DISTRIBUTIONS OF MODULUS OF ELASTICITY AND MODULUS OF RUPTURE IN FOUR MILL-RUN LUMBER POPULATIONS

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(Received December 2018)

Abstract. The modulus of elasticity (MOE) and modulus of rupture (MOR) of graded lumber populations are commonly modeled by normal, lognormal, or Weibull distributions, but recent research has cast doubt on the appropriateness of these models. Such modeling has implications for ultimate performance and efficiency of resource use. It has been shown mathematically that the distribution of MOR in a graded subpopulation does not have the same theoretical form as the full, ungraded (or "mill-run") population from which it was drawn; rather, its form is pseudo-truncated, exhibiting thinned tails. Although the phenomenon of pseudo-truncation in graded populations has been well substantiated, the form of the underlying full distribution—an essential factor in characterizing the distribution of the graded population—remains unsettled. The objective of this study was to characterize the distributions of both MOE and MOR in four diverse mill-run lumber populations to determine if and to what extent the distributions of strength and stiffness in mill-run lumber are similar from mill to mill. The authors collected a mill-run sample of 200 southern pine 2 × 4 specimens from each of four sawmills, for a total of 800 test pieces. After measuring MOE and MOR, they fit candidate distributions to those data by mill and evaluated each distribution for goodness of fit. Results suggest that perhaps none of the traditional distributions of normal, lognormal, or Weibull is adequate to model MOE or MOR across all four mills; rather, MOE and MOR in full lumber populations might be better modeled by skew normal or mixed normal distributions.

Keywords: Full lumber population, mill run, modulus of elasticity, modulus of rupture, normal distribution, Weibull, pseudo-truncated.

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Wood and Fiber Science, 51(2), 2019, pp. 183-192
https://doi.org/10.22382/wfs-2019-019

INTRODUCTION

Modulus of elasticity (MOE) and modulus of rupture (MOR) in graded structural lumber populations are commonly modeled by normal, lognormal, or Weibull distributions (Green and Evans 1987; Evans et al 1997; ASTM 2017a, 2017b). Research by Verrill et al (2012, 2013, 2014, 2015) has cast doubt on the appropriateness of these models. They noted that the distributional form of MOR in a graded lumber subpopulation depends on the MOR distribution of the full (or "mill-run") lumber population from which it is drawn. They demonstrated mathematically that the distribution of MOR in a graded subpopulation does not have the same theoretical form as the distribution for the corresponding full, ungraded population; rather, the subpopulation form is pseudo-truncated, exhibiting thinned tails. (For example, if the full population were a two-parameter Weibull distribution, the graded subpopulation would be a pseudo-truncated Weibull.) They also presented empirical evidence (Verrill et al 2013, 2014, 2019) that ignoring this pseudo-truncation can yield reliability calculations that seriously over- or underestimate the probability of lumber failure in service.

Although Verrill et al (2012, 2013, 2014, 2015) predicted and observed the phenomenon of pseudo-truncation in graded subpopulations, the exact form(s) of the underlying full distribution(s)—an essential factor in characterizing the distributions of the graded subpopulations—remains unsettled. Using MOE as a grading variable, Verrill et al (2012, 2015) derived a pseudo-truncated Weibull distribution for the MOR of a graded lumber subpopulation under the assumption that the MOE distribution of a full lumber population is a Gaussian (ie normal) and the MOR distribution is a two-parameter Weibull. This assumption of a Gaussian–Weibull bivariate distribution was investigated by Verrill et al (2017) and Owens et al (2018) on 200 mill-run specimens of southern pine 2 × 4 lumber sampled from a single mill on a single day. Results of the goodness-of-fit tests of this pilot study failed to support the Gaussian–Weibull bivariate distribution hypothesis. Whereas the MOE data were well fit by a normal distribution, the MOR data were *not* well fit by a two-parameter Weibull distribution; rather, the MOR data were well fit by both a skew normal distribution and a mixed normal distribution.

Because a definitive conclusion cannot be drawn on the basis of a single sample, the authors expanded this investigation by adding three samplings of 200 pieces each from three new mills for a total of 800 specimens, including the original pilot sample of 200. If the distributional fits among the four mills are similar, it might be possible to identify a single appropriate form for the pseudo-truncated distributions of the graded subpopulations. For example, if MOE and MOR for the mill-run populations are well fit by normal and skew normal distributions, respectively, then it might be appropriate to assume an underlying Gaussian–skew normal bivariate distribution that yields a pseudo-truncated skew normal distribution on grading by MOE binning. If the fits differ, it could suggest that mill-run distributions differ from mill to mill, which would mean that the associated pseudo-truncated distributions for the corresponding graded lumber subpopulations would also differ.

The objective of this study was to characterize the distributions of both MOE and MOR in four separate mill-run lumber populations to determine if and to what extent the distributions of strength and stiffness in mill-run lumber are similar from mill to mill. If distributions of full lumber populations can be more appropriately characterized and ultimately generalized, it may be possible to derive better models for graded lumber strength distributions and, thus, improve and enhance efficiency, safety, and resource conservation.

MATERIALS AND METHODS

Sampling

A mill-run sample of 200 pieces of 2 × 4 southern pine (*Pinus* spp.) dimension lumber was acquired from each of four large Mississippi sawmills for a total of 800 specimens. Rough and kiln-dried, all specimens measured approximately 1.7 × 3.7 in. (4.32 × 9.40 cm). Their nominal length was 8 ft (244 cm), with roughly 2.5 cm of overlength.

The 200 specimens from the first mill (hereafter referred to as "Mill 1") were procured in the summer of 2016 for a pilot study. The sampling scheme is detailed in Owens et al (2018). Mill 1 produces 2 × 4 through 2 × 12 southern pine dimension lumber. For the purposes of this study, it was classified as a "full-complement mill" (ie a mill that produces a full range of sizes).

Three new samplings of 200 pieces were conducted in the summer of 2017, each from a different sawmill, for a total of 600 additional specimens. In an attempt to account for variability among manufacturers, effort was made to sample from mills with a range of production profiles (Table 1). The second mill (hereafter "Mill 2") primarily procures small-diameter round wood and manufactures a preponderance of 2 × 4 lumber. For the purposes of this study, it was classified as a "small log mill." Like the mill from the pilot study, the third mill (hereafter "Mill 3") produces 2 × 4 through 2 × 12 lumber. It too was classified as a "full-complement mill." The fourth mill (hereafter "Mill 4") purchases relatively large logs and saws few 2 × 4s. It was classified as a "large log mill."

At each sawmill, a kiln package was randomly selected based on the weekly kiln output. The top course of lumber was removed. Then, the subsequent 200 pieces were collected sequentially and designated as test specimens. Finally, the remainder of the kiln package, along with its top course, was returned to production. The test specimens were removed from the production line after kiln-drying, but before the planing and grading stations. All materials were of sufficient character to make it through the optimizing edger and trimmer without breaking. Subject only to this condition, the quality of the pieces was unrestricted. That is, the pieces were drawn from the full lumber population rather than from a single grade. Each sampling of 200 pieces constituted a mill-run lumber sample. Although the material was not pulled in accord with a random sampling scheme, we believe that the mechanical shuffling of lumber before the unscrambler and the kiln stacker randomized the pieces. It can be argued that the material from a given mill represents a random sample from several hours of a day's production from that mill.

The material was transported to Mississippi State University where it was planed on all four sides to final dressed dimensions of 1.5 × 3.5 in. (3.81 × 8.89 cm). Although the material was pulled from production and tested as mill-run lumber, the material was graded after planing by a Southern Pine Inspection Bureau-certified inspector to provide additional data for future analyses. A visual grade was recorded for each piece. Each board was labeled with a unique identification number and premarked to indicate the randomized positioning of the specimen within the third-point bending fixture used in destructive testing. First, the positioning of the 59.5-in. (151.13 cm) test span within the 8-ft long specimen was determined by a randomly generated number and marked on the top edge of each test piece. This action ensured random placement of the maximum bending moment along the length of each specimen. Then, the corresponding load head positions were marked. Finally, the lumber was stacked unwrapped outside on wooden saw horses under a covered breezeway to protect it from the elements, aid in moisture equalization, and minimize further drying that is often associated with interior storage.

Testing

The MOE and bending strength (MOR) of each specimen were determined for subsequent

Table 1. Production profiles of each sawmill sampled.

	Typical log size	Primary lumber dimensions
Mill 1 (pilot mill)	Full range (small to large)	2 × 4 through 2 × 12
Mill 2	Small diameter	2 × 4
Mill 3	Full range (small to large)	2 × 4 through 2 × 12
Mill 4	Large diameter	Wide dimensions (few 2 × 4)

analysis. Each specimen was subjected to both nondestructive evaluation and a static bending test. The nondestructive testing devices used were fibre-gen's Director HM200[1] (hereafter Director) and Metriguard's E-computer Model 340[2] (hereafter E-computer).

The Director is a handheld device that estimates MOE by measuring the acoustic velocity (in feet per second or meters per second) of a longitudinal stress wave traveling through a specimen. For the Director test, each specimen was supported in a flatwise orientation by two sawhorses, thereby allowing approximately 30 cm of specimen overhang on each end. The device's sensor was held against one end of the specimen while a tap was administered to the same end with a hammer. The device generated an acoustic velocity output in feet per second (subsequently converted to meters per second) from which a dynamic MOE value in pascals was calculated with the following Eq 1, where E is the elasticity, ρ is the density, and V is the acoustic velocity (Ross 2015). The final value was converted to gigapascals and recorded for subsequent analysis.

$$E = \rho V^2 \tag{1}$$

The E-computer device estimates MOE by measuring the transverse vibration of each piece. For the E-computer test, each specimen was supported near its ends by two metal tripods. One tripod is topped with a transducer connected by a cord to a laptop computer. The transducer measures the transverse vibration of the test piece. All pieces were tested in a flatwise orientation. The ends were aligned with the tops of each tripod allowing for a 2.5-cm overhang at each end. The tripod measured and recorded the weight. Oscillation was initiated by lightly tapping each specimen near its mid-length. The transducer sensed the vibration, and the laptop generated a dynamic MOE output in million pounds per square inch (subsequently converted to gigapascals). The software calculates the elasticity value based on the following Eq 2, where E is the MOE, f is the frequency of the specimen's vibration, W is the weight of the specimen, S is the span, C is a constant, I is the moment of inertia, and g is the acceleration due to gravity (Ross 2015).

$$E = \left(f^2 W S^3\right) / (CIg) \tag{2}$$

Each static bending test was performed on an Instron universal testing machine as per the flexure test method under ASTM D 198-15 (ASTM 2015) (Fig 1). The specimens were loaded in an edgewise orientation. Although third-point loading and a span-to-depth ratio of 17:1 were used (59.5 in., or 151.13 cm) (Fig 2), the test pieces were not trimmed to this length. Instead, the specimens were placed in the fixture such that the randomly determined span boundaries and corresponding load head placement markers lined up with the reaction supports and load heads, respectively. Whatever overhang there was on either end was allowed to remain as per the ASTM guidance. The MC of each piece at the time of testing was measured from its face approximately halfway between the load head markings with a Wagner L601-3[3] handheld moisture meter. Before the zeroing of the extensometer (used to measure deflection), each specimen was loaded with approximately 222.4 N (50 lbs.) to ensure proper placement and seating of the load heads. The test was then applied until full rupture. The average length of time until rupture was approximately 5 min.

One lumber specimen from one of the mills had been sawn with a large knot occupying almost the entire width of the test piece at the midpoint. The specimen broke during handling and could not be tested. The piece was eliminated from the analysis by listwise deletion.

The testing resulted in four datasets for each mill: MOE values from the static bending test (hereafter "static MOE") in gigapascals, MOR values from the static bending test in megapascals, MOE values from the Director test (hereafter "Director

[1] Fibre-gen Limited, Christchurch, New Zealand. www.fibre-gen.com
[2] Metriguard Inc., Pullman, Washington, USA. www.metriguard.com
[3] Wagner Electronic Products Inc., Rogue River, Oregon, USA. www.wagnermeters.com

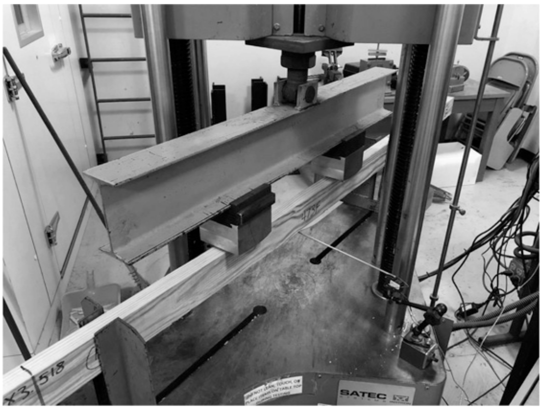

Figure 1. Static bending test setup as per ASTM D198-15.

E") in gigapascals, and MOE values from the E-computer test (hereafter "E-computer E") in gigapascals. Before analysis, all MOR and MOE values for all datasets were adjusted as per ASTM D 1990-16 (ASTM 2016) to make them comparable at a common 15% MC. The average MC before adjustment was 12.8% (SD 1.56).

Statistical Methods

Distributions were fit to each of the four datasets and evaluated for goodness of fit. Candidate distributions were selected based on previous research. Normal, lognormal, two-parameter Weibull, and three-parameter Weibull appear widely in the literature (Galligan et al 1986; Green and Evans 1987; Evans et al 1997; ASTM 2017a, 2017b). Skew normal and mixed normal distributions showed good fit in a previous study by the current authors (Verrill et al 2017; Owens

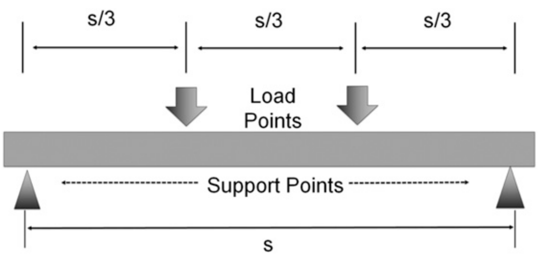

Figure 2. Third-point loading with a span-to-depth ratio of 17:1.

et al 2018). The probability density functions of the distributions are provided in the Appendix. The normal and lognormal fits and the Shapiro–Wilk tests of normality (and lognormality) reported in this article were performed in the R programming environment (R Core Team 2013) and made use of the nortest package (Gross and Ligges 2015). The maximum likelihood fits and goodness-of-fit tests for the other four distributions were performed primarily via Fortran programs written by the authors. (See http://www1.fpl.fs.fed.us/4mills.html for listings of these programs.) The Cramér–von Mises and Anderson–Darling goodness-of-fit tests for lognormal and normal distributions were based on Section 4.8 of D'Agostino and Stephens (1986). The Cramér–von Mises and Anderson–Darling goodness-of-fit tests for two-parameter Weibull distributions were based on Sections 4.10 and 4.11 of D'Agostino and Stephens (1986). The simulation-based Cramér–von Mises goodness-of-fit p-values for the skew normal, mixed normal, and three-parameter Weibull distributions were obtained via a "parametric bootstrap" (a particular type of computer simulation). For the two-parameter Weibull goodness-of-fit tests, we also made use of the EWGoF package (Krit 2017) in the R programming environment (R Core Team 2013).

RESULTS

Probability plots and histograms for the 96 cases (4 mills × 4 variables × 6 distributions) can be found at http://www1.fpl.fs.fed.us/4mills.plots.html. The results of the goodness-of-fit tests for Mills 1-4 appear in Tables 2-5, respectively. Tables 2-5 are further summarized in Table 6. For each of the four properties (static MOE, E-computer E, Director E, and MOR), Table 6 presents the number of mills for which a distribution was rejected by a goodness-of-fit test at a 0.05 significance level. This number can range from 0 to 4. Low numbers for a distribution suggest that it might be a good model for both stiffness and strength at multiple mills. The Table 6 results indicate that skew normal and mixed normal models perform relatively well, and, eg the two-parameter Weibull model does not.

Table 2. Goodness-of-fit p-values for Mill 1 (full-complement pilot mill).

Property	GOF test	Distribution					
		Normal	Lognormal	Two-par Weibull	Three-par Weibull	Skew normal	Mixed normal
Static MOE	Shapiro–Wilk	**0.371**	<0.001	—	—	—	—
	Cramér–von Mises	**0.054**	0.040	0.002	—	—	—
	Anderson–Darling	**0.095**	0.015	0.006	—	—	—
	CVM simulation[a]	—	—	—	0.017	**0.129**	**0.104**
E-computer E	Shapiro–Wilk	**0.318**	<0.001	—	—	—	—
	Cramér–von Mises	**0.252**	0.018	0.006	—	—	—
	Anderson–Darling	**0.202**	0.011	0.003	—	—	—
	CVM simulation[a]	—	—	—	0.040	**0.481**	**0.498**
Director E	Shapiro–Wilk	**0.185**	<0.001	—	—	—	—
	Cramér–von Mises	0.011	0.007	<0.001	—	—	—
	Anderson–Darling	0.024	0.004	<0.001	—	—	—
	CVM simulation[a]	—	—	—	0.001	0.020	**0.506**
MOR	Shapiro–Wilk	0.001	<0.001	—	—	—	—
	Cramér–von Mises	<0.001	<0.001	<0.001	—	—	—
	Anderson–Darling	<0.001	<0.001	<0.001	—	—	—
	CVM simulation[a]	—	—	—	0.001	**0.277**	**0.584**

[a] In cases where critical values for the Cramér–von Mises test were not available in D'Agostino and Stephens (1986), they were determined by simulation.
$N = 200$. Bold values indicate that a test failed to reject a distribution at a 0.05 significance level. GOF, goodness of fit; par, parameter; E-computer E, dynamic MOE as tested with the E-computer device; Director E, dynamic MOE as tested with the Director HM200 device; CVM, Cramér–von Mises; "—," the test was not performed.

DISCUSSION

For all three measures of elasticity (static MOE, E-computer E, and Director E), the normal distribution yielded poor fits to the mill-run data for mills 2 through 4. Thus, these datasets suggest that an assumption of a Gaussian (normal) mill-run MOE distribution is not justified. If one were to rank the distribution models from best to worst based on the number of mills for which they provided a reasonable fit (Table 6), they might be ordered as skew normal and mixed normal (a tie); lognormal; three-parameter Weibull; normal; and two-parameter Weibull.

For MOR, the two-parameter Weibull distribution seemed a poor fit for the mill-run data for

Table 3. Goodness-of-fit p-values for Mill 2 (small log mill).

Property	GOF test	Distribution					
		Normal	Lognormal	Two-par Weibull	Three-par Weibull	Skew normal	Mixed normal
Static MOE	Shapiro–Wilk	<0.001	**0.350**	—	—	—	—
	Cramér–von Mises	<0.001	**0.328**	<0.001	—	—	—
	Anderson–Darling	<0.001	**0.357**	<0.001	—	—	—
	CVM simulation[a]	—	—	—	0.018	**0.167**	**0.920**
E-computer E	Shapiro–Wilk	<0.001	**0.093**	—	—	—	—
	Cramér–von Mises	<0.001	**0.250**	<0.001	—	—	—
	Anderson–Darling	<0.001	**0.196**	<0.001	—	—	—
	CVM simulation[a]	—	—	—	0.030	**0.098**	**0.436**
Director E	Shapiro–Wilk	<0.001	0.013	—	—	—	—
	Cramér–von Mises	<0.001	0.043	<0.001	—	—	—
	Anderson–Darling	<0.001	0.028	<0.001	—	—	—
	CVM simulation[a]	—	—	—	0.029	**0.087**	**0.245**
MOR	Shapiro–Wilk	<0.001	**0.146**	—	—	—	—
	Cramér–von Mises	<0.001	**0.311**	0.001	—	—	—
	Anderson–Darling	<0.001	**0.273**	0.001	—	—	—
	CVM simulation[a]	—	—	—	**0.108**	**0.567**	**0.598**

[a] In cases where critical values for the Cramér–von Mises test were not available in D'Agostino and Stephens (1986), they were determined by simulation.
$N = 199$. Bold values indicate that a test failed to reject a distribution at a 0.05 significance level. GOF, goodness of fit; par, parameter; E-computer E, dynamic MOE as tested with the E-computer device; Director E, dynamic MOE as tested with the Director HM200 device; CVM, Cramér–von Mises; "—," the test was not performed.

Table 4. Goodness-of-fit *p*-values for Mill 3 (full-complement mill).

		Distribution					
Property	GOF test	Normal	Lognormal	Two-par Weibull	Three-par Weibull	Skew normal	Mixed normal
Static MOE	Shapiro–Wilk	<0.001	**0.392**	—	—	—	—
	Cramér–von Mises	<0.001	**0.365**	<0.001	—	—	—
	Anderson–Darling	<0.001	**0.463**	<0.001	—	—	—
	CVM simulation[a]	—	—	—	**0.080**	**0.181**	**0.088**
E-computer E	Shapiro–Wilk	<0.001	**0.724**	—	—	—	—
	Cramér–von Mises	0.008	**0.997**	<0.001	—	—	—
	Anderson–Darling	0.002	**0.976**	<0.001	—	—	—
	CVM simulation[a]	—	—	—	**0.469**	**0.942**	**0.764**
Director E	Shapiro–Wilk	<0.001	**0.597**	—	—	—	—
	Cramér–von Mises	<0.001	**0.727**	<0.001	—	—	—
	Anderson–Darling	<0.001	**0.723**	<0.001	—	—	—
	CVM simulation[a]	—	—	—	**0.111**	**0.563**	**0.404**
MOR	Shapiro–Wilk	**0.104**	<0.001	—	—	—	—
	Cramér–von Mises	**0.737**	<0.001	**0.784**	—	—	—
	Anderson–Darling	**0.423**	<0.001	**0.770**	—	—	—
	CVM simulation[a]	—	—	—	**0.405**	**0.332**	**0.521**

[a] In cases where critical values for the Cramér–von Mises test were not available in D'Agostino and Stephens (1986), they were determined by simulation.
$N = 200$. Bold values indicate that a test failed to reject a distribution at a 0.05 significance level. GOF, goodness of fit; par, parameter; E-computer E, dynamic MOE as tested with the E-computer device; Director E, dynamic MOE as tested with the Director HM200 device; CVM, Cramér–von Mises; "—," the test was not performed.

all but one mill. At least in the case of these datasets, it seems that the assumption that mill-run MOR is distributed as a two-parameter Weibull distribution is not justified. If one were to rank the distribution models from best to worst based on the number of mills for which they seemed a reasonable fit (Table 6), they might be ordered as skew normal; mixed normal; normal and three-parameter Weibull (a tie); and lognormal and two-parameter Weibull (a tie).

These results suggest that bivariate mill-run MOE-MOR distributions are not Gaussian–Weibulls, and, thus, the MOR distributions of graded subpopulations are not pseudo-truncated Weibulls. Instead, given the results from this

Table 5. Goodness-of-fit *p*-values for Mill 4 (large log mill).

		Distribution					
Property	GOF test	Normal	Lognormal	Two-par Weibull	Three-par Weibull	Skew normal	Mixed normal
Static MOE	Shapiro–Wilk	0.024	**0.217**	—	—	—	—
	Cramér–von Mises	0.020	**0.340**	0.002	—	—	—
	Anderson–Darling	0.023	**0.419**	0.002	—	—	—
	CVM simulation[a]	—	—	—	**0.094**	**0.314**	**0.075**
E-computer E	Shapiro–Wilk	<0.001	**0.777**	—	—	—	—
	Cramér–von Mises	0.003	**0.846**	<0.001	—	—	—
	Anderson–Darling	0.001	**0.706**	<0.001	—	—	—
	CVM simulation[a]	—	—	—	**0.272**	**0.766**	**0.306**
Director E	Shapiro–Wilk	<0.001	**0.621**	—	—	—	—
	Cramér–von Mises	0.006	**0.551**	<0.001	—	—	—
	Anderson–Darling	0.002	**0.420**	<0.001	—	—	—
	CVM simulation[a]	—	—	—	**0.382**	**0.339**	0.040
MOR	Shapiro–Wilk	**0.064**	<0.001	—	—	—	—
	Cramér–von Mises	**0.197**	<0.001	0.019	—	—	—
	Anderson–Darling	**0.144**	<0.001	0.030	—	—	—
	CVM simulation[a]	—	—	—	0.013	**0.096**	0.004

[a] In cases where critical values for the Cramér–von Mises test were not available in D'Agostino and Stephens (1986), they were determined by simulation.
$N = 200$. Bold values indicate that a test failed to reject a distribution at a 0.05 significance level. GOF, goodness of fit; par, parameter; E-computer E, dynamic MOE as tested with the E-computer device; Director E, dynamic MOE as tested with the Director HM200 device; CVM, Cramér–von Mises; "—," the test was not performed.

Table 6. Goodness-of-fit test summary score card.

Property	Two-par Weibull	Normal	Three-par Weibull	Lognormal	Mixed normal	Skew normal
Static MOE	4	3	2	1	0	0
E-computer E	4	3	2	1	0	0
Director E	4	4	2	2	1	1
MOR	3	2	2	3	1	0

The numbers in the table indicate the number of mills for which a distribution was rejected by a goodness-of-fit test at a 0.05 significance level for each of the four properties (static MOE, E-computer E, Director E, and MOR). These numbers can range from 0 to 4. MOE, modulus of elasticity; MOR, modulus of rupture; par, parameter; E-computer E, dynamic MOE as tested with E-computer device; Director E, dynamic MOE as tested with Director HM200 device.

mill-run study of four mills, one might speculate that the underlying mill-run MOE and MOR distributions are skew normal or mixed normal, and that the derived MOR distributions for graded lumber subpopulations might be pseudo-truncated skew normal or pseudo-truncated mixed normal.

The authors note, however, that skew normal distributions are theoretically constrained to have skewnesses that lie between −1 and 1. In 3 of the 16 cases in this study (4 mills × 4 variables), sample skewnesses were larger than 1. In 4 of the 16 cases in this study (3 stiffness, 1 MOR), the *p*-value of the skew normal goodness-of-fit test was less than 0.10. In 4 of the 16 cases in this study (3 stiffness, 1 MOR), the *p*-value of the mixed-normal goodness-of-fit test was less than 0.10. Thus, one cannot be fully confident that skew normal or mixed normal distributions are always appropriate. The authors note that Verrill et al (2018) conducted detailed analyses of the Mill 1 (pilot mill) data that strongly suggest that the mill-run stiffness–MOR data sets can be well modeled as mixtures of bivariate normal distributions which would be in accord with separate univariate mixed normal models for mill-run stiffness and MOR distributions.

The authors are currently engaged in analyzing new data from the same four mills. These data were taken in the "winter" rather than the "summer" so that the stability of stiffness and strength distributions over time could be investigated. Analyses of these new data sets should help researchers gain (or lose) additional confidence in pseudo-truncated skew normal and pseudo-truncated mixed normal models for the MOR distributions of grades of lumber.

CONCLUSION

The objective of this study was to investigate the distributions of both MOE and MOR in four diverse mill-run lumber populations to determine if and to what extent the distributions of MOR and MOE in mill-run lumber are similar from mill to mill. The authors collected a mill-run sample of 200 southern pine 2 × 4 specimens from each of four sawmills, for a total of 800 test pieces. After measuring MOR and MOE, they fit candidate distributions to those data by mill and evaluated each distribution for goodness of fit. Results suggest that perhaps none of the traditional distributions of normal, lognormal, or Weibull is adequate to model mill-run MOE or MOR across all four mills; rather, MOE and MOR in full lumber populations might be better modeled by skew normal or mixed normal distributions. We are currently analyzing additional data to see whether these models are stable over time.

ACKNOWLEDGMENTS

This research was made possible by funding from the US Department of Agriculture Forest Service Forest Products Laboratory under Agreement No. 17-JV-11111133-035. Any opinions, findings, conclusion, or recommendations expressed in this publication are those of the author(s) and do not necessarily reflect the view of the US Department of Agriculture. The authors wish to acknowledge and thank the Southern Pine Inspection Bureau for their gracious contributions to this research.

REFERENCES

ASTM (2015) Standard test methods of static tests of lumber in structural sizes. D198-15. Annual book of ASTM standards. ASTM, West Conshohocken, PA.

ASTM (2016) Standard practice for establishing allowable properties for visually-graded dimension lumber from in-grade tests of full-size specimens. D1990-16. Annual book of ASTM standards. ASTM, West Conshohocken, PA.

ASTM (2017a) Standard practice for sampling and data-analysis for structural wood and wood-based products.

D2915-17. Annual book of ASTM standards. ASTM, West Conshohocken, PA.

ASTM (2017b) Standard specification for computing reference resistance of wood-based materials and structural connections for load and resistance factor design. D5457-17. Annual book of ASTM standards. ASTM, West Conshohocken, PA.

D'Agostino RB, Stephens MA (1986) Goodness-of-fit techniques. Marcel Dekker, New York, NY.

Evans JW, Johnson RA, Green DW (1997) Goodness-of-fit tests for two-parameter and three-parameter Weibull distributions. Pages 159-178 *in* NL Johnson and N Balakrishnan, eds. Advances in the theory and practice of statistics: A volume in honor of Samuel Kotz. Wiley, New York, NY.

Galligan WL, Hoyle RJ, Pellerin RF, Haskell JH, Taylor JR (1986) Characterizing the properties of 2-inch softwood dimension lumber with regressions and probability distributions: Project completion report. U.S. Department of Agriculture, Forest Service, Forest Products Laboratory, Madison, WI.

Green DW, Evans JW (1987) Mechanical properties of visually graded lumber: Volumes 1-8. U.S. Department of Commerce, National Technical Information Service, Springfield, VA.

Gross J, Ligges U (2015) nortest: Tests for normality. R package version 1.0-4. https://CRAN.R-project.org/package=nortest. (31 May 2018).

Krit M (2017) EWGoF: Goodness-of-fit tests for the exponential and two-parameter Weibull distributions. R package version 2.2.1. https://CRAN.R-project.org/package=EWGoF. (31 May 2018).

Owens FC, Verrill SP, Kretschmann DE, Shmulsky R (2018) Distributions of MOE and MOR in a full lumber population. Wood Fiber Sci 50(3):265-279.

R Core Team (2013) R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. http://www.R-project.org (31 May 2018).

Ross RJ, (ed.) (2015) Nondestructive evaluation of wood: Second edition. General Technical Report FPL-GTR-238. U.S. Department of Agriculture, Forest Service, Forest Products Laboratory, Madison, WI. 169 pp.

Verrill SP, Evans JW, Kretschmann DE, Hatfield CA (2012) Asymptotically efficient estimation of a bivariate Gaussian-Weibull distribution and an introduction to the associated pseudo-truncated Weibull. U.S. Department of Agriculture, Forest Service, Forest Products Laboratory Research Paper FPL-RP-666, Madison, WI. 76 pp.

Verrill SP, Evans JW, Kretschmann DE, Hatfield CA (2013) An evaluation of a proposed revision of the ASTM D 1990 grouping procedure. U.S. Department of Agriculture, Forest Service, Forest Products Laboratory Research Paper FPL-RP-671, Madison, WI. 34 pp.

Verrill SP, Evans JW, Kretschmann DE, Hatfield CA (2014) Reliability implications in wood systems of a bivariate Gaussian-Weibull distribution and the associated univariate pseudo-truncated Weibull. ASTM J Test Eval 42(2):412-419.

Verrill SP, Evans JW, Kretschmann DE, Hatfield CA (2015) Asymptotically efficient estimation of a bivariate Gaussian-Weibull distribution and an introduction to the associated pseudo-truncated Weibull. Commun Stat Theor Methods 44:2957-2975.

Verrill SP, Owens FC, Kretschmann DE, Shmulsky R (2017) Statistical models for the distribution of modulus of elasticity and modulus of rupture in lumber with implications for reliability calculations. U.S. Department of Agriculture, Forest Service, Forest Products Laboratory Research Paper FPL-RP-692, Madison, WI. 51 pp.

Verrill SP, Owens FC, Kretschmann DE, Shmulsky R (2018) A fit of a mixture of bivariate normals to lumber stiffness-strength data. U.S. Department of Agriculture, Forest Service, Forest Products Laboratory Research Paper FPL-RP-696, Madison, WI. 44 pp.

Verrill SP, Owens FC, Kretschmann DE, Shmulsky R, Brown L (2019) Visual and MSR grades of lumber are not two-parameter Weibulls and why it matters (with a discussion of censored data fitting). Under review. http://www1.fpl.fs.fed.us/weib2.new.pdf (12 November 2018).

APPENDIX—PROBABILITY DENSITY FUNCTIONS

NORMAL DISTRIBUTION

The normal probability density function is given by

$$f(x;\ \mu,\sigma)=\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left(-(x-\mu)^2\Big/\left(2\sigma^2\right)\right)$$

for $x \in (-\infty, \infty)$, where μ is the mean and σ is the standard deviation. This distribution is denoted by the notation $N(\mu, \sigma^2)$.

LOGNORMAL DISTRIBUTION

The lognormal probability density function is given by

$$f(x;\ \mu,\sigma) =\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\frac{1}{x}\exp\left(-(\log(x)-\mu)^2\Big/\left(2\sigma^2\right)\right)$$

for $x \in (0, \infty)$, where μ is the mean and σ is the standard deviation of the log of the original data.

SKEW NORMAL

The skew normal distribution has a probability density function

$$f(x;\xi,\omega,\alpha)=\frac{2}{\omega}\times\phi\left(\frac{x-\xi}{\omega}\right)\times\Phi\left(\alpha\left(\frac{x-\xi}{\omega}\right)\right)$$

for $x \in (-\infty,\infty)$, where φ denotes the probability density function of a standardized normal; Φ denotes the cumulative distribution function of a standardized normal; and ξ, ω, and α are the parameters of the skew normal.

MIXED NORMAL

In this article, a "mixed normal distribution" refers to a mixture of *two* normal distributions. Such a mixture results when specimens are drawn with probability p from an N (μ_1, σ_1^2) distribution and with probability 1-p from an N (μ_2, σ_2^2) distribution. In this case, the probability density function is given by

$$f(x;\ \mu_1, \sigma_1, p,\ \mu_2, \sigma_2) = p \times \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma_1}\exp\left(-(x-\mu_1)^2 \big/ \left(2\sigma_1^2\right)\right) + (1-p) \times \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma_2}\exp\left(-(x-\mu_2)^2 \big/ \left(2\sigma_2^2\right)\right)$$

for $x \in (-\infty, \infty)$.

TWO-PARAMETER WEIBULL

The two-parameter Weibull has a probability density function

$$f(w; \gamma, \beta) = \gamma^\beta \beta w^{\beta-1} \exp\left(-(\gamma w)^\beta\right)$$

for $w \in [0, \infty)$, where β is the shape parameter and γ is the inverse of the scale parameter.

THREE-PARAMETER WEIBULL

The three-parameter Weibull has a probability density function

$$f(w; \gamma, \beta, c) = \gamma^\beta \beta (w-c)^{\beta-1} \exp\left(-(\gamma(w-c))^\beta\right)$$

for $w \in [c, \infty)$, where β is the shape parameter, γ is the inverse of the scale parameter, and c is the location parameter.