# TECHNICAL NOTE: STRESS ANALYSIS OF CELLULOSIC-MANURE COMPOSITES

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**Abstract.** Ability to determine stresses in loaded, perforated cellulosic-manure composites from recorded temperature information was demonstrated. Being able to stress analyze such green materials addresses several societal issues. These include providing engineering members fabricated from materials that are suitable for developed and developing nations, relieving a troubling by-product of agricultural regions and reducing demands on our landfills. Most engineering applications of these materials necessitate knowing their structural integrity eg capability to evaluate stresses.

Keywords: Cellulosic-manure composites, green materials, stress analysis, thermoelasticity.

#### INTRODUCTION

The State of Wisconsin alone produces 190 million pounds of manure per day (Statista 2016). In addition to a troubling agricultural by-product, this material adversely contributes to land nutrient imbalance and watershed contamination. With appropriate refining and/or processing with other materials like recycled paper pulped fibers, one can produce numerous value-added products. However, many engineering applications of such materials necessitate knowing their structural integrity ie require stress evaluation. Motivated by the above, this paper determines the stresses in the perforated cellulosic-manure composite plate (Fig 1) from the load-induced temperature information (ie thermoelastic stress analysis [TSA]). Processing the recorded thermal data with an Airy stress function provides the individual components of full-field stress, including on the edge of the hole.

#### MATERIALS AND METHODS

Eq 1 is a relevant Airy stress function satisfying the equation  $K^4 \Phi = 0$  equilibrium

$$\Phi = a_0 + b_0 \cdot \ln \mathbf{r} + c_0 \cdot r^2 + \sum_{n=2,4,6\dots}^N \left( a_n \cdot r^n + b_n \cdot r^{n+2} + c_n \cdot r^{-n} + d_n \cdot r^{-(n-2)} \right) \cdot \cos(n \cdot \theta)$$
(1)

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Figure 1. Perforated cellulosic-manure composite plate (R = 12.7 mm, w = 62 mm, t = 7.2 mm).

and compatibility (Khaja 2012). On differentiating Eq 1 to obtain the stresses, imposing tractionfree conditions  $\sigma_{rr} = \sigma_{r\theta} = 0$  on the edge of the hole of Fig 1 and determining the Airy coefficients of the stress function from recorded *S*\*, the individual stress components are available in both polar and rectangular coordinates.

Figure 1 shows the present central circularly perforate plate as machined from a larger panel. The panel was fabricated by compressing a solution of two cellulosic fiber sources: anaerobic digested bovine manure and recycled old corrugated containers to produce a naturally bonded in-plane randomly oriented (isotropic) cellulose-fiber panel. Its in-plane elastic modulus  $E \sim 7$  GPa (1.0 Mpsi) equals or exceeds that of most commercial hardboards. The plate was loaded sinusoidally between 445 and 890 N at a rate of 20 Hz. An infrared camera recorded the corresponding load-induced temperature information. Recognizing the recorded data is unreliable at edges, the 10,000 TSA values used as input originate at  $1.2 \le r/R \le 2$  away from the edge of the hole. The thermomechanical coefficient, K, was determined by S\*/far-field uniform stress ( $\sigma_o$ ), giving K =184 U/MPa where the symbol U denotes unit less. Comparing experimental and reconstructed information of S\* based on the Airy stress function indicate that one should use seven Airy coefficients.

### RESULTS

Based on the 10,000 measured values of  $S^*$  associated with Fig 2 and evaluating the seven Airy coefficients from the recoded thermal

information, the individual stresses are available throughout the plate. For example, Fig 3 is a full-field plot of the tangential stress,  $\sigma_{\theta}/\sigma_{o}$ where the loading is in the horizontal (x-axis) direction. The TSA-evaluated stress concentration factor,  $K_t$ , of 3.63 compares with that of 3.77 based on Young and Budynas (2002).

#### CONCLUSIONS

The ability to stress analyze members fabricated from green materials such as cellulosic-manure composites was demonstrated. In addition to creating a value-added material, this capability addresses a societal problem and eases a major challenge to our land and watersheds. Although the structural integrity of engineering applications



Figure 2. Thermogram, *S*, from vertically loaded plate with calibrated color bar (MPa).



Figure 3. Experimentally determined tangential stress,  $\sigma_{\theta} / \sigma_o$ .

of these materials necessitates knowing the stresses, the authors are unaware of any previous experimental stress analysis of cellulosicmanure composites. Although demonstrated presently to the case of a round hole in a finite isotropic composite tensile plate, the approach is applicable to much more complicated situations (eg bolted, pinned or nailed holes, side notches, or nonisotropic materials).

#### REFERENCES

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