LOWER TOLERANCE LIMIT APPROACH TO EQUATION-BASED RATIONAL DESIGN VALUES FOR L-SHAPED MORTISE AND TENON JOINTS

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Abstract. Statistical lower tolerance limits (LTLs) were computed for the ratios obtained by dividing the test values for 360 L-shaped rectangular mortise and tenon joints consisting of 72 different configurations of five specimens each by the corresponding values estimated by a nonlinear-regression expression fitted to the test data. LTLs were computed for the resulting ratios at the 75|75, 90|75, 75|90, and 90|90 confidence|proportion levels. At these levels, the corresponding LTLs amounted to 88.1%, 87.4%, 75.8%, and 74.9%, respectively, of the estimates. The percentages of values that fell below the above stated LTLs were 24.2%, 23.3%, 8.3%, and 7.5%. On average, 53% of the test values below a given tolerance limit fell in the range of 90-99% of that limit. Differences between 75|75 and 90|75 limits as well as between 75|90 and 90|90 limits were sufficiently small that the greater confidence level appears desirable. This study is too limited in scope to suggest the appropriate confidence|proportion level that might be used in determining design values for joints as a percentage of estimated values, but it does raise the question and emphasizes the importance of determining what percentage of failure is acceptable along with what level of confidence is appropriate for furniture design.

Keywords: Lower tolerance limits, nonlinear regression analysis, design values.

INTRODUCTION

In a previous paper (Kasal et al 2015), the bending moment capacities of nine geometric configurations of round-shouldered L-shaped mortise and tenon joints, Figs 1 and 2, constructed with two different wood species, two different adhesives, and subjected to two types of loading were presented along with relevant statistics.

To quantify the effects of these variables on the bending moment capacities of the specimens, a nonlinear regression expression was fitted to the individual test data points. This expression is of value to those designing furniture since it provides

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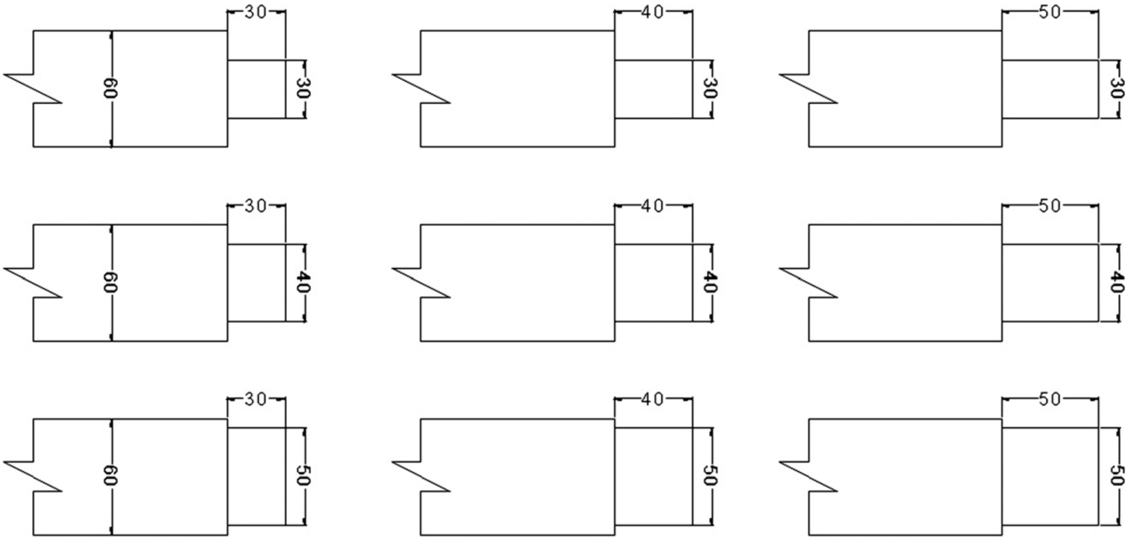

Figure 1. Rails were 21 mm thick by 60 mm wide. Tenons were 7 mm thick with a 2-mm top and bottom-edge radius.

a concise presentation of test results along with a means of estimating the capacities of joints similar to those included in the study. A factor that must be taken into consideration if this expression is used for design purposes, however, is that essentially half of the specimens on which this expression is based had capacities less than the estimated values. The immediate questions that follows are how much less are the test values than the corresponding estimated values and, for design purposes, to what degree should the estimated values be reduced to account for these differences?

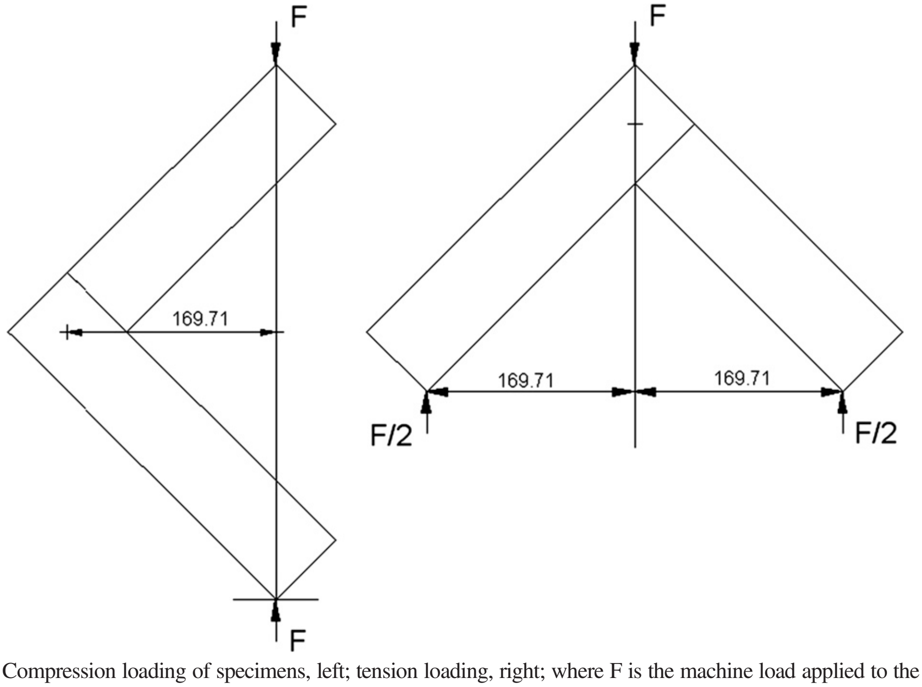

Figure 2. Compression loading of specimens, left; tension loading, right; where F is the machine load applied to the specimen.

Previous research directed toward determining reasonable design stresses for various wood species used in the front rails of sofas (Eckelman 1974) suggests that statistical lower tolerance limits (LTLs) may provide a rational procedure for answering such questions (Ireson, et al 1960; Natrella 1963; Ostle 1963; Link 1985). Ideally, use of such procedures enables designers to assume (with a specified degree of confidence) that a given proportion of joints of similar construction could be expected to have capacities equal to or greater than some fraction of the corresponding estimated values. If LTLs are used, however, a question to be decided is what confidence and reduction levels are appropriate for furniture construction both from an economic and a structural integrity (including safety) point of view.

The first objective of the work presented here, accordingly, was to explore whether a statistically based LTL procedure might provide a means of determining rational design values as a percentage of the values estimated by an expression fitted to the test results for a specified set of mortise and tenon joints. A second objective was to examine the effect of the selection of representative confidence|proportion levels on subsequent design values.

MATERIALS AND METHODS

Description of Specimens, Construction, and Testing

The geometries of the joints used in the study (Kasal et al 2015) are illustrated in Fig 1. Half of the joints were constructed of Turkish beech and half of Scotch pine; half of the joints of each species in turn were bonded with either a polyvinyl acetate (PVA) or a polyurethane (PU) adhesive; thus, 90 specimens were constructed of Scottish pine with a PVA adhesive, 90 of Scottish pine with a PU adhesive, 90 of European beech with a PVA adhesive, and 90 of European beech with a PU adhesive—for a total of 360 specimens. Subsequently, half (180) of the joints of each construction were loaded in tension and half (180) in compression as shown in Fig 2. Capacities of the specimens are given in Table 1 (compression) and 1b (tension).

Procedures

The procedure followed in this study was to divide each of the 360 test values given in Tables 1 and 2 by the corresponding estimated values that are given by the nonlinear regression expression (Kasal et al 2015)

$$M = 0.00227(WL)(0.229W + d)\,S^{0.42}k_1k_2 \quad (1)$$

where M refers to the moment capacity of the joint under compression or tension, *Nm*; W refers to the width and L to the length of the tenon, *mm*; d refers to the width of the shoulder, *mm*, ie d = (60-tenon width)/2; k_1 refers to loading: k_1 (tension) = 1, or, k_1 (compression) = 1.066; and k_2 refers to adhesive: k_2 (PU) = 1.0, or, k_2 (PVA) = 0.85. The coefficient of determination for this expression was 0.62.

A histogram, Fig 3, summarizes the distribution of ratios with a fitted normal curve. The symmetry as well as the similarity between the bars and the curve indicates no evidence against a normal assumption.

LTLs were then determined for the resulting data set by means of the relationship

$$\mathrm{LTL} = \overline{\mathrm{X}} - k \times s \quad (2)$$

where $\overline{X}$ is the average of the test/estimated ratios, s is the standard deviation of the ratios, and k is the corresponding tolerance factor (Ostle 1963).

A quantile-quantile plot, Fig 4, illustrates how the quantiles of the ratios correspond to the theoretical quantiles of a normal random variable. In this plot, no strong evidence exists against normality. Taken together, Figs 3 and 4 demonstrate no evidence against a normal assumption for the distribution of the 360 ratios; hence the LTLs were computed for k-factors based on 360 samples.

A "low" 75% confidence|75% proportion level and a "high"90% confidence|90% proportion

Table 1. Moment capacity of joints loaded in compression.

	Tenon width (mm)								
	30			40			50		
	Tenon length (mm)								
Wood species (adhesive)	30	40	50	30	40	50	30	40	50
	Moment capacity (nm)								
Pine (PVA)	68.3	93.2	169.2	81.6	124.9	143.2	121.5	128.2	183.5
	68.3	81.6	133.4	89.9	123.2	144.8	111.5	133.2	178.1
	73.3	101.6	113.2	89.9	139.8	128.2	91.6	134.9	189.5
	69.9	104.9	131.5	84.9	119.9	111.5	111.5	129.0	184.8
	69.9	95.3	119.9	79.9	138.2	131.9	121.5	119.9	181.5
Avg.	69.9	95.3	133.4	85.2	129.2	131.9	111.5	129.0	183.5
STD	2.0	9.0	21.6	4.6	9.2	13.4	12.2	5.8	4.2
CoV%	2.9	9.4	16.2	5.4	7.1	10.2	11.1	4.5	2.3
Pine (PU)	118.2	114.1	183.1	146.5	193.1	191.5	120.8	188.1	196.5
	105.2	116.5	144.8	155.7	183.1	178.1	119.9	209.8	228.1
	109.6	106.6	145.0	150.2	190.2	188.1	129.9	204.8	213.7
	101.6	112.9	169.8	163.2	194.8	206.4	119.9	229.8	213.7
	91.6	112.7	148.2	163.2	189.8	191.0	113.5	191.5	216.4
Avg.	105.2	112.7	158.2	155.7	190.2	191.0	120.8	204.8	213.7
STD	9.8	3.8	17.4	7.5	4.5	10.2	5.8	16.6	11.3
CoV%	9.4	3.4	10.4	4.8	2.4	5.3	4.8	8.1	5.3
Beech (PVA)	129.5	139.9	199.8	183.1	159.8	238.2	173.1	219.8	223.1
	122.7	151.5	204.8	146.5	153.2	238.2	154.5	211.4	223.1
	118.2	139.8	204.8	144.8	155.2	214.8	148.2	228.1	233.1
	118.3	145.3	196.9	181.5	141.5	216.8	154.5	229.8	218.1
	124.9	149.8	178.1	164.0	166.5	283.0	142.2	186.5	208.8
Avg.	122.7	145.3	196.9	164.0	155.2	238.2	154.5	215.1	221.2
STD	4.8	5.4	11.0	18.3	9.2	27.5	11.6	17.6	8.8
CoV%	3.9	3.7	5.6	11.2	5.9	11.5	7.5	8.2	4.0
Beech (PU)	159.8	163.2	199.8	169.8	143.2	204.8	183.1	211.4	279.1
	156.9	158.2	204.8	168.7	128.2	213.7	193.5	210.4	288.0
	174.8	159.8	204.8	171.5	146.8	233.1	203.1	225.3	254.7
	143.2	155.3	196.9	168.7	166.2	253.1	189.8	244.7	270.7
	149.8	140.2	178.1	164.8	149.8	208.1	198.1	234.7	303.0
Avg.	156.9	155.3	196.9	168.7	146.8	222.6	193.5	225.3	279.1
STD	11.9	8.9	11.0	2.5	13.6	20.3	7.7	14.8	18.1
CoV%	7.6	5.8	5.6	1.5	9.3	9.1	4.0	6.6	6.5

Data from original study by Kasal, et al 2015.

level were selected as starting points—under the assumption that at the lower level a substantial number of joints might be expected to have capacities less than the chosen LTL, whereas few joints might be expected to have less capacity than the LTL at the 90%|90% confidence|proportion level; however, LTL's at the 75|90 and 90|75 confidence|proportion levels were included in the study to provide added information concerning the implications of the selection of a particular set of levels on potential design values.

RESULTS

Test-Value/Estimated-Value Ratios

Individual test values along with their averages and standard deviations obtained from the initial study by Kasal et al (2015) are given in Tables 1 (compression) and 2 (tension). The test-capacity/estimated-capacity ratios for the four broad specimen groups (pine-PVA + pine-PU and beech PVA + beech PU in compression; and pine-PVA + pine-PU and beech-PVA + beech-PU in tension) are given in Tables 3 and 4.

Table 2. Moment capacity of joints loaded in tension.

	Tenon width (mm)								
	30			40			50		
	Tenon lengths (mm)								
	30	40	50	30	40	50	30	40	50
Wood species (adhesive)	Moment capacity (nm)								
Pine (PVA)	98.23	120.70	146.5	59.93	109.00	134.90	123.20	104.10	196.50
	88.51	139.80	165.7	73.25	124.00	148.20	143.20	134.90	191.30
	88.51	121.50	134.9	69.51	122.60	146.50	102.40	123.40	182.30
	72.42	102.40	123.2	69.09	133.20	116.50	110.70	124.90	202.60
	94.90	119.00	123.2	75.75	124.00	136.50	146.50	129.90	184.00
Avg.	88.51	120.70	138.70	69.51	122.60	136.50	125.20	123.40	191.30
STD	9.93	13.28	17.91	6.02	8.66	12.62	19.44	11.72	8.51
CoV%	11.21	11.00	12.91	8.66	7.06	9.25	15.53	9.50	4.45
Pine (PU)	109.00	134.40	155.70	141.50	198.40	194.80	151.50	218.30	248.30
	106.60	128.20	191.50	131.80	215.30	173.10	166.50	211.90	220.40
	87.40	128.20	173.10	122.70	229.80	165.70	166.50	220.60	217.30
	97.19	147.30	165.70	123.20	229.80	181.50	155.50	222.30	228.60
	85.74	134.00	181.50	139.80	203.10	155.70	137.40	195.90	228.60
Avg.	97.19	134.40	173.50	131.80	215.03	174.10	155.50	213.80	228.60
STD	10.67	7.82	13.84	8.89	14.58	14.95	12.11	10.73	12.06
CoV%	10.98	5.82	7.98	6.74	6.77	8.59	7.79	5.02	5.27
Beech (PVA)	144.80	177.60	245.50	115.70	159.80	155.70	92.68	169.00	203.10
	140.10	173.10	233.10	111.00	159.80	194.80	70.76	145.70	212.30
	136.50	177.60	271.40	100.70	174.80	171.70	106.60	155.70	219.80
	162.30	213.10	232.10	111.00	160.70	162.30	100.70	147.30	209.10
	116.50	146.50	245.50	116.50	144.00	174.00	92.68	160.70	201.40
Avg.	140.10	177.60	245.50	111.00	159.80	171.70	92.68	155.70	209.10
STD	16.46	23.70	15.84	6.29	10.90	14.86	13.58	9.64	7.38
CoV%	11.76	13.35	6.45	5.67	6.82	8.66	14.65	6.19	3.53
Beech (PU)	103.20	194.60	224.50	95.07	134.60	270.00	99.89	119.90	241.40
	101.60	189.80	214.80	105.40	155.70	288.00	94.90	123.20	219.80
	112.40	193.10	223.10	95.07	126.50	264.70	118.20	131.20	249.70
	104.30	212.30	225.60	90.73	140.70	257.20	104.90	150.70	273.00
	99.89	183.10	234.70	89.07	115.70	270.00	106.60	131.20	223.10
Avg.	104.30	194.60	224.50	95.07	134.60	270.00	104.90	131.20	241.40
STD	4.83	10.83	7.12	6.35	15.02	11.36	8.73	11.95	21.64
CoV%	4.63	5.56	3.17	6.68	11.16	4.21	8.32	9.11	8.97

Data from original study by Kasal et al 2015.

The average value of the 360 ratios was 1.023 with a standard deviation of 0.199. To better visualize the distribution of the individual ratios above and below average, the values of the ratios along with the average are illustrated in Figs 5-8. As is illustrated, 177 out of the 360 specimens (49.2%), ie 91 in compression plus 86 in tension, had ultimate/estimated ratios less than 1.0.

Lower Tolerance Limit for Ratios

The tolerance factors, k, for 360 specimens at the 75|75, 90|75, 75|90, and 90|90 confidence|proportion levels were 0.715, 0.751, 1.331, and 1.377, respectively. At the 75|75 confidence-proportion level, the LTL for the entire collection of transformed data (ie specimen-capacity/estimated-capacity) using the k-factor for 360 specimens of 0.715 was

$$\mathrm{LTL}(75|75) = 1.023 - 0.715 \times 0.199 = 0.881 \tag{3}$$

Thus, the LTL for the transformed data at this confidence-proportion level amounts to 0.881 of the rounded average of the ratios so that the

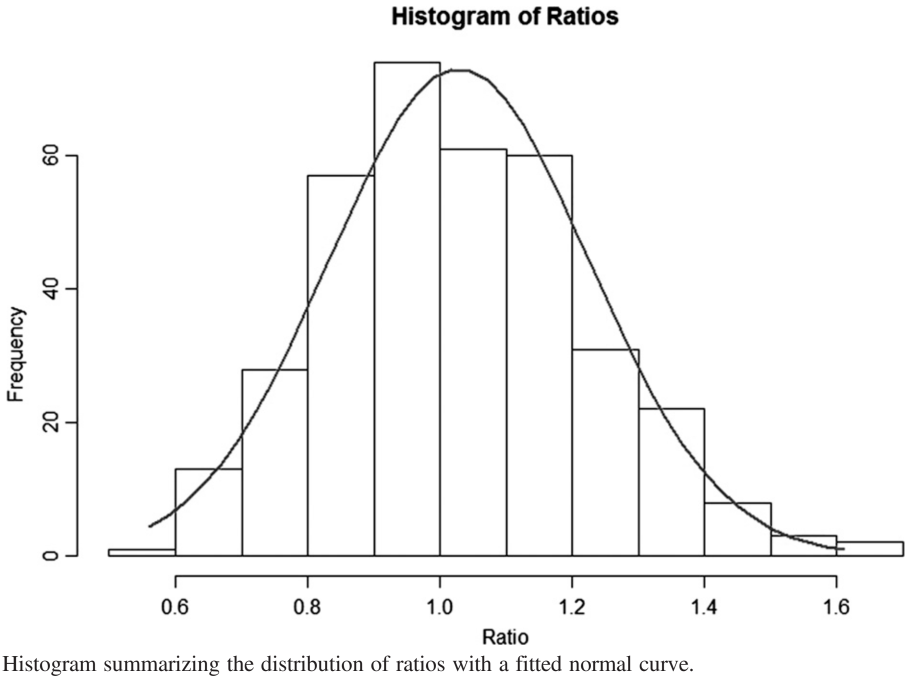

Figure 3. Histogram summarizing the distribution of ratios with a fitted normal curve.

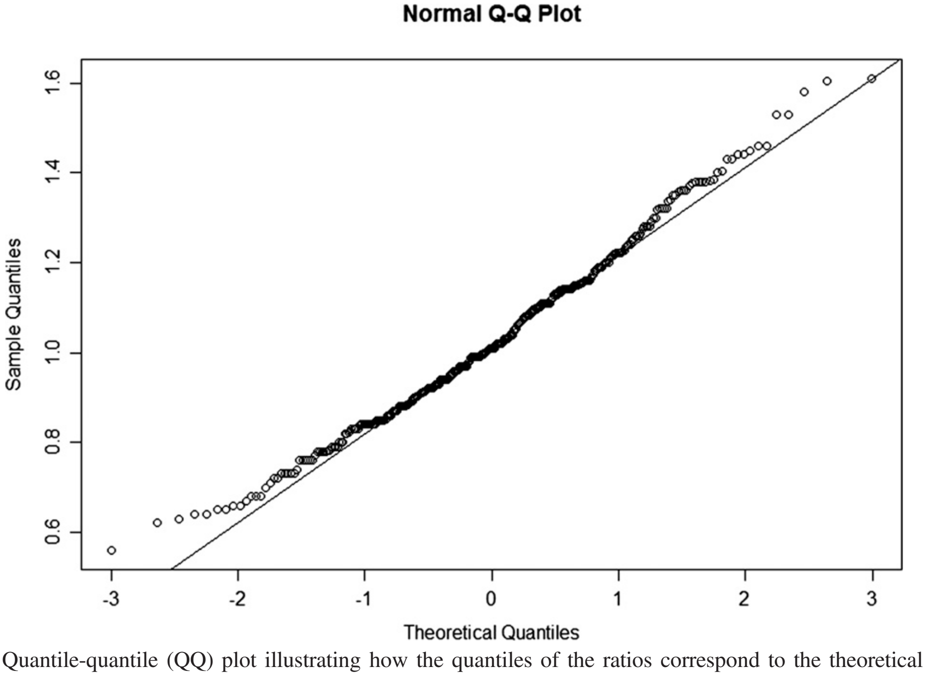

Figure 4. Quantile-quantile (QQ) plot illustrating how the quantiles of the ratios correspond to the theoretical quantiles of a normal random variable.

Table 3. Ultimate/estimated ratios for joints loaded in compression.

	Tenon width (mm)								
	30			40			50		
	Tenon length (mm)								
	30	40	50	30	40	50	30	40	50
Wood species (adhesive)	Ratio—test value/estimated value								
Pine (PVA)	0.783	0.802	1.164	0.801	0.920	0.840	1.110	0.880	1.007
	0.783	0.702	0.918	0.883	0.907	0.850	1.020	0.914	0.978
	0.840	0.874	0.779	0.883	1.030	0.760	0.840	0.925	1.040
	0.802	0.902	0.905	0.834	0.883	0.660	1.020	0.885	1.015
	0.802	0.820	0.825	0.785	1.018	0.780	1.110	0.823	0.996
Avg.	0.802	0.816	0.857	0.837	0.952	0.777	1.021	0.885	1.007
STD	0.023	0.078	0.066	0.045	0.068	0.079	0.112	0.040	0.023
CoV%	2.916	9.517	7.726	5.423	7.093	10.188	10.968	4.516	2.283
Pine (PU)	1.153	0.840	1.070	1.220	1.210	0.960	0.940	1.097	0.917
	1.026	0.850	0.850	1.300	1.150	0.890	0.930	1.224	1.064
	1.068	0.780	0.850	1.250	1.190	0.940	1.010	1.195	0.997
	0.990	0.830	0.990	1.360	1.220	1.030	0.930	1.340	0.997
	0.893	0.820	0.870	1.360	1.190	0.960	0.880	1.117	1.010
Avg.	1.026	0.824	0.925	1.300	1.191	0.957	0.939	1.195	0.997
STD	0.096	0.028	0.102	0.063	0.028	0.051	0.045	0.097	0.053
CoV%	9.351	3.359	10.998	4.830	2.347	5.314	4.838	8.110	5.294
Beech (PVA)	1.201	0.970	1.110	1.450	0.950	1.130	1.280	1.219	0.990
	1.138	1.050	1.140	1.160	0.910	1.130	1.140	1.173	0.990
	1.096	0.970	1.140	1.150	0.920	1.020	1.100	1.265	1.034
	1.097	1.010	1.100	1.440	0.840	1.030	1.140	1.274	0.968
	1.158	1.040	0.990	1.300	0.990	1.350	1.050	1.034	0.926
Avg.	1.138	1.010	1.095	1.302	0.924	1.134	1.143	1.193	0.982
STD	0.044	0.038	0.061	0.146	0.055	0.131	0.086	0.098	0.039
CoV%	3.892	3.741	5.590	11.179	5.941	11.533	7.521	8.180	3.998
Beech (PU)	1.260	0.960	0.940	1.150	0.720	0.830	1.150	0.997	1.053
	1.237	0.930	0.970	1.140	0.650	0.870	1.220	0.992	1.086
	1.378	0.940	0.970	1.160	0.740	0.940	1.280	1.062	0.961
	1.128	0.920	0.930	1.140	0.840	1.020	1.190	1.154	1.021
	1.181	0.830	0.840	1.110	0.760	0.840	1.250	1.107	1.143
Avg.	1.237	0.918	0.931	1.138	0.743	0.901	1.217	1.062	1.053
STD	0.094	0.053	0.052	0.017	0.069	0.082	0.048	0.070	0.068
CoV %	7.591	5.752	5.590	1.453	9.284	9.108	3.960	6.585	6.499

corresponding LTL for Eq (1) amounts to 88.1% of the estimated values, ie,

$$M(75|75) = 0.881 \times \left[0.00227(\mathrm{WL})(0.229W + d)S^{0.42}k_1k_2\right] \tag{4}$$

Referring to Figs 5-8 and Table 5, it can be seen that at the 75|75 confidence-proportion level, 87 ratios (24.2% of 360) were less than the LTL of 0.881. The distribution of these ratios below the LTL, Table 5, was as follows: 45 (51.7% of 87) had values that were within the range of 90-100% of the LTL, 24 (27.6% of 87) within the range of 80-90%, 16 (18.4% of 87) within the range of 70-80%, and 2 (2.3% of 87) within the range of 60-70% of LTL. Thus, the majority of the values were clustered just below the LTL.

At the highest confidence|proportion level examined, ie the 90|90 level, the tolerance factor, k, is 1.377 so that the corresponding LTL is

$$\mathrm{LTL}(90|90) = 1.023 - 1.377 \times 0.199 = 0.749 \tag{5}$$

Table 4. Ultimate/estimated ratios for joints loaded in tension.

	Tenon width (mm)								
	30			40			50		
	Tenon length (mm)								
	30	40	50	30	40	50	30	40	50
Wood species (adhesive)	Ratio—test value/estimated value								
Pine (PVA)	1.201	1.110	1.070	0.630	0.860	0.850	1.200	0.761	1.150
	1.082	1.280	1.220	0.770	0.970	0.930	1.400	0.987	1.120
	1.082	1.110	0.990	0.730	0.960	0.920	1.000	0.903	1.067
	0.886	0.940	0.900	0.720	1.050	0.730	1.080	0.913	1.186
	1.160	1.090	0.900	0.790	0.970	0.860	1.430	0.950	1.077
Avg.	1.082	1.107	1.018	0.728	0.962	0.857	1.221	0.903	1.120
STD	0.121	0.122	0.131	0.063	0.068	0.079	0.190	0.086	0.050
CoV%	11.213	11.002	12.913	8.657	7.064	9.247	15.526	9.496	4.447
Pine (PU)	1.133	1.048	0.970	1.260	1.320	1.040	1.260	1.357	1.235
	1.108	0.999	1.190	1.170	1.440	0.920	1.380	1.318	1.096
	0.908	0.999	1.080	1.090	1.530	0.880	1.380	1.372	1.081
	1.010	1.149	1.030	1.100	1.530	0.970	1.290	1.382	1.137
	0.891	1.045	1.130	1.240	1.360	0.830	1.140	1.218	1.137
Avg.	1.010	1.048	1.082	1.173	1.437	0.930	1.289	1.329	1.137
STD	0.111	0.061	0.086	0.079	0.097	0.080	0.100	0.067	0.060
CoV%	10.980	5.821	7.976	6.744	6.773	8.587	7.791	5.017	5.275
Beech (PVA)	1.431	1.320	1.460	0.980	1.010	0.790	0.730	0.999	0.961
	1.384	1.280	1.380	0.940	1.010	0.990	0.560	0.861	1.004
	1.349	1.320	1.610	0.850	1.110	0.870	0.840	0.920	1.040
	1.604	1.580	1.380	0.940	1.020	0.820	0.790	0.871	0.989
	1.152	1.090	1.460	0.990	0.910	0.880	0.730	0.950	0.953
Avg.	1.384	1.316	1.456	0.939	1.014	0.872	0.731	0.920	0.989
STD	0.163	0.176	0.094	0.053	0.069	0.075	0.107	0.057	0.035
CoV%	11.755	13.346	6.454	5.671	6.821	8.655	14.654	6.190	3.531
Beech (PU)	0.867	1.226	1.130	0.680	0.730	1.160	0.670	0.602	0.971
	0.853	1.196	1.080	0.760	0.840	1.240	0.640	0.619	0.884
	0.944	1.217	1.120	0.680	0.680	1.140	0.790	0.660	1.004
	0.876	1.337	1.140	0.650	0.760	1.110	0.700	0.757	1.098
	0.839	1.154	1.180	0.640	0.620	1.160	0.710	0.660	0.897
Avg.	0.876	1.226	1.132	0.684	0.726	1.165	0.703	0.660	0.971
STD	0.041	0.068	0.036	0.046	0.081	0.049	0.059	0.060	0.087
CoV%	4.634	5.565	3.171	6.682	11.157	4.207	8.324	9.096	8.966

Thus, the LTL for the ratios at this confidence-proportion level amounted to 0.749 of the rounded average value of 1.0 so that the corresponding LTL for Eq (1) amount to 74.9% of the estimated values, ie,

$$M(90|90) = 0.749 \times \left[0.00227(WL)(0.229\ \ W + d)S^{0.42}\mathrm{k}_1\mathrm{k}_2\right] \quad (6)$$

At this (90|90) confidence|proportion level (Figs 5-8; Table 5), only 27 (7.5% of 360) specimens had ratios less than the LTL of 0.749. The distribution of ratios below the 90|90 LTL was as follows: 14 (51.9% of 27) had values that were within the range of 90-100% of the LTL, 12 (44.4% of 27%) within the range of 80-90%, and 1 (3.7% of 27) within the range of 70-80%. Thus, there is a substantial reduction in the number of test/estimated ratios below the LTL (27 vs 87) but at a substantial

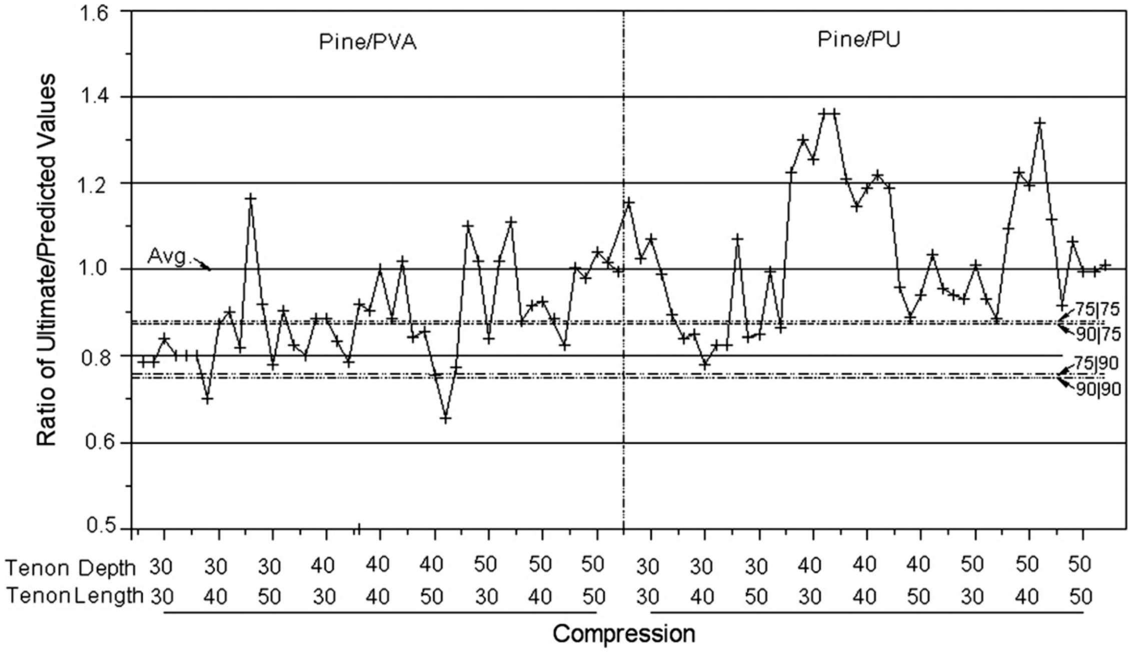

Figure 5. Ratios of test/predicted values are shown as +'s. Arrows point to the rounded average (1.0) and the 75|75 (0.881), 90|75 (0.873), 75|90 (0.758), and 90|90 (0.749) confidence|proportion levels for the ratios.

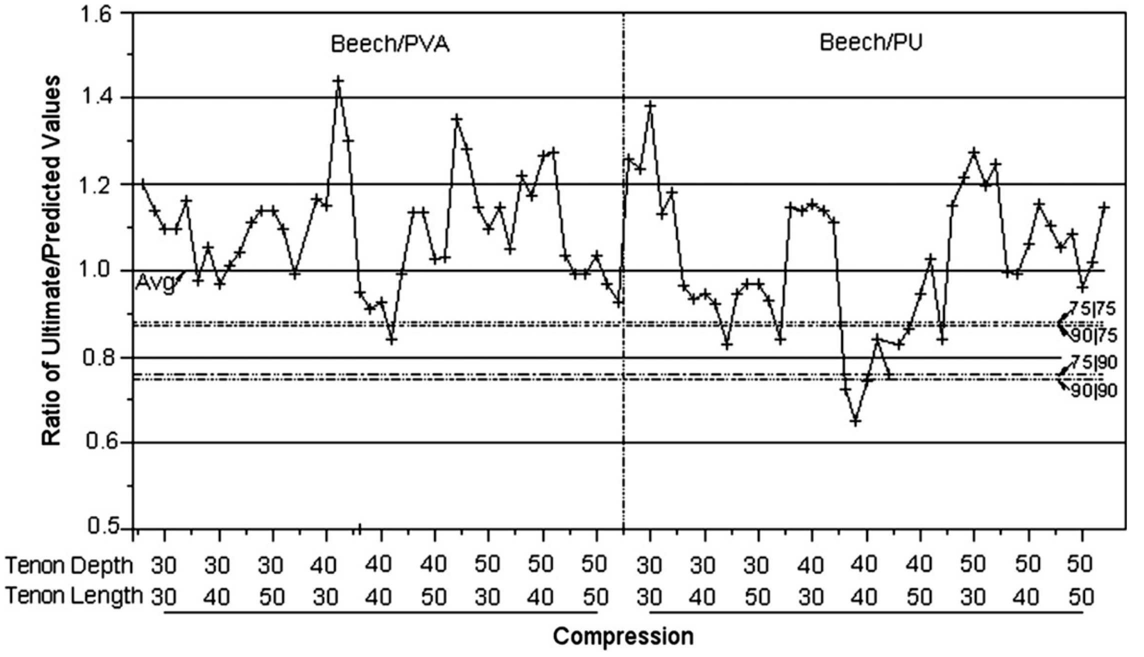

Figure 6. Ratios of test/predicted values are shown as +'s. Arrows point to the rounded average (1.0) and the 75|75, 90|75, 75|90, and 90|90 confidence|proportion levels for the ratios.

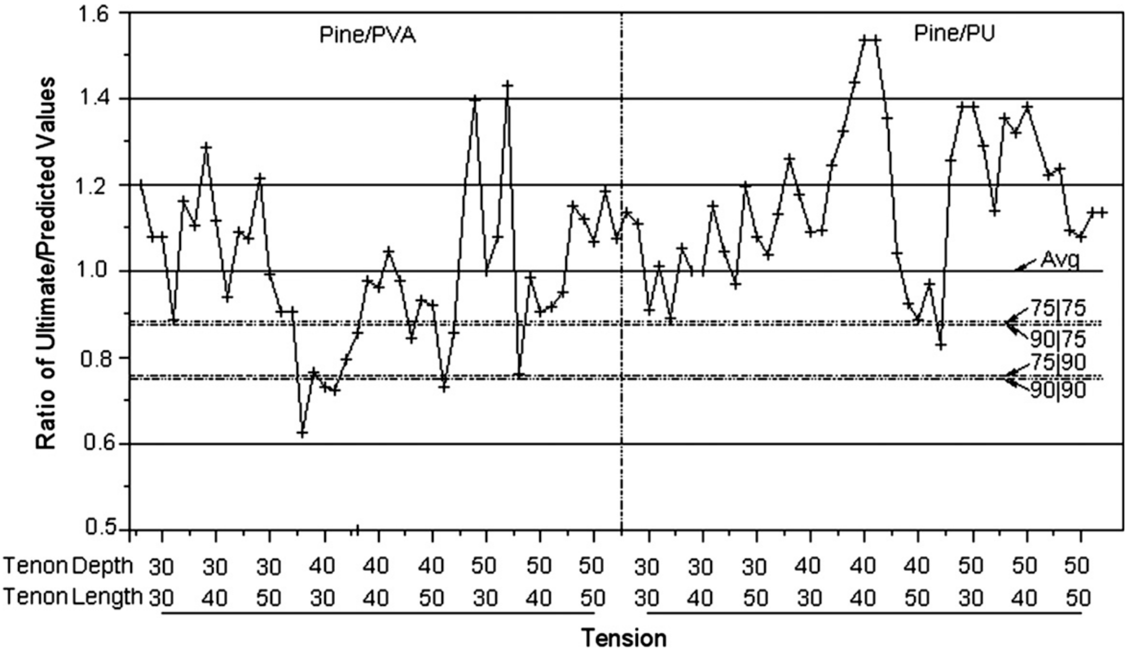

Figure 7. Ratios of test/predicted values are shown as +'s. Arrows point to the rounded average (1.0) and the 75|75, 90|75, 75|90, and 90|90 confidence-proportion levels for the ratios.

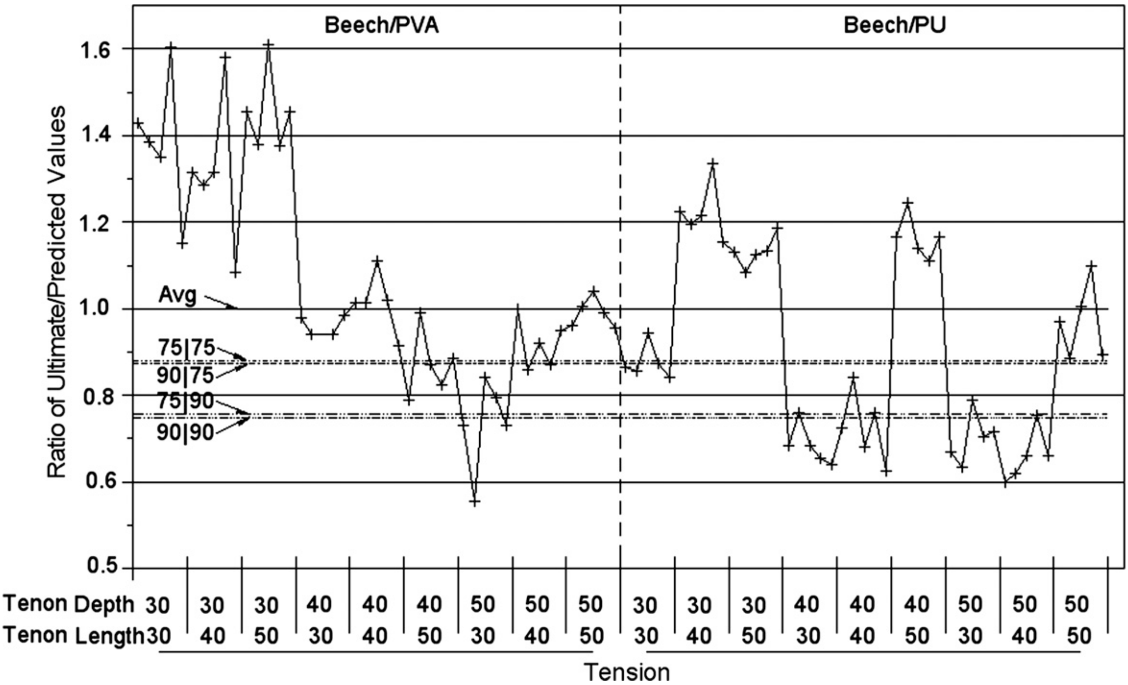

Figure 8. Ratios of test/predicted values are shown as +'s. Arrows point to the rounded average (1.0) and the 75|75, 90|75, 75|90, and 90|90 confidence|proportion levels for the ratios.

Table 5. Distribution of test/estimated ratios below specified LTL's.

LTL	75|75 LTL	% of 87	75|90 LTL	% of 84	75|90 LTL	% of 30	90|90 LTL	% of 27
No. of values below LTL	87	100	84	100	30	100	27	100
Between 90% and 100% of LTL	45	51.7	44	52.4	17	56.7	14	51.9
Between 80% and 90% of LTL	24	27.6	24	28.6	11	36.7	12	44.4
Between 70% and 80% of LTL	16	18.4	14	16.7	2	6.67	1	3.7
Between 60% and 70% of LTL	2	2.3	2	2.4	0	0	0	0

LTL, lower tolerance limits.

15% reduction in the value of the LTL (0.749 vs 0.881). This result emphasizes the importance of determining what percentage of failure is acceptable along with an appropriate level of confidence.

Additionally, as can be seen in Figs 5-8, at the intermediate 90|75 and 75|90 LTL levels examined, the 90|75 LTL amounts to 0.873, which is only slightly less (0.9%) than the 75|75 LTL of 0.881. Likewise, the 75|90 LTL amounts to 0.758, which is only slightly greater (1.2%) than the 90|90 LTL of 0.749. The distribution of ratios below the LTL's for these confidence| proportion levels is essentially the same as for the 75|75 and 90|90 ratios, Table 5.

Thus, overall, the test/estimated ratios less than a given LTL tended to be clustered relatively closely to the LTL—essentially 50% of the values below the LTL's lie in the range of 90-100% of the LTL and 80% within the range of 80-100% of LTL; even in the case of the 75|75 and 90|75 confidence-proportion levels, 80% of the values are clustered within 20% of the LTL.

Finally, it should be noted that in considering the broader application of these techniques, the variability of the test-values for the individual sets of data are of interest. Analyzing the data presented in Tables 1 and 2, the coefficients of variation for the sets (of five specimens) ranged from a low of 1.45% to a high of 16.22%. Presumably, these values are representative for such joints; however, it should be noted that the material from which these joints were constructed presumably was largely defect free, and the specimens were constructed under closely controlled conditions. Had the specimens been constructed under less closely controlled conditions, presumably with accompanying increases in standard deviation, the LTL's would be expected to be correspondingly lower.

CONCLUSIONS

Statistical LTL techniques provide a useful means of rationally examining potential joint capacity design values as a fraction of the estimated values provided by nonlinear regression analyses of test data. Specifically, LTL values based on the mean and standard deviation of the ratios formed by dividing individual test values by their corresponding estimated values provides the information needed to calculate the percentage of specimens that might be expected to have less capacity than a specified fraction of the estimated values along with specified degrees of confidence in those fractions.

Proportion levels affected an LTL more than confidence levels—the LTL's for 75|75 and 90|75 confidence|proportion levels differed little from one another whereas the LTL's for 90|75 and 75|90 confidence|proportion levels differed substantially from one another. Thus, designers should investigate whether use of a greater confidence level substantially reduces accompanying potential design values.

The results of this study alone do not provide definitive answers to the question of what are appropriate confidence|proportion levels to be used in deriving joint capacity design values from nonlinear regression expressions fitted to results of test data. Determination of widely applicable rational design values for mortise and tenon joints based on statistical LTLs will require extensive sampling of data related to the capacity of joints constructed under a variety of quality control scenarios. Ideally, a nonlinear regression expression would be fitted to the data obtained by numerous researchers and statistical LTLs determined for the estimates provided by that expression.

In addition, results of extensive tests of joints constructed under "normal" manufacturing conditions (where both economic and safety factors must be considered) as well as laboratory conditions are needed. Of particular importance, extensive tests of joints of a single configuration are needed that provide factual background data as well as enhance user confidence in the LTL procedures.

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