes the probability density function of a standardized normal, Φ denotes the cumulative distribution function of a standardized normal, and $\xi, \omega,$ and α are the parameters of the skew normal.

MIXED NORMAL

In this article, a "mixed normal distribution" refers to a mixture of *two* normal distributions. Such a mixture results when specimens are drawn with probability p from an $N(\mu_1, \sigma_1^2)$ distribution and with probability $1-p$ from a $N(\mu_2, \sigma_2^2)$ distribution. In this case, the probability density function is given by

$$\begin{aligned} f(x;\ \mu_1,\sigma_1,p,\ \mu_2,\sigma_2) &= p\times\frac{1}{\sqrt{2\pi}}\,\frac{1}{\sigma_1}\,\exp\left(-(x-\mu_1)^2\Big/\left(2\sigma_1^2\right)\right) \\ &+ (1-p)\times\frac{1}{\sqrt{2\pi}}\,\frac{1}{\sigma_2}\,\exp\left(-(x-\mu_2)^2\Big/\left(2\sigma_2^2\right)\right) \end{aligned}$$

for $x \in (-\infty, \infty)$.

TWO-PARAMETER WEIBULL

The two-parameter Weibull has probability density function

$$f(w;\gamma,\beta) = \gamma^{\beta}\beta w^{\beta-1}\exp\left(-(\gamma w)^{\beta}\right)$$

for $w\in[0,\infty)$, where β is the shape parameter and γ is the inverse of the scale parameter.

THREE-PARAMETER WEIBULL

The three-parameter Weibull has probability density function

$$f(w;\gamma,\beta,c) = \gamma^{\beta}\beta(w-c)^{\beta-1}\exp\left(-(\gamma(w-c))^{\beta}\right)$$

for $w\in[c,\infty)$, where β is the shape parameter, γ is the inverse of the scale parameter, and c is the location parameter.