DISTRIBUTIONS OF MOE AND MOR IN A FULL LUMBER POPULATION

Frank C. Owens†

Assistant Research Professor
Department of Sustainable Bioproducts
Mississippi State University
Mississippi State, MS 39762-5724
E-mail: fco7@msstate.edu

Steve P. Verrill†

Mathematical Statistician
USDA Forest Products Laboratory
Madison, WI 53726
E-mail: sverrill@fs.fed.us

Rubin Shmulsky†

Professor and Department Head
Department of Sustainable Bioproducts
Mississippi State University
Mississippi State, MS 39762-5724
E-mail: rs26@msstate.edu

David E. Kretschmann†

Research General Engineer (Retired)
USDA Forest Products Laboratory
Madison, WI 53726
E-mail: dkretschmann@alsc.org

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Abstract. Reliability calculations for lumber products ultimately depend on the statistical distributions that we use to model lumber stiffness and strength. Fits of statistical distributions to empirical data allow researchers to estimate the probability of failure in service. For these fits to be useful, the theoretical statistical distributions must be good matches for the empirical lumber property populations. It has been common practice to assume that the MOE of a grade of lumber is well-fit by a normal distribution, and the MOR of a grade of lumber is well-fit by a normal, lognormal, or two-parameter Weibull distribution. Recent theoretical results and empirical tests have cast significant doubt on these assumptions. The exact implications of the theoretical results depend on the distributions of full (mill-run) MOE and MOR populations. Mill-run data have not yet appeared in the literature. Instead, studies have focused on subpopulations formed by visual or machine stress rated (MSR) grades of lumber. To better understand the implications of the recent theoretical results, we have investigated the statistical distributions of mill-run MOE and MOR data. An ungraded mill-run sample of 200 southern pine 2 × 4 s produced at a single mill on a single day was subjected to both nondestructive (transverse vibration and longitudinal stress wave) evaluation and static bending tests to determine its MOE and MOR values. Various distributions were fit to the MOE and MOR data and evaluated for goodness-of-fit. The results suggest that mill-run MOE might be adequately modeled by a normal distribution or a mixture of two normal distributions, mill-run MOR might be adequately modeled by a skew normal distribution or a mixture of two normals, and neither mill-run MOE nor mill-run MOR is well-fit by a Weibull distribution.

Keywords: Mill-run lumber, Weibull distribution, normal distribution, modulus of elasticity, modulus of rupture.

* Corresponding author
† SWST member

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INTRODUCTION

Reliability calculations for lumber products ultimately depend on the statistical distributions that we use to model lumber stiffness and strength. Fits of statistical distributions to empirical data allow researchers to estimate the probability of failure in service. For these fits to be useful for reliability-based design, the theoretical statistical distributions must be good matches for the empirical lumber property populations.

Although it has been common practice to assume that the MOE of a grade of lumber is well-modeled by a normal distribution, and the MOR of a grade of lumber is well-modeled by a normal, lognormal, or two-parameter Weibull distribution (Green and Evans 1987; Evans et al 1997; ASTM 2010, 2015), recent theoretical results and empirical tests have cast considerable doubt on these assumptions.

The formation of a lumber grade begins with a full lumber population that comprises not only in-grade pieces but also every other piece of lumber that is produced when logs are sawn. (Colloquially, one might refer to this as a "mill-run" population.) Verrill et al (2012, 2015) argue that lumber grade subpopulations are formed by selecting a subset of the full population based on a grading (or predictor) variable that is positively correlated with MOR. For example, in the case of machine grading, a subpopulation can be chosen based on a range of stiffness values. (This statement is somewhat of an oversimplification, as MSR grading also involves visual grade and additional edge knot restrictions.) In the case of visual grading, the predictor variable is implicit. Following a particular set of grading rules, a human grader classifies the lumber into quality categories based on the presence or absence of certain strength-reducing characteristics, appearance attributes, and features related to fitness-for-use (eg a sufficient nailing edge).

Verrill et al (2012, 2015) demonstrate with mathematical proofs that the MOR population associated with a specific grade of lumber will not have the same theoretical form as the full (mill-run) MOR population from which the grade's subpopulation is drawn. Instead, it will be pseudo-truncated, exhibiting thinned or tightened tails (Verrill et al 2012, 2015). They note that such tightened tails can be observed in probability plots of in-grade data (Verrill et al 2013, 2014). They also demonstrate with computer simulations that if one fits a non-pseudo-truncated distribution (eg a Weibull distribution) to pseudo-truncated data (eg pseudo-truncated Weibull data), probability estimates can be seriously in error (Verrill et al 2013, 2014). Thus, there is strong theoretical and empirical evidence that the MOR distributions of grades of lumber are not Weibulls and that this matters.

Verrill et al (2012, 2015) calculated the form of pseudo-truncated MOR grade data under the assumptions that the generating MOE-MOR mill-run data have a bivariate Gaussian–Weibull distribution (so the mill-run MOE distribution is a normal and the mill-run MOR distribution is a Weibull), and pieces of lumber are placed in grades based on hard limits on MOE values or, as in the case of visual grades, an implicit strength predictor. Such a process yields MOR grade distributions that are pseudo-truncated Weibulls. But what if the MOE mill-run data are not normal, or the MOR mill-run data are not Weibull? MOR grade data will still have thinned tails but the probability density function calculated by Verrill et al (2012, 2015) will not be exactly right. Thus, before strength distribution calculations can be precisely made for lumber graded by MOE values, we need to investigate the actual univariate distributions of mill-run MOE and mill-run MOR populations and their joint (bivariate) distribution.

The empirical investigations described in this article were designed to take a preliminary look at these distributions. Here, we restrict ourselves to the population of lumber produced at a single mill on a single day. We realize that populations of lumber produced over multiple days at multiple mills in multiple regions are almost certain to have a more complicated structure. However, we felt that it made sense to begin by addressing the fundamental question of whether a "simple" mill-run population could be modeled by a "simple" distribution such as a normal or two-parameter Weibull. Thus, in this article, we report the MOE-MOR data and distribution fits for a sample of 200 pieces of mill-run 2 × 4 lumber obtained from a single mill on a single day.

MATERIALS AND METHODS

Sampling

Two hundred kiln-dried, rough southern yellow pine (*Pinus* spp.) 2 × 4 s were procured from a large regional sawmill in central Mississippi. The dimension mill that donated the lumber has a single line primary breakdown followed by a curve gang resaw. It produces 2 × 4 through 2 × 12 pine dimension lumber from its log supply. Its annual production is approximately 200 million board feet. The mill is optimized to a large degree throughout with the intention of maximizing board foot recovery from each log. The rough dry target dimensions for the mill were 1.7 × 3.7 inches (4.32 × 9.40 cm). The nominal length of the specimens was 8 feet (244 cm) with approximately 2.5 cm of overlength.

The mill managers (who were unaware of the objectives of the research) were asked to pull 200 pieces of 2 × 4 lumber as the material was removed from the kiln and taken off sticks. The material was removed from the production line after kiln-drying but before the planing and grading stations. All material was of sufficient character to make it through the optimizing edger and trimmer without breaking. Subject only to this condition, the quality of the pieces was unrestricted (the pieces were drawn from the full lumber population rather than from a single grade), and the resulting specimens constituted a full mill-run sample. Although the material was not pulled in accord with a random sampling scheme, it is believed that the mechanical shuffling of lumber before the unscrambler and the kiln stacker randomized the pieces. It can be argued that the material represents a random sample from several hours of a day's production.

The material was transported to Mississippi State University where it was planed on all four sides to final dressed dimensions of 1.5 × 3.5 inches (3.81 × 8.89 cm). Although the material was pulled from production and tested as mill-run lumber, the material was graded after planing at Mississippi State University by a Southern Pine Inspection Bureau–certified inspector to provide additional data for future analyses. A visual grade was recorded for each piece. Each board was labeled with a unique identification number and premarked to indicate the randomized positioning of the specimen within the third-point bending fixture used in destructive testing. First, the positioning of the 59.5-inch (151.13 cm) test span within the 8-foot long specimen was determined by a randomly generated number and marked on the top edge of each test piece. This action ensured random placement of the maximum bending moment along the length of each specimen. Then, the corresponding load head positions were marked. Finally, the lumber was stacked unwrapped outside on wooden sawhorses under a covered breezeway to protect it from the elements, aid in moisture equalization, and minimize further drying that is often associated with interior storage.

Testing

The elasticity (MOE) and strength (MOR) of each test sample was assessed for subsequent analysis. Each specimen was subjected to both nondestructive evaluation and a static bending test. The nondestructive testing devices used were Fiber-gen's Director HM200 (hereafter Director) and Metriguard's E-computer Model 340 (hereafter E-computer).

The Director is a handheld device that estimates MOE by measuring the acoustic velocity (in feet per second or meters per second) of a longitudinal stress wave traveling through a specimen. For the Director test, each specimen was supported in a flatwise orientation by two sawhorses, thereby allowing approximately 30 cm of specimen overhang on each end. The device's sensor was held against one end of the specimen while a tap was administered to the same end with a hammer. The device generated an acoustic velocity output in ft/s (subsequently converted to m/s) from which a dynamic MOE value in Pascals was calculated with Eq (1), where E is elasticity, ρ is density, and V is acoustic velocity (Ross and Pellerin 1994). The final value was converted to GPa and recorded for subsequent analysis.

$$E = \rho V^2 \tag{1}$$

The E-computer device estimates MOE by measuring the transverse vibration of each piece. For the E-computer test, each specimen was supported near its ends by two metal tripods. One tripod is topped with a transducer connected by a cord to a laptop computer. The transducer measures the transverse vibration of the test piece. All pieces were tested in a flatwise orientation. The ends were aligned with the tops of each tripod allowing for a 2.54 cm overhang at each end. The tripod measured and recorded the weight. Oscillation was initiated by lightly tapping each specimen near its midlength. The transducer sensed the vibration, and the laptop generated a dynamic MOE output in million psi (subsequently converted to GPa). The software calculates the elasticity value based on Eq (2), where E is MOE, f is the frequency of the specimen's vibration, W is the weight of the specimen, S is the span, C is a constant, I is the moment of inertia, and g is the acceleration due to gravity (Ross and Pellerin 1994).

$$E = \left(f^2\, W\, S^3\right) / (C\, I g) \tag{2}$$

Each static bending test was performed on an Instron universal testing machine per the flexure test method under ASTM D 198-15 (ASTM 2015) (Fig 1). The specimens were loaded in

Figure 1. A southern pine 2 × 4 undergoing a static bending test per ASTM D 198-15, flexure method.

an edgewise orientation. Although third-point loading and a span-to-depth ratio of 17:1 were used (59.5 in, or 151.13 cm), the test pieces were not trimmed to this length. Instead, the specimens were placed in the fixture such that the randomly-determined span boundaries and corresponding load head placement markers lined up with the reaction supports and load heads, respectively. Whatever overhang there was on either end was allowed to remain per ASTM guidance. The MC of each piece at the time of testing was measured from its face approximately halfway between the load head markings with a Delmhorst J-88 pin-type moisture meter to a depth of approximately 8 mm. Before the zeroing of the extensometer (used to measure deflection), each specimen was loaded with approximately 222.4 N (50 lbs.) to ensure proper placement and seating of the load heads. The test was then applied until full rupture. The average length of time until rupture was approximately 5 min.

The testing resulted in four datasets: MOE values from the static bending test (hereafter abbreviated "sb-MOE") in GPa, MOR values from the static bending test in MPa, MOE values from the Director test (hereafter abbreviated "Dir-E") in GPa, and MOE values from the E-computer test (hereafter abbreviated "Ecomp-E") in GPa. Before analysis, all MOR and MOE values for all datasets were adjusted per ASTM D 1990-16 (ASTM 2016) to make them comparable at a common 15% MC. The average MC before adjustment was 13.3% (SD 1.88).

Statistical Methods

To identify statistical distributions that yielded good models for stiffness and MOR in the mill-run lumber population sampled, various candidate distributions had to be fitted to each of the four datasets and subsequently evaluated. We fit four univariate models—two-parameter Weibull, three-parameter Weibull, normal, and mixed normal—to each of the stiffness measures. We fit six univariate models—two-parameter Weibull, three-parameter Weibull, normal, three-parameter beta, skew normal, and mixed normal—to the MOR values. The MOR dataset was left-skewed and this affected our choice of potential models. (The p-value for a [right-skewed] lognormal model was 7e-10.) We also fit one bivariate model, a mixture of bivariate normal distributions, to each of the stiffness-MOR pairs. An analysis of this bivariate model is provided in Verrill et al (2018). The probability density functions of the univariate distributions are provided in the Appendix. We followed the fits with formal tests of goodness-of-fit, and also created and studied diagnostic plots—probability plots and histograms overlaid with fitted probability density functions. Only a subset of these plots appears in the current article. A complete set can be found in Verrill et al (2017).

The normal fits were performed in the R programming environment (R Core Team 2013). The maximum likelihood fits for the other distributions were performed via Fortran programs written by the authors. The Cramér–von Mises (CVM) goodness-of-fit tests for normal distributions were performed via the nortest package of Gross and Ligges (2015) in the R programming environment. The CVM test for a two-parameter Weibull distribution is based on sections 4.10 and 4.11 of D'Agostino and Stephens (1986) and makes use of their Table 4.17 to calculate critical values for the test. The simulation-based Cramér–von Mises (CVM-sim) goodness-of-fit p-values were obtained via a "parametric bootstrap" (a particular type of computer simulation). The CVM test for a two-parameter Weibull, the CVM-sim goodness-of-fit tests, and the likelihood ratio tests were all performed via Fortran programs written by the authors. The source code for the Fortran programs written by the authors can be found at http://www1.fpl.fs.fed.us/mordist.html.

RESULTS

Parameter Estimates

Parameter estimates (represented by Greek letters except for R, c, and p) for the fitted distributions are shown in Table 1. The parameter p for the mixed normal distribution represents the

Table 1. Parameter estimates.

	Weibull, two-parameter		Weibull, three-parameter			Normal		Three-parameter beta			Skew normal		
	Sc.	Shape	Sc.	Shape	Loc.	Mean (Loc.)	SD (Sc.)	Shape	Shape	Scale	Loc.	Sc.	Shape
Data	$\hat{\lambda} = {}^{1}/_{\hat{\gamma}}$	$\hat{\beta}$	$\hat{\lambda} = {}^{1}/_{\hat{\gamma}}$	$\hat{\beta}$	$\hat{c}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{R}$	$\hat{\xi}$	$\hat{\omega}$	$\hat{\alpha}$
sb-MOE (GPa)	10.8	4.45	8.38	3.42	2.28	9.8	2.41	—	—	—	—	—	—
Ecomp-E (GPa)	12.2	4.74	9.15	3.51	2.94	11.2	2.55	—	—	—	—	—	—
Dir-E (GPa)	11.8	4.74	9.27	3.68	2.46	10.8	2.48	—	—	—	—	—	—
MOR(MPa)	59.8	3.80	59.9	3.80	0.00	54.1	16.4	4.35	3.91	102.0	72.4	24.4	−2.46

	Mixed normal				
	Loc.	Sc.	Prop. (p)	Loc.	Sc.
Data	$\hat{\mu}_1$	$\hat{\sigma}_1$	$\hat{p}$	$\hat{\mu}_2$	$\hat{\sigma}_2$
sb-MOE (GPa)	9.2	2.00	0.85	13.2	1.45
Ecomp-E (GPa)	10.5	1.38	0.23	11.4	2.76
Dir-E (GPa)	10.1	0.55	0.17	11.0	2.69
MOR(MPa)	48.0	17.9	0.59	62.9	8.34

All parameter values were estimated via maximum likelihood fits. The parameters are listed at the top of the table. The corresponding probability density functions are provided in the Appendix. The type of parameter (location, scale, or shape) is indicated above the parameter. SD, standard deviation; Loc., location; Sc., scale; Prop. (p), proportion of the left normal in the mixture. Subscript numerals 1 and 2 correspond to the left and right normals in the mixture. A dash indicates no parameter was estimated. sb-MOE, static MOE from bending test; Ecomp-E, dynamic MOE from the E-computer test; Dir-E, dynamic MOE from the Director test.

proportion attributed to the "leftmost" of the two distributions that form the mixture.

Results of the Goodness-of-Fit Tests

The results of the goodness-of-fit tests for each candidate distribution are summarized in the following text. All *p*-values appear in Table 2. Histograms with fitted probability density function overlays appear in Figures 2-5. Probability plots for the MOR data appear in Figure 6.

SB-MOE

Weibull, Two-Parameter

A CVM test rejects the null hypothesis that the data come from a two-parameter Weibull

Table 2. *p*-values from goodness-of-fit tests.

		Tests		
Data	Distribution	Simulation-based Cramér–von Mises *p*-values	Cramér–von Mises *p*-values	Likelihood ratio *p*-values
sb-MOE	Weibull, two-parameter	—	0.01	—
	Weibull, three-parameter	0.032	—	—
	Normal	0.052	0.055	0.351
	Mixed normal	0.08	—	—
Ecomp-E	Weibull, two-parameter	—	0.01	—
	Weibull, three-parameter	0.050	—	—
	Normal	0.279	0.252	0.463
	Mixed normal	0.61	—	—
Dir-E	Weibull, two-parameter	—	0.01	—
	Weibull, three-parameter	0.002	—	—
	Normal	0.011	0.011	0.012
	Mixed normal	0.67	—	—
MOR	Weibull, two-parameter	—	0.01	—
	Weibull, three-parameter	0.001	—	—
	Normal	0.0002	0.0002	0.001
	Three-parameter beta	0.01	—	—
	Skew normal	0.65	—	—
	Mixed normal	0.66	—	—

p-values of 0.01 and 0.001 may not be exact. (The true value could be smaller.) sb-MOE, static MOE from bending test; Ecomp-E, dynamic MOE from the E-computer test; Dir-E, dynamic MOE from the Director test. A dash indicates that the test was not performed.

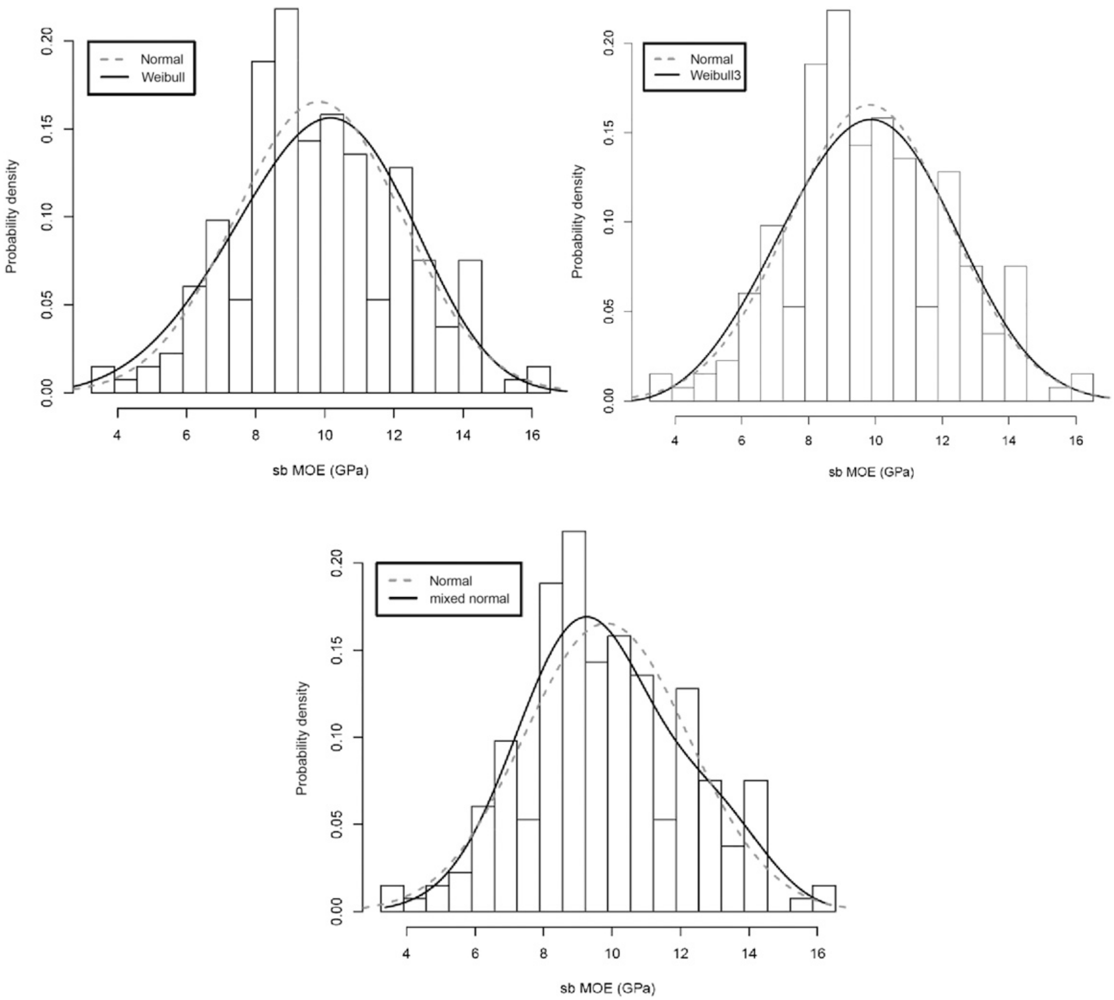

Figure 2. Histograms of the sb-MOE data overlaid with fitted normal, two-parameter Weibull, three-parameter Weibull, and mixed normal probability density functions.

distribution (p-value = 0.01) indicating that a two-parameter Weibull is not a good fit for the data.

Weibull, Three-Parameter

A CVM-sim test of the null hypothesis that the data come from a three-parameter Weibull distribution yields a p-value of 0.032, indicating that a three-parameter Weibull is probably not a good fit for the data.

Normal

CVM-sim and CVM tests of the null hypothesis that the data come from a normal distribution yield p-values of 0.052 and 0.055, respectively, indicating that a normal distribution might not be a good fit for the data. On the other hand, a likelihood ratio test does not reject the null hypothesis that a normal distribution is adequate to model the data vs the alternative hypothesis that a mixed normal is needed (p-value = 0.351).

Mixed Normal

A CVM-sim test does not reject the null hypothesis that the data come from a mixed normal

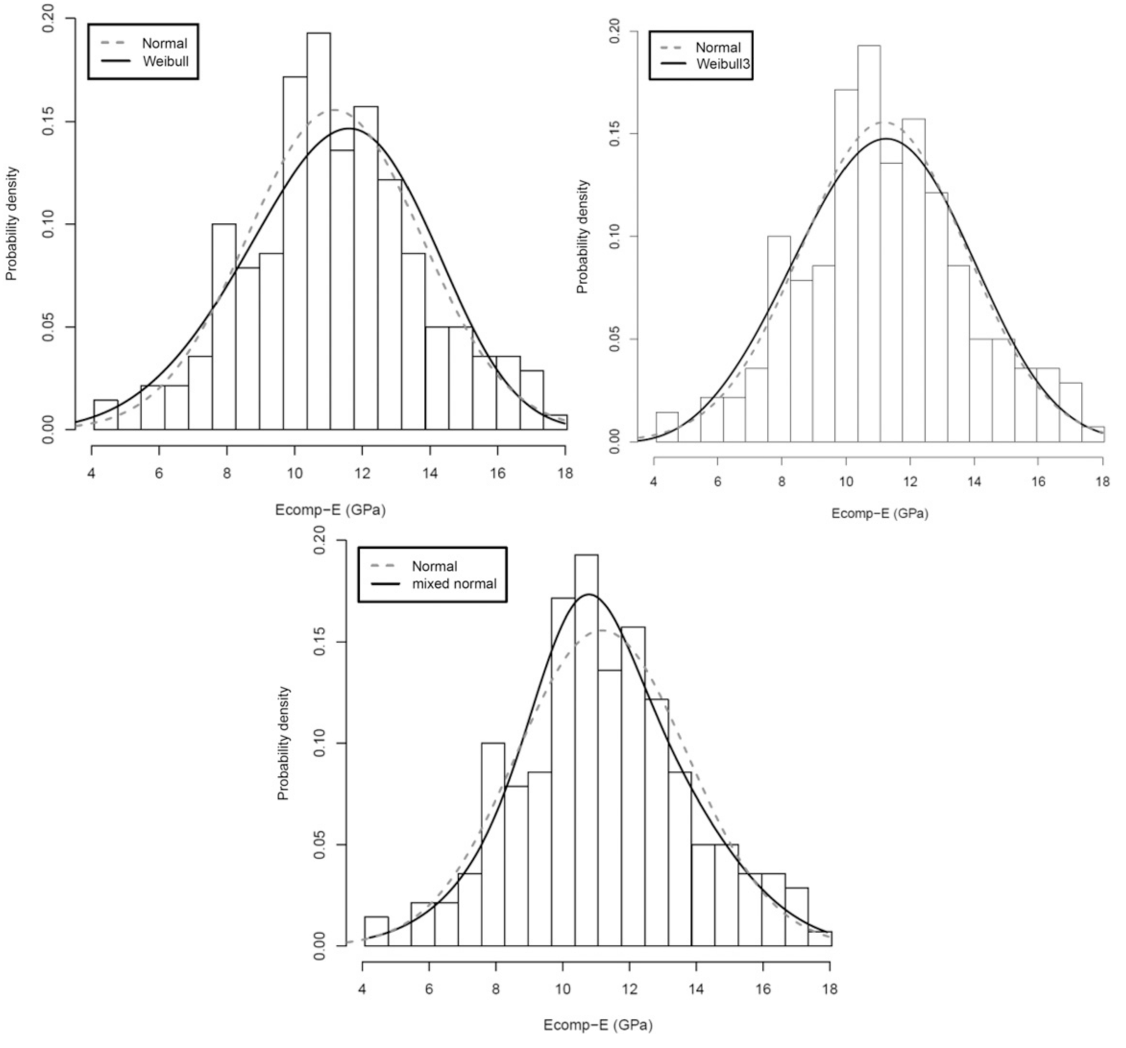

Figure 3. Histograms of the Ecomp-E data overlaid with fitted normal, two-parameter Weibull, three-parameter Weibull, and mixed normal probability density functions.

distribution (p-value = 0.08), indicating that a mixed normal distribution might be a reasonable fit for the data.

ECOMP-E

Weibull, Two-Parameter

A CVM test rejects the null hypothesis that the data come from a two-parameter Weibull distribution (p-value = 0.01), indicating that a two-parameter Weibull is not a good fit for the data.

Weibull, Three-Parameter

A CVM-sim test of the null hypothesis that the data come from a three-parameter Weibull distribution yields a p-value of 0.050, indicating that a three-parameter Weibull is probably not a good fit for the data.

Normal

CVM-sim and CVM tests of the null hypothesis that the data come from a normal distribution yield

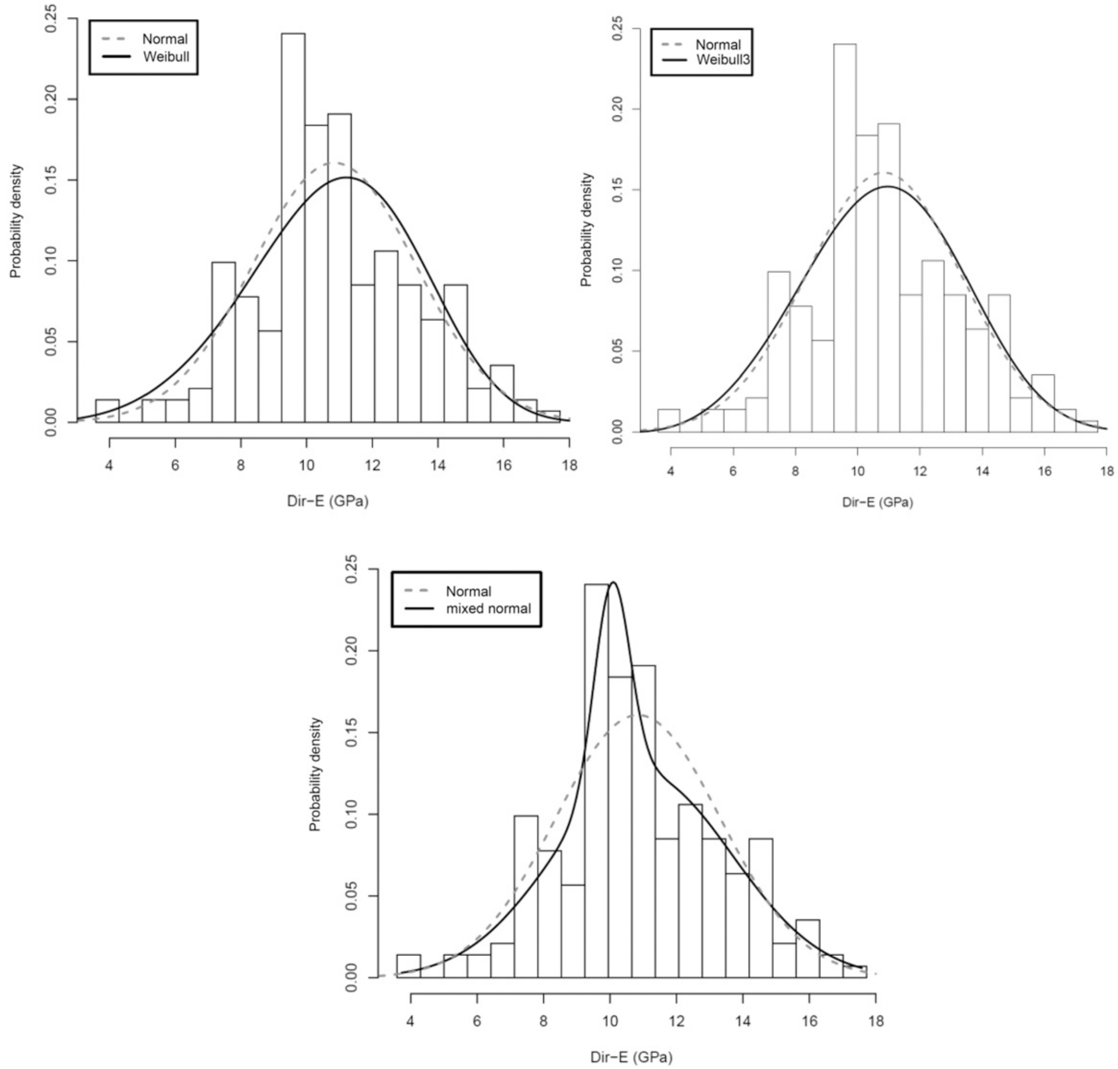

Figure 4. Histograms of the Dir-E data overlaid with fitted normal, two-parameter Weibull, three-parameter Weibull, and mixed normal probability density functions.

p-values of 0.279 and 0.252, respectively, indicating that a normal distribution might be a reasonable fit for the data. Also, a likelihood ratio test does not reject the null hypothesis that a normal distribution is adequate to model the data vs the alternative that a mixed normal is needed (p-value $= 0.463$).

Mixed Normal

A CVM-sim test does not reject the null hypothesis that the data come from a mixed normal distribution (p-value $= 0.61$), indicating that a mixed normal distribution might be a reasonable fit for the data.

DIR-E

Weibull, Two-Parameter

A CVM test rejects the null hypothesis that the data come from a two-parameter Weibull distribution (p-value $= 0.01$), indicating that a two-parameter Weibull is not a good fit for the data.

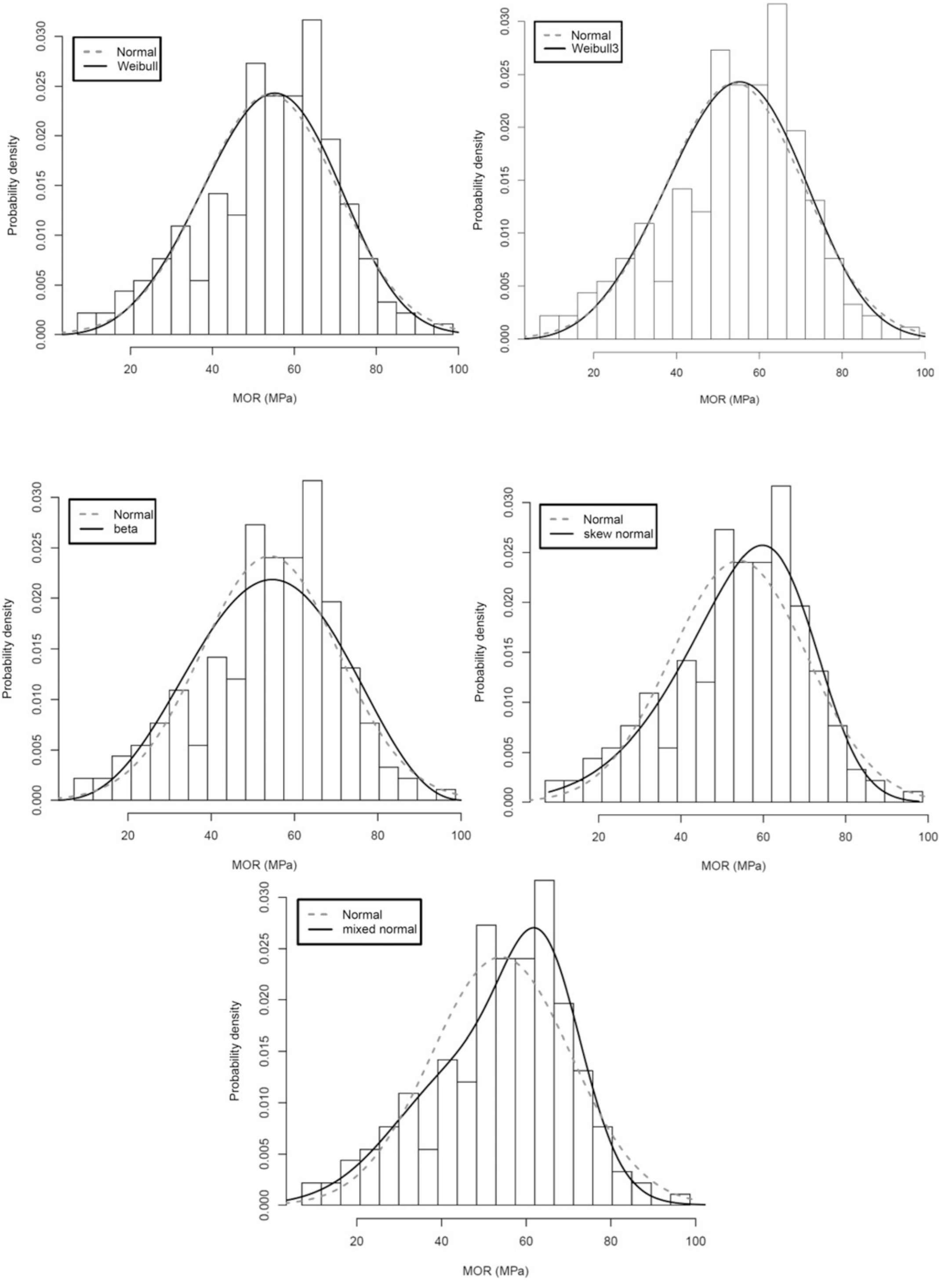

Figure 5. Histograms of the MOR data overlaid with fitted normal, two-parameter Weibull, three-parameter Weibull, three-parameter beta, skew normal, and mixed normal probability density functions.

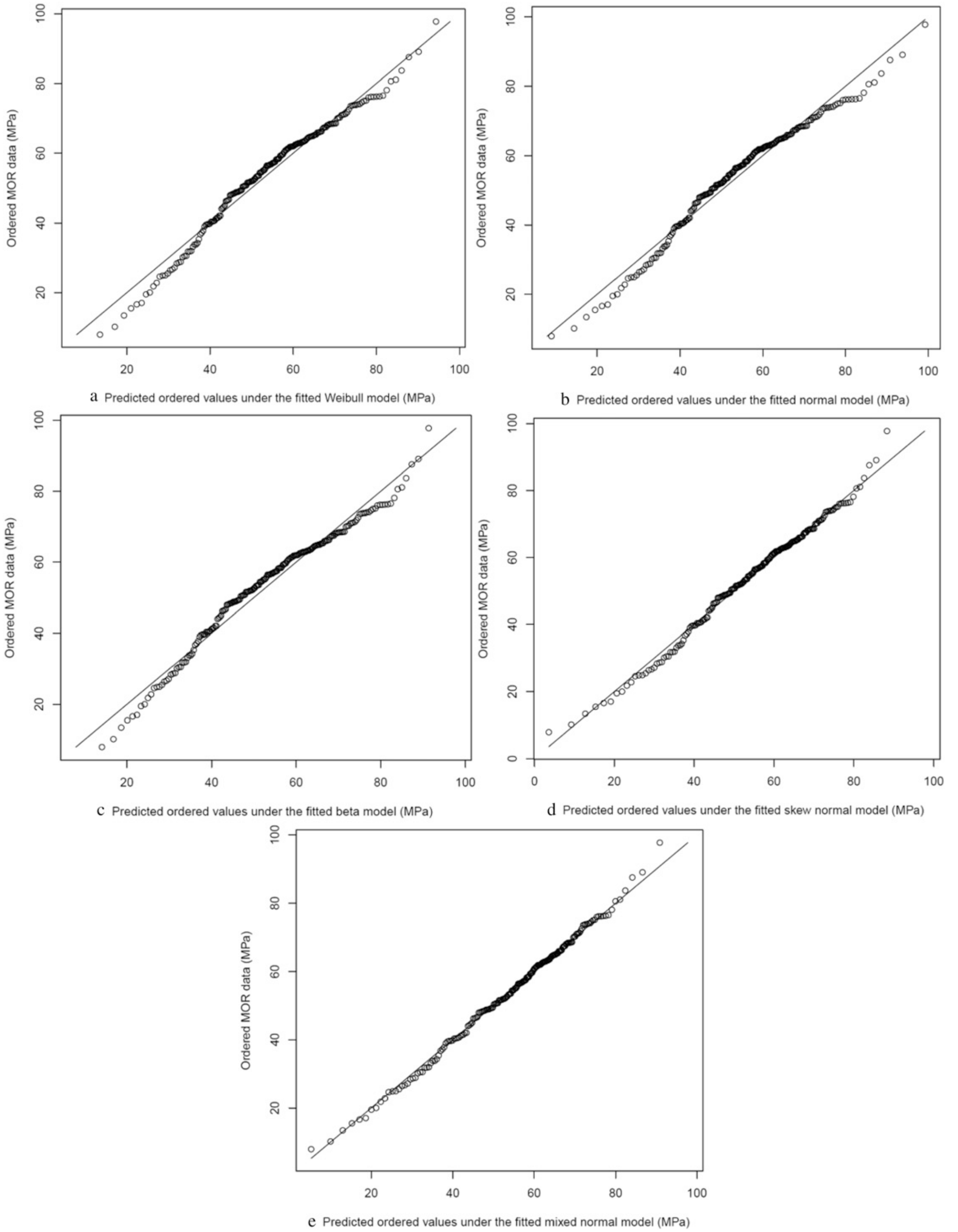

Figure 6. Probability plots of MOR data fitted with two-parameter Weibull, three-parameter Weibull, normal, three-parameter beta, skew normal, and mixed normal distributions. (The best-fit two-parameter Weibull and best-fit three-parameter Weibull are equivalent and represented by the same plot (a) above.) Ordered observed data (Y axis) plotted against expected scores (X axis). The reference line $y = x$ indicates perfect fit.

Weibull, Three-Parameter

A CVM-sim test of the null hypothesis that the data come from a three-parameter Weibull distribution yields a p-value of 0.002 indicating that a three-parameter Weibull is not a good fit for the data.

Normal

CVM-sim and CVM tests of the null hypothesis that the data come from a normal distribution both yield p-values of 0.011, indicating that a normal distribution is not a good fit for the data. Furthermore, a likelihood ratio test rejects the null hypothesis that a normal distribution is adequate to model the data vs the alternative that a mixed normal is needed (p-value $= 0.012$).

Mixed Normal

A CVM-sim test does not reject the null hypothesis that the data come from a mixed normal distribution (p-value $= 0.67$), indicating that a mixed normal distribution might be a reasonable fit for the data.

MOR

Weibull, Two-Parameter

A CVM test rejects the null hypothesis that the data come from a two-parameter Weibull distribution (p-value $= 0.01$). This result is further supported by the probability plot in Figure 6a. Clear deviation of the plot from the reference line in the figure indicates that a two-parameter Weibull is not a good fit for the data.

Weibull, Three-Parameter

A CVM-sim test of the null hypothesis that the data come from a three-parameter Weibull distribution yields a p-value of 0.001. This result is further supported by the probability plot in Figure 6a. (The best-fit two-parameter Weibull and best-fit three-parameter Weibull are equivalent and represented by the same plot.) Clear deviation of the plot from the reference line in the figure indicates that a three-parameter Weibull is not a good fit for the data.

Normal

CVM-sim and CVM tests of the null hypothesis that the data come from a normal distribution both yield p-values of 0.0002. This result is further supported by the probability plot in Figure 6b. Clear deviation of the plot from the reference line in the figure indicates that a normal distribution is not a good fit for the data. Also, a likelihood ratio test rejects the null hypothesis that a normal distribution is adequate to model the data vs the alternative that a mixed normal is needed ($p = 0.001$). These results suggest that a mixed normal is likely a better fit than a normal.

Three-Parameter Beta

A CVM-sim test rejects the null hypothesis that the data come from a three-parameter beta distribution (p-value $= 0.01$). This result is further supported by the probability plot in Figure 6c. Clear deviation of the plot from the reference line in the figure indicates that a three-parameter beta is not a good fit for the data.

Skew Normal

A CVM-sim test does not reject the null hypothesis that the data come from a skew normal distribution (p-value $= 0.65$). This result is further supported by the probability plot in Figure 6d. Relative conformity of the plot to the reference line in the figure indicates that a skew normal distribution might be a reasonable fit for the data.

Mixed Normal

A CVM-sim test does not reject the null hypothesis that the data come from a mixed normal distribution (p-value $= 0.66$). This result is further supported by the probability plot in Figure 6e.

Relative conformity of the plot to the reference line in the figure indicates that a mixed normal distribution might be a reasonable fit for the data.

RESULTS SUMMARY

The goodness-of-fit tests suggest that for the mill-run lumber population sampled, sb-MOE, Ecomp-E, and Dir-E are not distributed as two- or three-parameter Weibulls. The tests do not rule out the possibility that a single normal distribution might be a suitable model for sb-MOE and Ecomp-E, and that a mixed normal distribution might be a suitable model for all three measures of elasticity.

The goodness-of-fit tests further suggest that for the mill-run lumber population sampled MOR is not distributed as a two- or three-parameter Weibull, a normal, or a three-parameter beta. The tests do not rule out the possibility that a skew normal or a mixed normal might be a suitable model for the MOR distribution.

DISCUSSION

In the Introduction we discussed the fact that the exact implications of the pseudo-truncation associated with the formation of a visual or MOE-truncated grade of lumber will depend on the form of the mill-run population that is being pseudo-truncated. The experiment reported in this article was intended to constitute an initial investigation into the statistical forms of mill-run MOE and MOR distributions.

For the specific mill under consideration in the current article, we have found that the distributions of mill-run MOE measures might be normal or mixed normal, whereas the distribution of mill-run MOR might be skew normal or mixed normal. Thus, this initial investigation suggests that the MOE distributions of MOE-truncated grades of lumber might be truncated normal or truncated mixed normal, and the corresponding distributions of MOR might be pseudo-truncated skew normal or pseudo-truncated mixed normal.

In principle, the probability density functions of the proposed pseudo-truncated distributions can be calculated and fit in a manner similar to that reported for pseudo-truncated Weibulls in Verrill et al (2012, 2015). (For example, Verrill et al (2018) assume a MOE-MOR distribution that is a mixture of two bivariate normals, and derive the distribution of the corresponding pseudo-truncated MOR distribution.)

At this point, we are not suggesting that the standards community should adopt pseudo-truncated distributions as the basis for reliability calculations. Before this could be contemplated, studies of additional mills in additional regions at additional times must be made to determine whether mill-run stiffness and strength distributions are, in any sense, stable.

We are currently engaged in such research. However, given the fact that actual distributions may be complicated mixtures of base distributions that vary from mill to mill, region to region, time to time, size to size, and species to species, it may be that no satisfactory theoretical form(s) can be identified to form the basis of improved reliability models.

We suspect that ultimately, if reliability engineers want to obtain accurate and precise reliability estimates, they will need to develop detailed predictive models that yield real-time, in-line estimates of lumber strength based on measurements of stiffness, specific gravity, knot size and location, slope of grain, and other strength predictors.

CONCLUSIONS

The experiment reported in this article was intended to constitute an initial investigation into the statistical forms of mill-run MOE and MOR distributions. For the specific mill under consideration in the current article, the results showed that the distributions of mill-run MOE measures might be normal or mixed normal, whereas the distribution of mill-run MOR might be skew normal or mixed normal. Thus, this initial investigation suggests that the MOE distributions of MOE-truncated grades of lumber might be truncated normal or truncated mixed normal, and the corresponding distributions of MOR might be pseudo-truncated skew normal or pseudo-truncated mixed normal.

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APPENDIX—PROBABILITY DENSITY FUNCTIONS

NORMAL DISTRIBUTION

The normal probability density function is given by

$$f(x;\, \mu, \sigma) = \frac{1}{\sqrt{2\pi}}\,\frac{1}{\sigma}\exp\left(-\,(x-\mu)^2 \Big/ \left(2\sigma^2\right)\right)$$

for $x \in (-\infty, \infty)$, where μ is the mean and σ is the standard deviation. This distribution is denoted by the notation $N(\mu, \sigma^2)$.

MIXED NORMAL

In this article, a "mixed normal distribution" refers to a mixture of two normal distributions. Such a mixture results when specimens are drawn with probability p from a $N(\mu_1, \sigma_1^2)$ distribution and with probability 1-p from a $N(\mu_2, \sigma_2^2)$ distribution. In this case, the probability density function is given by

$$f(x;\, \mu_1, \sigma_1, p,\, \mu_2, \sigma_2) = p \times \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_1} \exp\left(-(x-\mu_1)^2 \Big/ \left(2\sigma_1^2\right)\right)$$

$$+ (1-p) \times \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_2} \exp\left(-(x-\mu_2)^2 \Big/ \left(2\sigma_2^2\right)\right)$$

for $x \in (-\infty, \infty)$.

TWO-PARAMETER WEIBULL

The two-parameter Weibull has probability density function

$$f(w; \gamma, \beta) = \gamma^\beta \beta w^{\beta-1} \exp\left(-(\gamma w)^\beta\right)$$

for $w \in [0, \infty)$, where β is the shape parameter and γ is the inverse of the scale parameter.

THREE-PARAMETER WEIBULL

The three-parameter Weibull has probability density function

$$f(w; \gamma, \beta, c) = \gamma^\beta \beta (w-c)^{\beta-1} \exp\left(-(\gamma(w-c))^\beta\right)$$

for $w \in [c, \infty)$, where β is the shape parameter, γ is the inverse of the scale parameter, and c is the location parameter.

THREE-PARAMETER BETA

The three-parameter beta has probability density function

$$f(x; \alpha, \beta, R) = \frac{x^{\alpha-1} \; (R-x)^{\beta-1}}{R^{\alpha+\beta-1}} \times \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$$

for $x \in [0, R]$, where Γ denotes the gamma function.

SKEW NORMAL

The skew normal distribution has probability density function

$$f(x; \xi, \omega, \alpha) = \frac{2}{\omega} \times \phi\left(\frac{x-\xi}{\omega}\right) \times \Phi\left(\alpha\left(\frac{x-\xi}{\omega}\right)\right)$$

for $x \in (-\infty, \infty)$, where ϕ denotes the probability density function of a standardized normal, Φ denotes the cumulative distribution function of a standardized normal, and $\xi, \omega,$ and α are the parameters of the skew normal distribution.