WITHIN-MILL VARIATION IN THE MEANS AND VARIANCES OF MOE AND MOR OF MILL-RUN LUMBER OVER TIME

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(Received April 2019)

Abstract. The literature related to the phenomenon of pseudo-truncation has emphasized that the mechanical property distributions of graded lumber subpopulations are determined by the mechanical property distributions of the mill-run (or full) lumber populations from which the subpopulations are formed. Whereas previous studies have shown that the means and variances of mechanical properties in the same visual grade of lumber can vary from mill to mill, there have been no studies on the stability of the means and variances of modulus of elasticity (MOE) and modulus of rupture (MOR) in mill-run lumber populations at the same mill over time. The objective of this study was to investigate if statistically significant differences between the means and variances of MOE and MOR in mill-run lumber populations at the same mill could be observed across samples taken several months apart. Two mill-run samples of 200 pieces of rough, dry 2 × 4 southern pine lumber were taken from each of four Mississippi sawmills: one in the summer and one in the winter. For each mill, the summer and winter means and variances of flexural MOR and MOE were compared. Whereas no significant differences were found between the mean MOR or mean MOE of the summer and winter samples from Mills 2 and 4, significant differences in mean MOE and/or MOR were found between the summer and winter samples from Mills 1 and 3. In addition, a Levene's test on the MOR of Mill 1 showed significant differences in the variance between the summer and winter samples. Further analysis revealed that in addition to the fact that the winter mill-run sample from Mill 3 was made up of a larger percentage of lower grade material than the summer sample, there were pronounced strength differences between the summer and winter samples both around the median and at the lowest (near-minimum) percentiles *within* each grade. This reinforces the notion that changes in mill-run MOR distributions over time can have an important effect on the overall strength of a given mill's visual grades over time. A theory of mixed distributions could account for these differences.

Keywords: Mill-run, full lumber population, modulus of rupture, modulus of elasticity, mean, variance, mixed distribution.

INTRODUCTION

A full, or "mill-run," lumber population includes every piece of lumber sawn from logs. Unlike a graded population, it includes all qualities from

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Wood and Fiber Science, 51(4), 2019, pp. 387-401
https://doi.org/10.22382/wfs-2019-037
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"best" to "worst." It may include pieces that would normally be end trimmed, might not make grade, or might otherwise be ground into chips.

The literature related to the statistical phenomenon of pseudo-truncation (Verrill et al 2012, 2013, 2014, 2015, 2017, 2019; Owens et al 2018, 2019) has emphasized that the bending strength distributions of graded lumber subpopulations are determined by the bending strength distributions of the mill-run (or full) lumber population from which the subpopulations are formed.

Although the impact that a mill-run population has on the distributional form(s) of modulus of rupture (MOR) in its graded subpopulation(s) has important implications for reliability calculations (Verrill et al 2013, 2014, 2018), it may not seem particularly relevant to those outside the engineering community. More important to everyday producers and consumers of structural lumber might be how mill-run populations influence basic properties of graded lumber such as mean and near-minimum bending strength.

The mechanical properties of visually graded lumber are known to vary. For example, Galligan and Snodgrass (1970) showed that lumber of the same species and visual grade can exhibit considerable differences in mechanical properties from mill to mill.

Variation is also understood to occur within the same mill over time. Bender and Woeste (2012) write:

> Because of differences in forests due to factors such as management practices, climate, soils, species mix within a species grouping and log processing variables, the strength of the material from different sawmills will vary from mill to mill and from week to week. This type of variation has been recognized as a natural part of the visual grading system since it was developed nearly a century ago. (p. 37)

Although previous studies have shown that the means and variances of mechanical properties in the same visual grade of lumber can vary from mill to mill, to the authors' knowledge, there have been no studies on the stability of the means and variances of mill-run lumber at the same mill over time. As mill-run lumber is known to impact the properties of graded lumber, this should be investigated.

The objective of this study was 2-fold. First, it seems necessary to determine if meaningful differences in mean modulus of elasticity (MOE) and MOR can be observed in mill-run lumber populations at the same mill over time. Whereas the presumption is that (slight) variation can and does occur from day to day, week to week, and month to month for any combination of the reasons Bender and Woeste (2012) mentioned previously, it seems less likely that these differences would be large under normal circumstances because that could drastically impact visual grade yield, product performance quality, machine stress–rated lumber yield/mix, and overall mill profitability. In other words, it is seemingly in a mill's best interest to source their raw material in a way that minimizes mechanical property variation over time (to the extent possible). Therefore, the first objective was to investigate if statistically significant differences between the means and variances of MOE and MOR can be observed at the same mill over time. If significant differences in means and variance can be found, it would suggest that meaningful (as opposed to negligible) differences in mechanical performance can be seen across time at the same sawmill.

Although some variation in mechanical properties undoubtedly occurs in mill-run lumber from week to week and even day to day because of variations in raw materials, it might be reasonable to assume that large variations are more likely to occur over a period of months than over a period of days. If the span of months is approximately six, one might also expect influence from seasonal variables such as log availability, forest tract access, etc. For these reasons, two samples of sawn material were obtained from each mill—one in the summer (June through July production) and one in the winter (December through January production).

It is important to note that this study did not aim to generalize how or determine why means and variances of mechanical properties of mill-run lumber populations might vary across seasons per se; rather, the aim was merely to determine if nontrivial differences can be observed in real mill-run lumber populations sampled several months apart. Summer and winter sampling was intended to maximize potential differences in mechanical properties (so they could be more easily detected) under the assumption that the changes in log availability, forest access, etc. that typically occur between summer and winter months might yield larger differences than those that might typically occur between (eg) consecutive days or weeks. It should also be understood that no claim is being made that basic lumber quality and mechanical properties somehow change depending on the season the trees were harvested or the logs milled.

The second objective of this study was to investigate how significant differences in bending strength in mill-run lumber populations (should they be found) affect the properties of the visually graded lumber extracted from them. For example, a reduction in mean strength in a mill-run population, presumably caused by "lower quality" baseline or parent raw material, might result in increased proportions of lower grade lumber, but is that the only kind of change one might expect? How might a change in mean strength at the mill-run level impact the strength within each visual grade?

To investigate these questions, two mill-run samples of 200 pieces of rough, dry 2 × 4 southern pine lumber were taken from each of four Mississippi sawmills: one in the summer and one in the winter. For each mill, the means and variances of flexural MOR and MOE of the two samples were compared. If significant differences in mean mechanical properties were found between the summer and winter mill-run samples, analysis continued at the level of visual grades to determine how those differences might have impacted both the grade yield and the strength properties of each individual grade.

As the bending strength of visually graded lumber is not typically monitored and tracked on a daily basis by sawmills (Bender and Woeste 2012), it is important that mill managers understand that a mean strength reduction in a mill-run population of lumber might affect the strength performances of individual grades.

MATERIALS AND METHODS

Sampling

In total, 1,600 pieces of mill-run 2 × 4 lumber were provided by four regional sawmills in Mississippi. Each mill provided 200 pieces of lumber sawn from summer (June or July) production and 200 pieces sawn from winter (December or January) production. For each sampling, a kiln package was randomly selected from the weekly dry kiln output. After removing the top course of lumber (to avoid potential and excessive warp), 200 rough dry pieces were selected sequentially. Full details of the sampling method appear in Owens et al (2019). The size of lumber after planing was approximately 1.5 × 3.5 × 96 inches (3.81 × 8.89 × 243.84 cm). Although the material was pulled from production and tested as mill-run lumber, the material was graded after planing by a Southern Pine Inspection Bureau (SPIB)–certified inspector to provide data for additional analyses.

Among the four mills, the first mill was classified as a "full complement" mill as it processes a full range of log sizes (no minimum butt size; maximum butt size of 24 inches [60.96 cm]). It sawed more or less a full complement of dimension lumber sizes (2 × 4 to 2 × 12). The second mill was classified as a small log mill because it saws mostly small logs (maximum butt size of 15 inches [38.10 cm]; no minimum butt size so long as the top is at least 4 inches [10.16 cm]). It sawed mainly 2 × 4 and a small proportion of 2 × 6. The third mill was also classified as a full complement mill. The fourth mill was classified as a large log mill because its log population is mainly large logs (maximum butt size of 28 inches [71.12 cm]; minimum butt size of 12 inches [30.48 cm]). It sawed mainly 2 × 8 to 2 × 12 with very little 2 × 4 and 2 × 6.

Testing

Both nondestructive and destructive tests were used to collect the data. For each specimen, MOR and three measures of MOE were recorded. Two nondestructive tests were performed to measure dynamic MOE. Metriguard's E-computer device (Model 340, Metriguard, Inc., Pullman, WA) estimated the MOE by measuring the transverse vibration in the sample. Each test piece was supported at its two ends. A transducer at one end of the specimen measured the frequency of the oscillation after a slight tap was applied to the midspan. The computer calculated the MOE according to the following equation (Ross and Pellerin 1994):

$$E = \frac{f^2 W S^3}{C I g},$$

where E = modulus of elasticity, S = span, W = weight of specimen, f = resonant frequency, I = moment of inertia, g = acceleration due to gravity, and C = constant.

Fibre-gen's device (Director HM200, Fibre-gen Limited, Christchurch, New Zealand) was used to measure acoustic velocity and calculate the MOE. The specimen was placed across two sawhorses. The device's sensor was held against one end of the test piece. The acoustic wave produced by a hammer tap traveled from one end to the other. Then the device measured the acoustic velocity and calculated the MOE based on the following equation (Ross and Pellerin 1994):

$$E = \rho V^2,$$

where E = modulus of elasticity, ρ = density of the specimen, and V = acoustic velocity.

A destructive third-point static bending test per ASTM D198-15 (ASTM 2015) was performed to measure the static MOE and MOR. Before testing, the MC of each specimen was measured by a Wagner L 601-3 handheld moisture meter (Wagner Electronic Products Inc., Rogue River, OR). The average MC of the test specimens was 13.3% (SD = 1.70). The span-to-depth ratio was held constant at 17:1. The lengthwise location of the 59.5-inch (151.13 cm) test span within each 96-inch specimen was randomly determined. Each specimen was placed into the fixture in an edgewise orientation. An extensometer was placed under the bottom edge of the midspan where the greatest deflection occurred. The load heads applied force until the test piece achieved full failure. The average testing time was approximately 5 min. All MOE and MOR values were adjusted to a common MC of 15% per ASTM 1990-16 (ASTM 2016) before analysis.

In the summer sample from Mill 2 and the winter sample from Mill 4, there was one broken piece each, before testing. These two pieces were not testable by any method, so the total number of data points for all properties was reduced to 199 each. In addition, among the winter samples, there were two pieces from Mill 1, two pieces from Mill 2, and one piece from Mill 3 for which the Director device did not produce a reading even after multiple attempts. The sample size for these specimens was reduced only for analyses that required Director data.

Statistical Methods

Mean comparisons of MOE and MOR were performed with *t*-tests on both the MOE and MOR of the summer and winter data sets of each mill. Levene's tests based on the median (Brown–Forsythe tests) were performed to assess homogeneity of variance. SPSS 25 (IBM Corp. 2017) was used to run the *t*-tests and Levene's tests. Minitab 18 (Minitab, Inc. 2017) was used to generate the smoothed curves for the cumulative percentage graphs (degree of smoothing = 0.5; number of steps = 2).

RESULTS

Mean Comparisons for MOR

Table 1 compares the mean MOR values of the summer and winter samples by mill. The results are presented graphically in Fig 1. For Mill 1 (the full complement pilot mill), the mean MOR values for summer and winter were 54.11 MPa and 53.91 MPa, respectively. The difference was not significant ($t\,[380.623] = 0.105, p = 0.916$) at a 0.05 level. For Mill 2 (the small log mill), the mean MOR values for summer and winter were 42.34 MPa and 43.89 MPa, respectively. The difference was not significant (t [397] = −0.890,

Table 1. Results of *t*-tests comparing the adjusted MOR of the summer and winter mill-run samples by mill.

Mill code	Mill type	Season	*n*	Mean (MPa)	*t*	df	*p* value (*t* test)	SD (MPa)	*p* value (Levene's)[a]	Fifth percentile (MPa)
1	Pilot (full complement)	Summer	200	54.11	0.105	380.623[b]	0.916	16.51	0.001	22.90
		Winter	200	53.91				20.51		17.11
2	Small log	Summer	199	42.34	−0.890	397	0.374	17.43	0.830	17.93
		Winter	200	43.89				17.22		20.32
3	Full complement	Summer	200	54.95	4.405	398	<0.001	21.61	0.526	20.90
		Winter	200	45.71				20.31		14.87
4	Large log	Summer	200	57.95	0.701	397	0.484	23.96	0.153	17.42
		Winter	199	56.38				20.77		19.11

All MOR values were adjusted to a common MC of 15% per ASTM 1990.

[a] The Levene's test was based on the median.

[b] A Levene's test was performed to assess homogeneity of variances. Equal variances were assumed for Mills 2, 3, and 4 but not for Mill 1.

$p = 0.374$). For Mill 3 (the second full complement mill), the mean MOR values for summer and winter were 54.95 MPa and 45.71 MPa, respectively. The difference was significant ($t\,[398] = 4.405, p < 0.001$). For Mill 4 (the large log mill), the mean MOR values for summer and winter were 57.95 MPa and 56.38 MPa, respectively. The difference was not significant ($t\,[397] = 0.701, p = 0.484$).

The histograms in Figs 2-5 graphically compare the summer and winter frequency distributions of MOR by mill. Figure 4 exhibits a clear leftward (or, in this case, downward) shift of the winter distribution relative to the summer distribution, as shown by the lower median and 5th percentile.

Levene's Test for MOR

For Mill 1, a Levene's test rejected the null hypothesis that the summer and winter population variances were equal ($\alpha = 0.05, p = 0.001$). For all other mills, the Levene's tests failed to reject the null hypothesis ($p > 0.05$).

Mean Comparisons for MOE

Table 2 compares the MOE values of the summer and winter samples by mill. All three measures of elasticity—the static MOE from the bending test (MOE-stat), the dynamic MOE from the Director test (Dir-E), and the dynamic MOE from the E-computer test (Ecomp-E)—appear in the table.

Error bars: 95% confidence interval. All MOR values were adjusted to a common MC of 15% per ASTM 1990-16.

Figure 1. Means of adjusted MOR for summer and winter mill-run samples by mill.

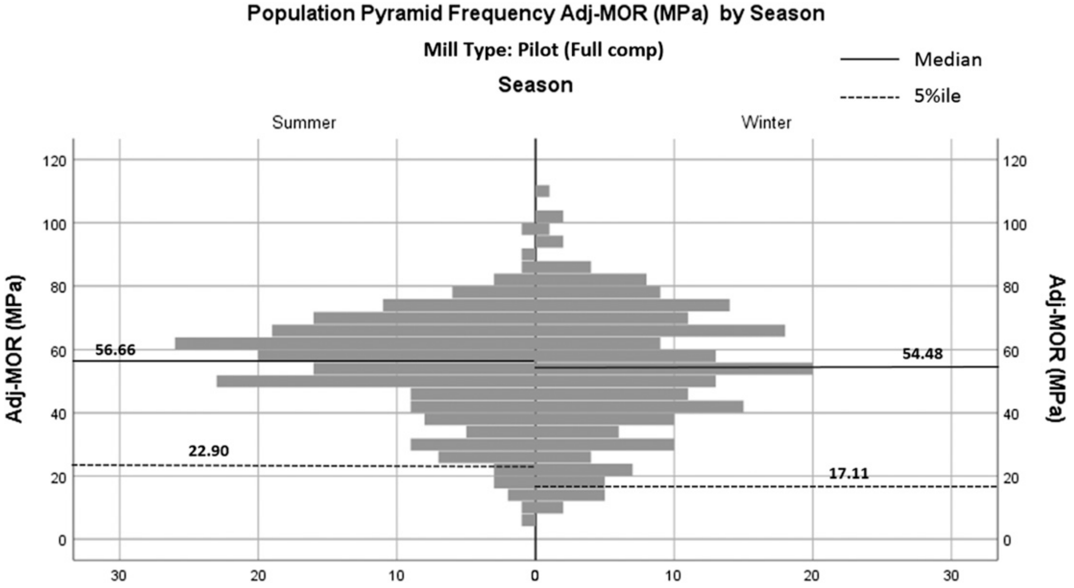

Figure 2. Frequency comparison of adjusted MOR (summer vs winter) for Mill 1.

For Mill 1 (the full complement pilot mill), the mean MOE-stat values for summer and winter were 9.82 GPa and 10.30 GPa, respectively. The difference was not significant (t [398] $= -1.885$, $p = 0.060$) at the 0.05 level. The mean Dir-E values for summer and winter were 10.84 GPa and 11.36 GPa, respectively. The difference was significant (t [396] $= -2.022$, $p = 0.044$). The mean Ecomp-E values for summer and winter were 11.18 GPa and 10.00 GPa, respectively. The difference was significant (t [398] $= 4.927$, $p = < 0.001$).

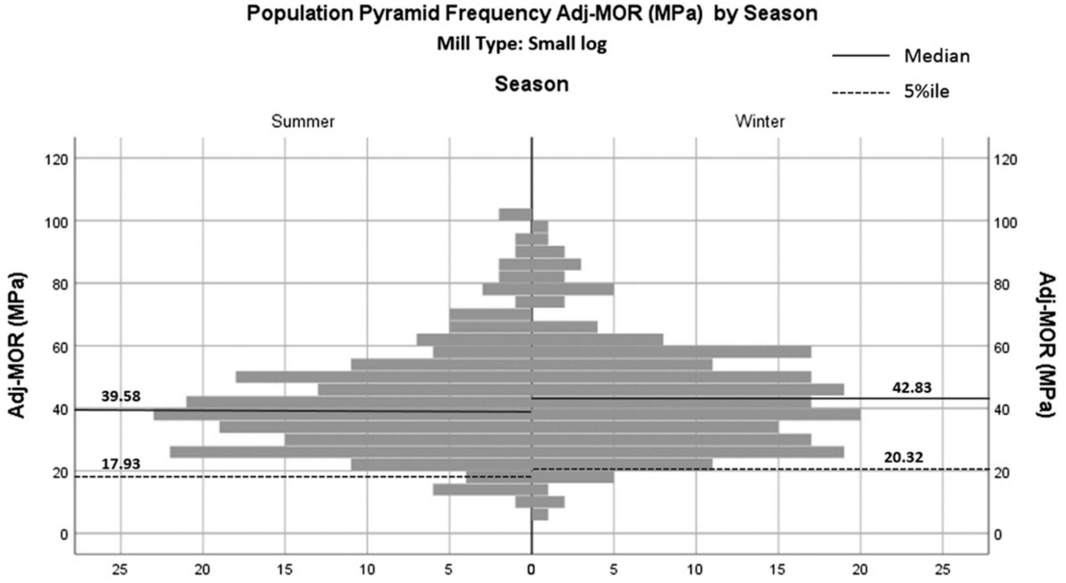

Figure 3. Frequency comparison of adjusted MOR (summer vs winter) for Mill 2.

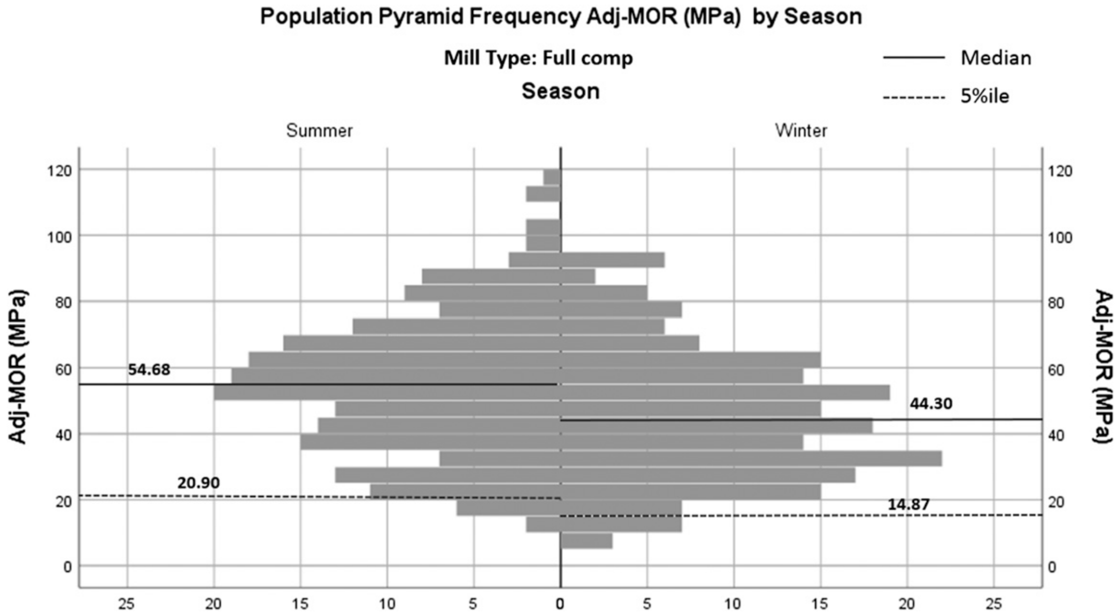

Figure 4. Frequency comparison of adjusted MOR (summer vs winter) for Mill 3.

For Mill 2 (the small log mill), the mean MOE-stat values for summer and winter were 8.99 GPa and 9.26 GPa, respectively. The difference was not significant (t [397] $= -1.055$, $p = 0.292$) at the 0.05 level. The mean Dir-E values for summer and winter were 9.24 GPa and 9.49 GPa, respectively. The difference was not significant (t [395] $= -0.931$, $p = 0.353$). The mean Ecomp-E

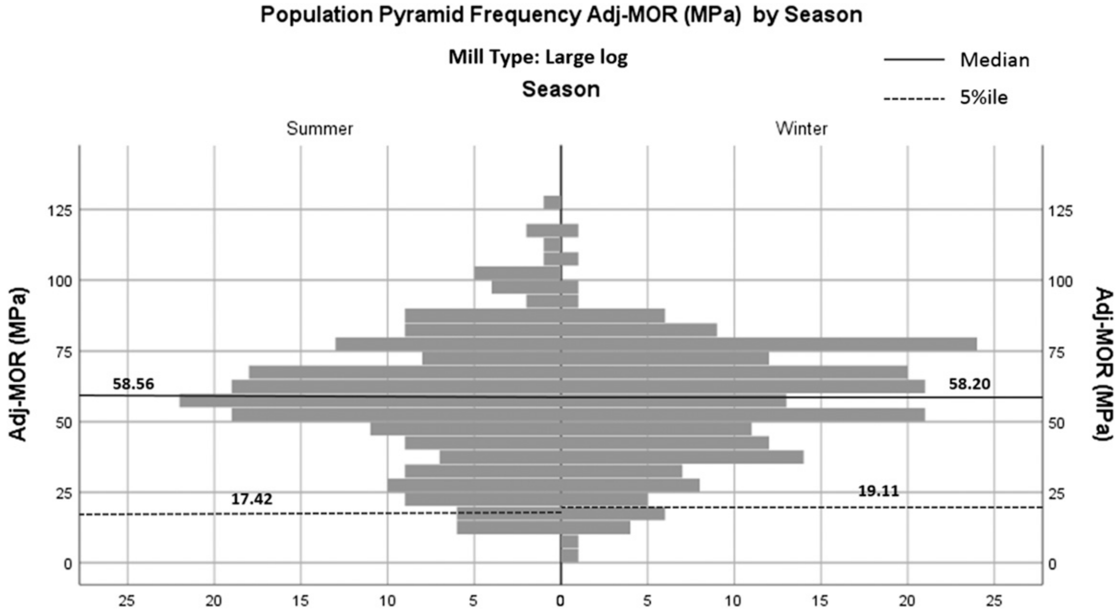

Figure 5. Frequency comparison of adjusted MOR (summer vs winter) for Mill 4.

Table 2. Results of *t*-tests comparing the adjusted MOE of the summer and winter mill-run samples by mill.

Mill code	Mill type	Data	Season	*n*	Mean (GPa)	*t*	df	*p* value (*t* test)	SD (GPa)	*p* value (Levene's)[a]
1	Pilot (full complement)	MOE-stat	Summer	200	9.82	−1.885	398	0.060	2.41	0.279
			Winter	200	10.30				2.59	
		Dir-E	Summer	200	10.84	−2.022	396	0.044	2.48	0.152
			Winter	198	11.36				2.66	
		Ecomp-E	Summer	200	11.18	4.927	398	<0.001	2.56	0.248
			Winter	200	10.00				2.25	
2	Small log	MOE-stat	Summer	199	8.99	−1.055	397	0.292	2.73	0.242
			Winter	200	9.26				2.41	
		Dir-E	Summer	199	9.24	−0.931	395	0.353	2.88	0.110
			Winter	198	9.49				2.44	
		Ecomp-E	Summer	199	8.86	1.078	369.038[b]	0.282	2.71	0.009
			Winter	200	8.60				2.05	
3	Full complement	MOE-stat	Summer	200	10.59	3.794	398	<0.001	2.74	0.085
			Winter	200	9.51				2.95	
		Dir-E	Summer	200	11.39	4.645	397	<0.001	2.90	0.255
			Winter	199	10.00				3.11	
		Ecomp-E	Summer	200	10.32	5.761	398	<0.001	2.54	0.193
			Winter	200	8.81				2.71	
4	Large log	MOE-stat	Summer	200	10.88	−0.173	397	0.863	2.81	0.627
			Winter	199	10.93				2.73	
		Dir-E	Summer	200	11.86	0.551	397	0.582	2.89	0.671
			Winter	199	11.70				2.87	
		Ecomp-E	Summer	200	10.76	−0.396	397	0.692	2.53	0.854
			Winter	199	10.86				2.47	

All MOE values were adjusted to a common MC of 15% per ASTM 1990. MOE-stat, static MOE from the bending test; Dir-E, dynamic MOE from the Director test; Ecomp-E, dynamic MOE from E-computer test.

[a] The Levene's test was based on the median.

[b] A Levene's test was performed to assess homogeneity of variances. Equal variances were not assumed for the Ecomp-E data from Mills 2.

values for summer and winter were 8.86 GPa and 8.60 GPa, respectively. The difference was not significant (t [369.038] = 1.078, p = 0.282).

For Mill 3 (the second full complement mill), the mean MOE-stat values for summer and winter were 10.59 GPa and 9.51 GPa, respectively. The difference was significant (t [398] = 3.794, $p <$ 0.001) at the 0.05 level. The mean Dir-E values for summer and winter were 11.39 GPa and 10.00 GPa, respectively. The difference was significant (t [397] = 4.645, $p <$ 0.001). The mean Ecomp-E values for summer and winter were 10.32 GPa and 8.81 GPa, respectively. The difference was significant (t [398] = 5.761, $p <$ 0.001).

For Mill 4 (the large log mill), the mean MOE-stat values for summer and winter were 10.88 GPa and 10.93 GPa, respectively. The difference was

Table 3. Comparison of grade breakdowns for summer and winter mill-run samples from Mills 1 to 4.

Mill no.	Sample	Select structural (%)	No. 1 (%)	No. 2 (%)	No. 3 (%)	Low grade (%)	Total (%)
1	Summer	15.5	3.5	30.5	30.0	20.5	100
	Winter	16.0	4.5	21.0	21.0	37.5	100
2	Summer	7.0	10.6	36.7	26.1	19.6	100
	Winter	7.5	12.0	37.5	17.0	26.0	100
3	Summer	11.0	9.0	27.5	25.0	27.5	100
	Winter	10.0	6.0	19.0	27.5	37.5	100
4	Summer	31.0	7.0	25.0	20.5	16.5	100
	Winter	22.1	8.5	30.2	13.6	25.6	100

"Low grade" refers to any specimen that graded below No. 3 (Southern Pine Inspection Bureau).

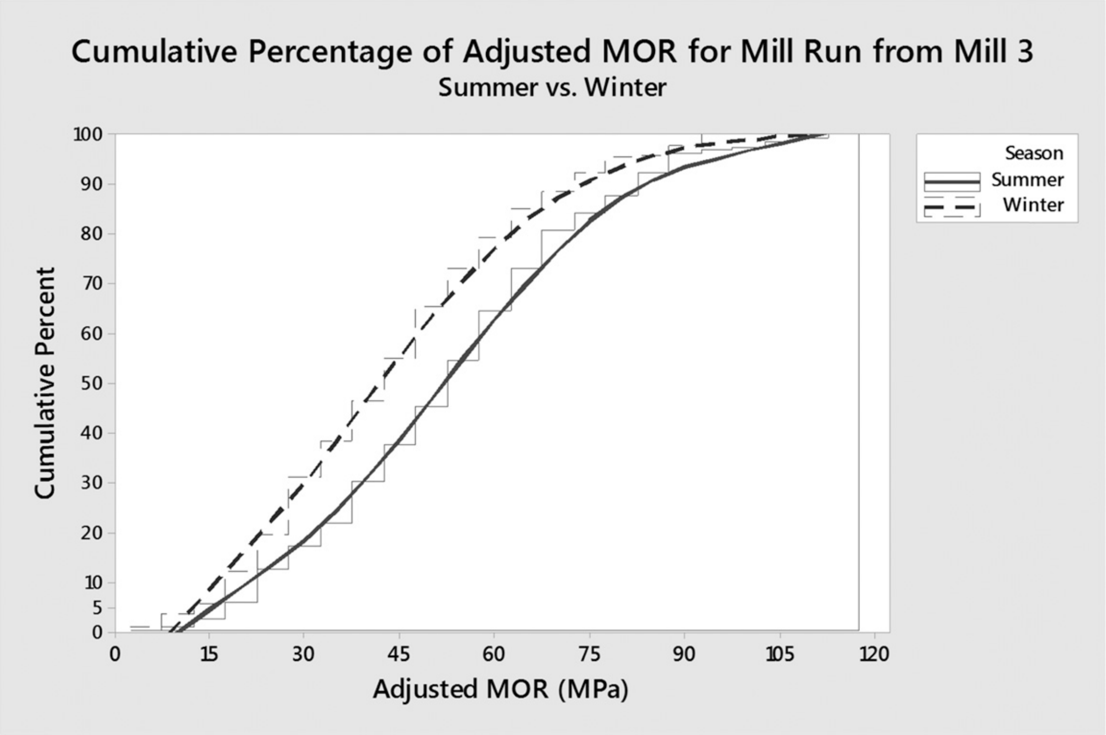

Figure 6. Cumulative percentage (summer vs winter) of adjusted MOR for the entire mill-run population (all developing grades) of Mill 3.

not significant (t [397] = −0.173, p = 0.863) at the 0.05 level. The mean Dir-E values for summer and winter were 11.86 GPa and 11.70 GPa, respectively. The difference was not significant (t [397] = 0.551, p = 0.582). The mean Ecomp-E values for summer and winter were 10.76 GPa and 10.86 GPa, respectively. The difference was not significant (t [397] = −0.396, p = 0.692).

Levene's Test for MOE

For the Ecomp-E of Mill 2, a Levene's test rejected the null hypothesis that the summer and winter population variances were equal (α = 0.05, p = 0.009). As for the MOE-stat and the Dir-E of Mill 2, the Levene's tests failed to reject the null hypothesis ($p > 0.05$). For all measures of MOE at all other mills (Mill 1, Mill 3, and Mill 4), the Levene's tests failed to reject the null hypothesis ($p > 0.05$).

DISCUSSION

Based on the results of the testing, it is possible to make some basic observations.

1. No significant differences were found between the mean mill-run MOR or mean mill-run MOE of the summer and winter samples from Mills 2 and 4. This finding suggests that the average strength and stiffness of the raw material (ie the logs) at these two mills was consistent between the summer and winter samplings. Mechanical properties at the same mill are believed to vary from day to day, week to week, and month to month, yet this result suggests that, in the case of some mills, those variations might be slight and have little meaningful impact on the overall strength and stiffness of the mill-run lumber population. In other words, with stable procurement of consistent material, MOR and MOE might be quite stable over time. This is not to say that these

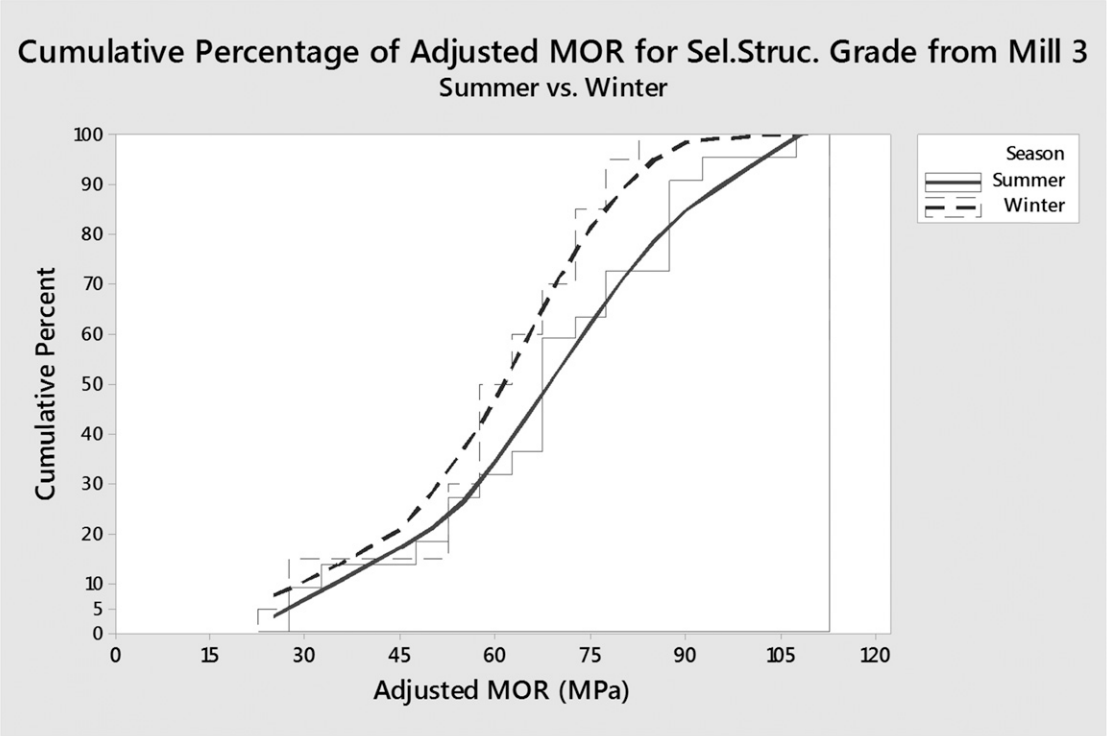

Figure 7. Cumulative percentage (summer vs winter) of adjusted MOR for the portion of the mill-run population from Mill 3 that graded select structural.

> mills are necessarily sourcing all their logs from the same stands all the time—which they are most likely not; rather, even under the assumption of daily, weekly, and monthly variations, stable availability of consistent raw materials from whatever source can make these variations negligible.

It should be noted that the failure of the *t*-tests to detect a significant difference between the summer and winter samplings of Mills 2 and 4 could have been a mere coincidence. It might be that there was significant variation in the MOR and MOE between every other day, week, and month within that 6-mo interval; however, the temporal distance between summer and winter samplings probably minimized that likelihood.

2. On the other hand, significant differences in mean mill-run MOE and/or MOR were found between the summer and winter samples from Mills 1 and 3. In addition, the Levene's test for the MOR of Mill 1 showed significant differences in the variance between summer and winter. These results suggest that the raw material at these two mills changed somehow over time. Although determining the exact cause is outside the scope of this study, this change might have been brought about by, for example, a change in forest accessibility brought on by local, seasonal fluctuations in precipitation levels, muddy terrain, or other disruptions to some (but not other) log supply sources.

Although only four mills in total were sampled, there seems to be preliminary evidence suggesting that mechanical properties of mill-run lumber produced several months apart (or at least on different days) are consistent at *some* mills but perhaps not at others. It is worth noting that the two mills that showed significant

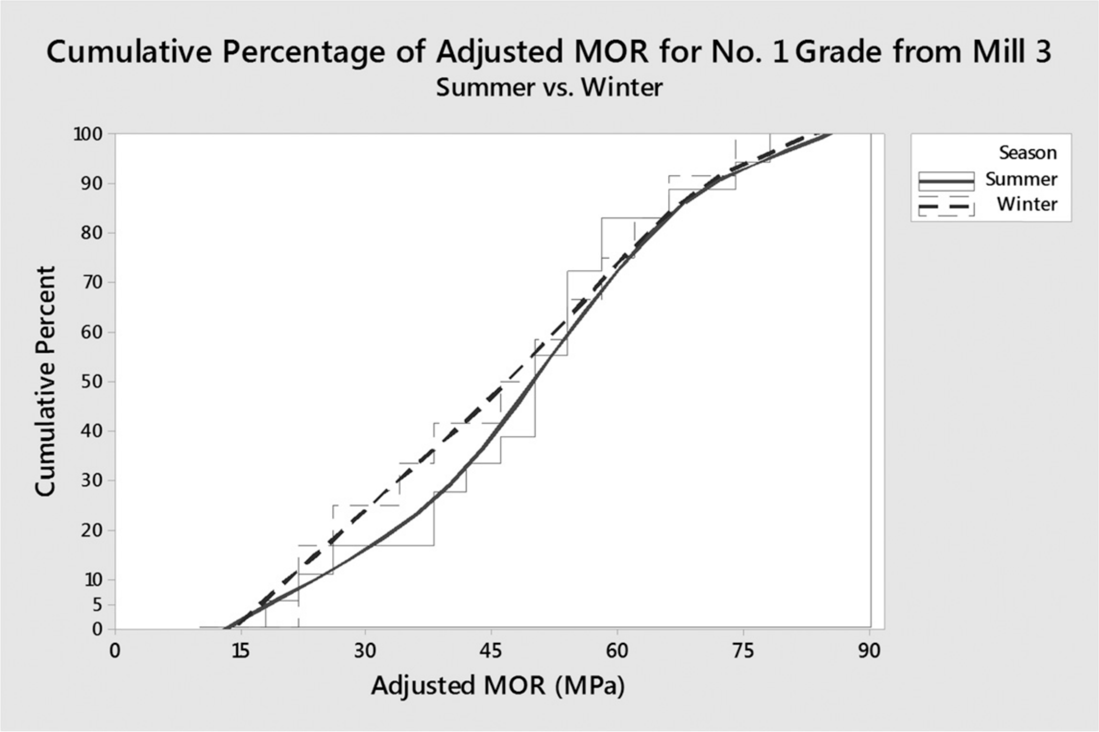

Figure 8. Cumulative percentage (summer vs winter) of adjusted MOR for the portion of the mill-run population from Mill 3 that graded No. 1.

differences in mean MOR and/or MOE between the summer and winter samples (ie Mills 1 and 3) were both full complement mills. Because the range of log sizes at these mills was greater than the range of log sizes at the small (Mill 2) and large (Mill 4) log mills, there was more opportunity for log size to vary between the summer and winter samplings. Alternatively, for example, because most of the logs were small and relatively similar in size at the small log mill, there was less opportunity for the difference in the log sizes to be great enough to give rise to significant differences in mean MOE and MOR between the summer and winter samples. The same logic can be applied to the large log mill.

In addition, the significant leftward (or downward) shift in the mill-run MOR distribution of Mill 3's winter sample relative to the summer sample leads one to wonder how such a shift in the mill-run distribution might affect the distributions of the graded lumber subpopulations extracted from it.

1. First, it is important to look at the grade breakdown to understand whether this leftward shift was influenced by a higher percentage of lower grade material. The grade breakdown for the mill-run summer and winter samples of all four mills is shown in Table 3. For Mill 3, the winter sample exhibits a higher percentage of pieces in the lowest grades (No. 3 and below). This indicates that the material from the winter sample contains larger grade-reducing defects than the summer samples, which undoubtedly contributes to the overall lower MOR and MOE of the mill-run population. In the case of Mill 3, the mill-run winter sample was made up of a higher percentage of lower grade (and lower value) material.
2. It is also important to consider whether each winter grade exhibits the same sort of leftward shift of the MOR distribution observed in the mill-run population. Figures 6-11 show summer and winter cumulative percentage comparisons for each grade taken from the mill-run

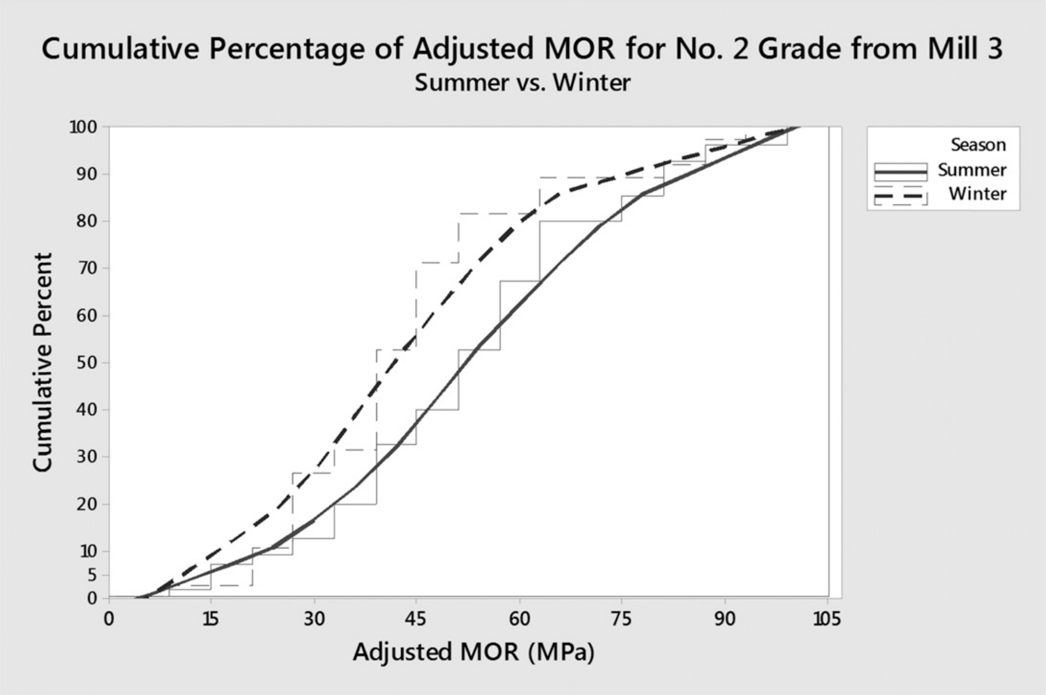

Figure 9. Cumulative percentage (summer vs winter) of adjusted MOR for the portion of the mill-run population from Mill 3 that graded No. 2.

population of Mill 3. Each grade shows a leftward distribution shift both near the median and at the lowest percentiles. This finding indicates that, in addition to the fact that the winter mill-run sample from Mill 3 was made up of a larger percentage of lower grade material than the summer sample, there were pronounced strength differences between the summer and winter samples both around the median and at the lowest (near-minimum) percentiles within each grade. This finding reinforces the reality that changes in mill-run MOR distributions over time can have an important effect on the overall strength of a mill's visual grades over time. As the bending strength of visually graded lumber is not typically monitored and tracked on a daily basis, sawmills need to be aware that, similar to the differences observed between mechanical properties from mill to mill, the strength of individual grades themselves (not just the grade breakdown) can change as a result of what happens at the mill-run level. In other words, although their grades may exhibit the same range of grade-reducing characteristics, they may not all perform in the same manner.

Finally, it is worth considering how a theory of mixed distributions could potentially account for significant differences in means and variances of MOE and MOR in mill-run populations at the same mill over time. In their analysis of a mill-run population of lumber sampled from a single sawmill on a single day, Verrill et al (2018) demonstrate that the MOE-MOR bivariate distribution could be well modeled by a mixture of bivariate normal distributions (in contrast to a single distribution) each representing a distinct underlying subpopulation, for example, mature wood vs juvenile wood (small logs vs large logs), two different subspecies within the southern pine group (for example *Pinus taeda* vs *Pinus palustris*), or pine taken from lowlands (less moisture stress) vs highlands (potentially more

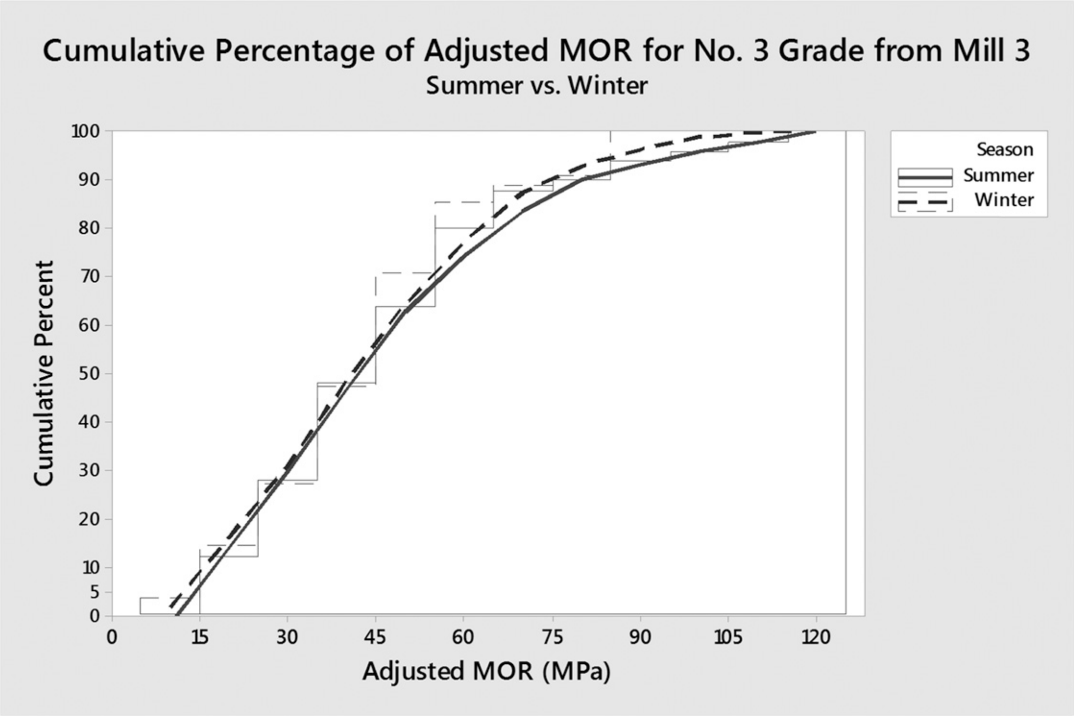

Figure 10. Cumulative percentage (summer vs winter) of adjusted MOR for the portion of the mill-run population from Mill 3 that graded No. 3.

moisture stress) as is common seasonal practice. In the case of a bivariate mixture of normal distributions, there is an added variable "*p*" that indicates the proportion of each of the two subpopulations comprising the mixture. (As per the probability density function for a mixture of bivariate normal distributions in Appendix A of Verrill et al [2018], the variable *p* quantifies the proportion of one [the leftmost] of the component distributions. The proportion of the other component distribution can be calculated by 1-*p*.) If the MOE–MOR bivariate distribution of a mill-run lumber population is indeed a mixture of two bivariate component distributions, then it might be possible that the respective proportions of those component distributions (or populations) change over time. For example, seasonal fluctuations in precipitation levels might change access to certain forest tracts, which, in turn, could alter the otherwise usual mix of small logs vs large logs and/or clear boles vs more knotty boles. Such a change could impact both mean MOE and MOR as well as the variance of the bivariate mixture even if the parameters of the component distributions remain constant.

A prudent next step would be to test these data for evidence of distribution mixtures. Owens et al (2019) have already investigated the MOR and MOE distributions of four mill-run summer data sets. They will also investigate MOR and MOE distributions of four mill-run winter data sets sampled at the same mills and assess whether MOR and MOE distributions (univariate and bivariate) are well modeled as mixtures.

CONCLUSION

This study investigated whether statistically significant differences between the means and variances of MOE and MOR in mill-run lumber populations at the same mill could be observed across samples taken several months apart. Two mill-run samples of 200 pieces of rough, dry 2 × 4 southern pine lumber were taken from each of

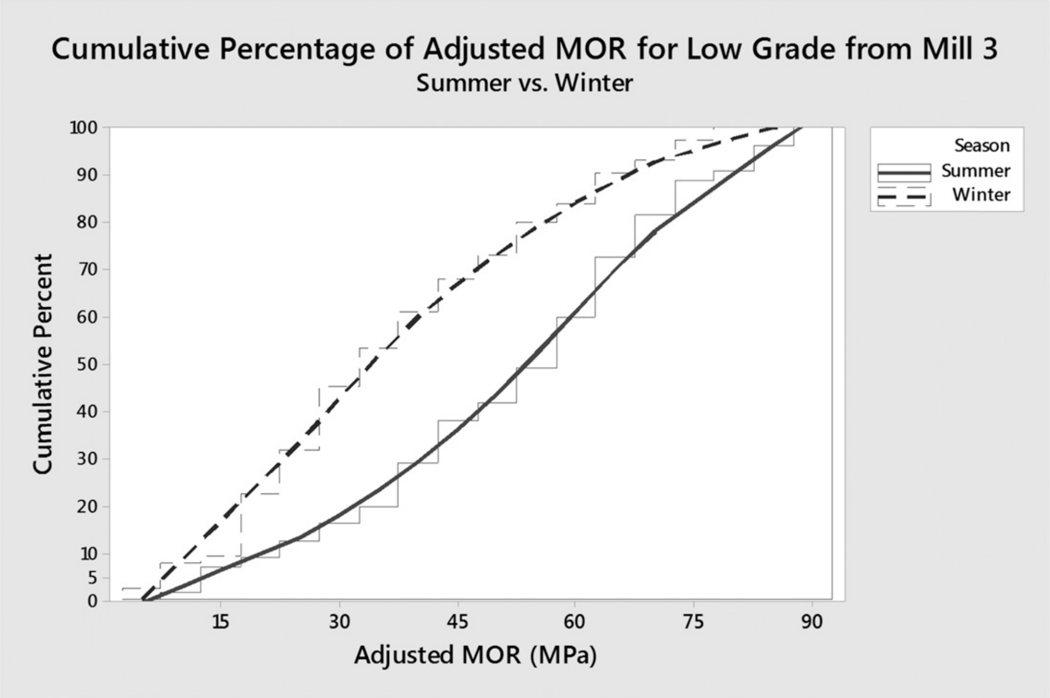

Figure 11. Cumulative percentage (summer vs winter) of adjusted MOR for the portion of the mill-run population from Mill 3 that graded below No. 3 (low grade).

four Mississippi sawmills: one in the summer and one in the winter. For each mill, the summer and winter means and variances of flexural MOR and MOE were compared. Whereas no significant differences were found between the mean MOR or mean MOE of the summer and winter samples from Mills 2 and 4, significant differences in mean MOE and/or MOR were found between the summer and winter samples from Mills 1 and 3. In addition, a Levene's test on the MOR of Mill 1 showed significant differences in the variance between the summer and winter samples. Further analysis revealed that in addition to the fact that the winter mill-run sample from Mill 3 was made up of a larger percentage of lower grade material than the summer sample, there were pronounced strength differences between the summer and winter samples both around the median and at the lowest (near-minimum) percentiles within each grade. This reinforces the reality that changes in mill-run MOR distributions over time can have an important effect on the overall strength of a mill's visual grades over time. A theory of mixed distributions could account for these differences.

ACKNOWLEDGMENTS

This research was made possible by funding from the US Department of Agriculture Forest Service Forest Products Laboratory under Agreement No. 17-JV-11111133-035. Any opinions, findings, conclusion, or recommendations expressed in this publication are those of the author(s) and do not necessarily reflect the view of the US Department of Agriculture. The authors wish to acknowledge and thank the Southern Pine Inspection Bureau (SPIB) for their gracious contributions to this research and Steve Verrill from the USDA Forest Products Laboratory for his valuable feedback throughout.

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