THE CONTINUUM OF CONNECTION RIGIDITY IN TIMBER STRUCTURES[1]

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ABSTRACT

The use of timber in rigid frames has been hampered by the debate surrounding the rigidity of the moment connections. Joint stiffness is a function of beam flexural stiffness, as well as of the rotational stiffness of the connection. The level of joint rigidity, which is predictable from joint stiffness, significantly affects the bending moments and forces that are transferred through the connection. We used joint-test data from the literature and computer models to assess the effect of various parameters on joint stiffness. There is a continuum of joint stiffness for moment-resisting connections where the deformed shapes of the beams in beam-to-column connections are described by pinned, semi-rigid, and rigid behavior. Engineers can assess the level of joint rigidity during the design process so that the resulting connections and frames meet performance expectations. It seems unlikely that a fully rigid joint can be designed for use in timber portal frames because of stiffness orthotropy. However, moment-resisting joints that are less than 50% rigid can be used in timber frames to develop frame-like behavior.

Keywords: Frames, semi-rigid connections, deformed shapes, timber, rigid connections, pinned connections, computer modeling.

INTRODUCTION

Rigid unbraced frames resist lateral forces by transferring reaction forces and bending moments of the beams through the beam-to-column connections to the columns and then to the foundation. Portal frames are useful architectural forms in that large open volumes can be developed, allowing for functional opportunities, flow of people, and architectural expression. A simple unbraced portal frame is shown in Fig. 1. Unbraced portal frames are widely used in both steel and concrete systems because rigid joints are easily designed with these materials. Some engineers have argued that timber cannot be used in unbraced frames because rigid joints cannot be designed for timber.

Timber portal frames can be created by using glulam rivets, nailed steel-gusset plates,

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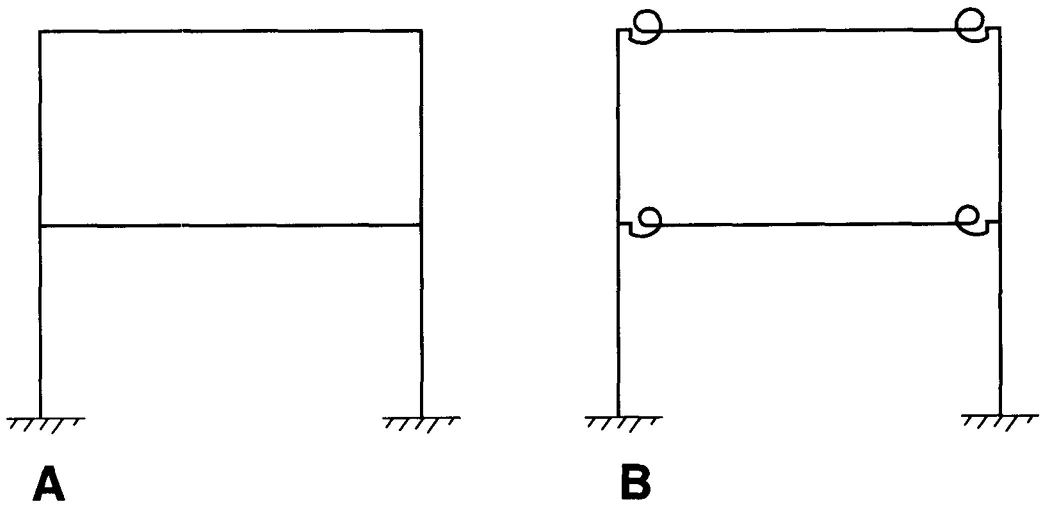

FIG. 1. A two-story, one-bay unbraced frame where (A) all connections are rigid; and (B) the beam-column connections are semi-rigid.

glued bars, and other technologies. Traditional post-and-beam joints also provide a frame-like structure. While the new and traditional connection systems work for many applications, they either are not acceptable for exposed architecture or fail to meet the demands of commercial-scale architecture. It is an architectural preference to minimize the visual impact of the connection, which simultaneously improves the skeletal expression of the structural system and represents an engineering solution that maximizes the material efficiency.

One of the main issues in developing commercial-scale architecture using timber frames is the design of joints with a predictable level of rigidity and the analysis of the structural system at the stated level of connection rigidity. In the timber engineering literature, connections being tested have been identified as *rigid* and *semi-rigid.* These reports give test data and ultimately present moment-rotation relationships for various connection designs. However, several fundamental issues have remained begging:

- What constitutes joint stiffness?
- What are the boundaries for the conditions that are referred to as rigid and semi-rigid?
- How are these connections differentiated from pinned connections?

The objective of this research is to establish a basis for quantifying the degree of joint rigidity in rigid timber frames. It will be shown that a continuum of joint rigidity exists for moment-resisting connections. This continuum logically can be partitioned using deformed shapes from beams having pinned, semi-rigid, and rigid boundaries.

TECHNICAL BACKGROUND

Basic structural analysis defines a *rigid joint* as one in which the structural members in the joint do not change position with respect to each other as the connected structural members are subjected to deformation—e.g., moment is transferred through the connection and the joint rotates. This is in contrast to a *pinned joint*—one in which no moment is transferred through the connection and the connected structural members are free to rotate with respect to each other under the forces that cause structural deformation. The *semi-rigid* joint is an intermediate condition where there is transfer of moment and there may be member rotation and limited joint rotation. The bending

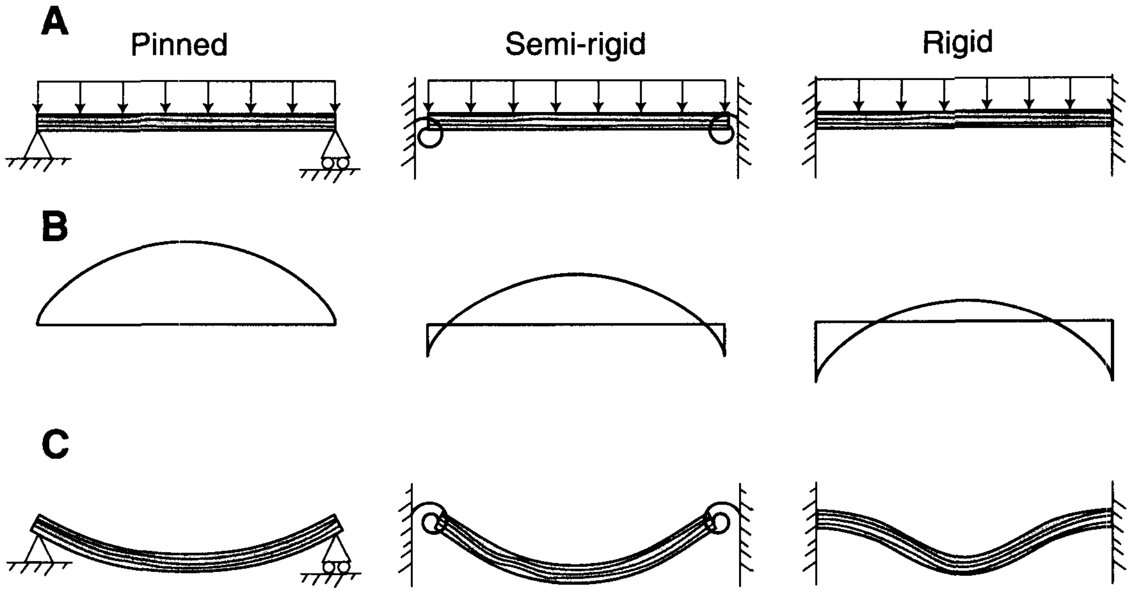

FIG. 2. Diagrams of beams having pinned, semi-rigid, and rigid end fixity: (A) loading, (B) qualitative bending moment, and (C) qualitative deformed shape.

moment in a beam is affected by the degree of connection rigidity, as is the deformed shape (Fig. 2).

In the analysis of a structure having semi-rigid joints, the connector flexibility can be accounted for by modifying the slope-deflection equations for a beam element (Dhillon and O'Malley 1999). The spring is assigned a rotational stiffness, k = force $\times$ length/radian. The columns are generally continuous and as a result do not have internal flexible connections.

The interaction of frame geometry and rotational stiffness of semi-rigid joints in symmetrically and asymmetrically deformed timber portal frames was investigated by Kikuchi (1991). His work showed relationships between the moment of inertia of the columns (I_c) and the beams (I_b), the ratio of the joint stiffness at the foundation and the beam-to-column connection, and the length of the beam (L) relative to the height of the column (H). His results seemed to indicate that frame sway was controlled by the rotational stiffness of the connections, while the stresses in the columns and the beams were a complicated function of rotational stiffness of the connections and geometric properties of the beams and columns.

McGuire (1995) examined the issue of semi-rigid connections in dynamically loaded beams of an unspecified material. His investigation focused on the minimum natural frequency for beams of uniform mass as affected by the joint stiffness—that is, whether the beam was connected by pinned, semi-rigid, or rigid connections. Using a finite-element program to study joint stiffness, McGuire demonstrated that the natural frequency shifted up or down, depending on the mass and the flexural stiffness (EI/L) of the beam, but that the curve shapes were the same. Ultimately, he developed a continuum for joint stiffness versus normalized minimum natural frequency (Fig. 3). Joint stiffness, a_{ij}, was defined as

$$a_{ij} = kL/E_b I_b \tag{1}$$

where k = rotational stiffness of the connection, L = beam length, E_b = modulus of elasticity of the beam, and I_b = moment of inertia of the beam (Fig. 3). Furthermore, McGuire demonstrated that semi-rigid pinned behavior occurs when $a_{ij} < 1.0$ because the deformed shape is

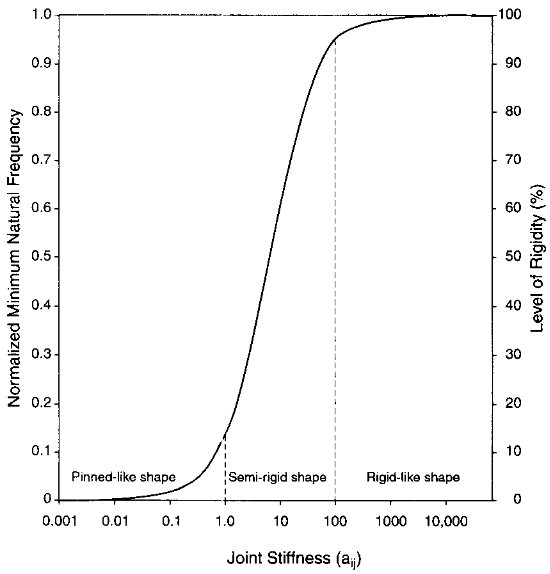

FIG. 3. The continuum of joint stiffness versus normalized minimum natural frequency (McGuire 1995) and the level of rigidity.

like that of a simply supported beam. He identified rigid behavior when $a_{ij} > 100$, because the deformed shape looked like that of a beam having fixed ends. Semi-rigid behavior was in the range $1.0 \leq a_{ij} \leq 100$, because the deformed shape was not like that of a beam with either pinned or fixed boundary conditions.

Recently, Dhillon and O'Malley (1999) developed an interactive design method for steel frames having semi-rigid connections. By balancing span and end moments in a beam, semi-rigid connections resulted in greater economy. Furthermore, story drift increased with connection flexibility.

In portal frames, when $I_b \gg I_c$ and the beam-column joint is rigid, the column inflection point develops at approximately the midheight. When $I_c \leq I_b$ and the beam-column joint is rigid, the inflection point moves from column midheight toward the beam-to-column joint. In most timber design scenarios, $I_c \geq I_b$ and the beam-column joint is not fully rigid; therefore, the frame deformation may depart from the previously stated conditions.

The literature demonstrates that, when alternative structural scenarios are modeled, the assumptions of joint rigidity will affect bending moments in the beams and, to a lesser extent, in the columns, in addition to story drift. Then, for analyses of alternative structural scenarios to be comparable, equivalent levels of joint rigidity are necessary for model input.

APPLICATION OF THE STIFFNESS CONTINUUM TO TIMBER FRAMES

For a joint in a timber frame to behave as a rigid joint, it must have the same deformation characteristics as the ideal rigid joint—bending of the connected members, rigid rotation of the joint, and transfer of moment. Clearly, the character of the joint can be manipulated by changing the length, stiffness, or cross-sectional geometry of the beam or by changing the rotational stiffness of the connection. Inasmuch as the dynamically deformed shapes of beams are approximately those of statically deformed beams, and the equations of motion can be reduced to the static case, the results of McGuire (1995) can be used for static analysis in timber portal frames. The level of rigidity on the right vertical axis of Fig. 3 was added to reflect this conclusion.

Information from physical models

Several groups have tested moment-resisting connections between timber beams and columns (Inayami and Sakamoto 1989; Komatsu 1989; Malhotra and Jin 1989; Batchelar and Hunt 1991; Komatsu et al. 1991; Cheng 1996; Hyde 1996). These connections were subjected to static and cyclic loading cycles. Only the static moment-rotation relationship is pertinent to this discussion. Table 1 shows approximate rotational stiffnesses determined from the figures and data of these reports. Also shown are some of the relevant details for the test specimens, as well as the joint stiffness determined from Eq. (1).

Although joints with high moment capacities (as much as 40,000 kNm/r) could be designed and fabricated, none of the joints behaved as though fully rigid when joint stiffness was defined by Eq. (1). Judging by the

TABLE 1. *Summarized results of rigid and semi-rigid connections testing.*

Investigator	E (MPa)	I (10^6 mm^4)	L (mm)	k (kNm/r)	α_{ij}	Comments
Batchelar and Hunt (1991)	9,000	—	5,600	27,024	0.04	matched beam and column, nailed steel gusset
Cheng (1996)	10,300	596	980	1,718	0.3	glulam rivet, steel side plates
Hyde (1996)	—	—	—	8,500	—	glued bars
Inayami and Sakamoto (1989)	12,000	195	1,400	4,800	2.8	column to steel foundation boot
Komatsu (1989)	9,000	1,562	3,000	11,300	2.4	dowels through steel plate
Komatsu et al. (1991)	9,000	1,562	3,000	39,650	8.4	nail plate
Malhotra and Jin (1989)	—	—	—	3	—	traditional mortise-tenon

level of rigidity shown in Fig. 3, the nail-plate joint by Komatsu was approximately 50% rigid, given the geometry and material stiffness of the system tested. Rigidity of the dowel beam-to-column joint of Komatsu (1989) was similar to that of the foundation connection of Inayami and Sakamoto (1989), 20 to 25% rigid, even though the rotational stiffness for the connections differed by a factor of approximately three. The rotational stiffness of the nail-plate connection by Batchelar and Hunt (1991) was larger than the other reported values (Table 1), but it had a pinned-like deformed shape because the *EI* of the beam was large. The test report by Hyde (1996) does not give material stiffness or beam geometry details. Hence, the joint stiffness cannot be determined, even though the rotational stiffness of the joint could be calculated from the moment-displacement diagrams.

In order for the rotational stiffness of the connection to be assessed correctly, the beam and column must be oversized, so that the deformation occurs in the connection, not as bending deflection in the connected members. The deformations of the Batchelar and Hunt test (1991) probably met this criteria because of the pinned-like deformation of the joint. It is easy to show from Eq. (1) that the connection by Batchelar and Hunt could achieve a high degree of semi-rigid behavior if the *EI* of the beam was reduced or the beam length was sufficiently increased.

Investigation by computer modeling

For the purpose of illustrating the interaction of beam flexural stiffness and connection stiffness in the joint stiffness, a simple beam-to-column model was devised. A structural analysis program (Multiframe®) was used as the computational tool. This software uses a linear stiffness method of analysis and provides rotational springs so that semi-rigid joints can be modeled. Because the linear elastic approach assumes that the deformations remain small, geometric and material nonlinearities were not allowed. This strategy yields an indication of what would be expected in the design range.

In all, five analyses were carried out on the beam-to-column assembly. Three analyses were conducted to demonstrate the effect of the beam on the joint stiffness, assuming a relatively rigid column. Figure 4 shows the geometry and the boundary conditions of the model; Table 2 gives the numerical values for member stiffness and size. The flexural rigidity of the column was made to approximate infinity, and the I_b was manipulated to demonstrate the effect of beam E_bI_b on the joint stiffness and the degree of rigidity. In Case 1, the joint stiffness was in the midrange of semi-rigid behavior; in Case 2, the joint stiffness was at the boundary where the joint would appear to have a fixed deformed shape. The same frame geometry was used in Case 3, but the high I_b would cause the joint to approach a pinned deformed shape. The rotational stiffness k = 10 MNm/r was arbitrarily selected and used in all five models because it was an attainable level of stiffness in timber engineering.

In application, the deformation and rotation of the joint are affected by both the beam and

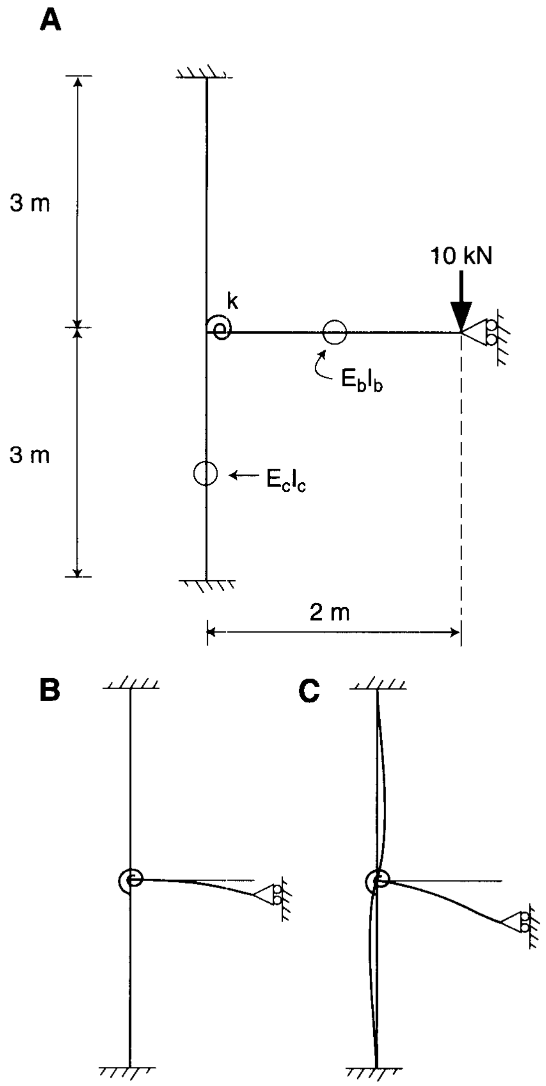

FIG. 4. The model used to assess the effect of beam and column stiffness on the joint stiffness and frame behavior: (A) column-and-beam frame model having a rotational spring, showing the load and boundary conditions for the five examples, (B) the deformed shape for Case 1, and (C) the deformed shape for Case 5.

the column and the rotational stiffness of the connection, k, which we know from Kikuchi (1991) and basic analysis. We cannot isolate the rotational stiffness of the connection and the beam, as did McGuire (1995), and still have a correct solution to the problem, because the column may be less than rigid relative to the beam. Two additional models were developed to demonstrate the effect of flexible columns, such as those that might be found in a design solution for a timber portal frame. In Case 4, the column was 250 × 250 mm, a design solution for a frame we were recently examining. Here the E_cI_c was approximately five times greater than the E_bI_b. In Case 5, the I_c of the column was reduced so that the $E_cI_c = E_bI_b$; the column was 160 × 160 mm. Both Cases 4 and 5 used the same beam geometry and stiffness as in Case 1. The a_{ij} for Case 5 is shown in Table 2 as >6 because it was expected that the frame would attain a more rigid-like shape than the frame of Case 1; however, the actual a_{ij} could not be determined because the boundary conditions departed from those of Eq. (1).

The results of the analyses are summarized in Table 3. In Cases 1, 2, and 3, the same moments occurred at the beam-to-column connections and at the foundation connections for all three models. This is because the rotational stiffness of the joint was the same in each model and governed the beam-to-column moment. However, the vertical deflection of the beam at the load point was nearly zero for the beam having a pin-like deformation (Case 3) and large for the beam having rigid-deformation characteristics (Case 2). This indicates that the greater percentage of vertical displacement results from beam bending, and not necessarily from joint rotation. Interestingly, in order for the joint to deform as though fixed, the beam flexural stiffness must be low—so low that the member would fail both resistance and serviceability functions of most design situations.

In Cases 4 and 5, the column was made less than infinitely rigid relative to the beam. When the column was five times stiffer than the beam (Case 4), the column appeared to be nearly rigid relative to the beam—hence, the small joint rotation. This led to a condition in which nearly all deformation occurred as beam bending, as in Case 1, with some additional deflection caused by joint rotation through column bending. Here the estimate of

TABLE 2. *The material and geometric parameters used in the structural models; k = 10M Nm/r for all models. Column size (mm) for each case appears in parentheses under* I_b *or* I_c.

Case	E_b (MPa)	I_b (10^6 mm^4)	E_c (MPa)	I_c (10^6 mm^4)	a_{ij}
1	11,500	58 (210 × 75)	rigid	—	6
2	11,500	44 (220 × 50)	rigid	—	100
3	11,500	34,952 (1,280 × 200)	rigid	—	0.01
4	11,500	58 (210 × 75)	11,500	326 (250 × 250)	~6
5	11,500	58 (210 × 75)	11,500	58 (160 × 160)	>6

joint stiffness from Eq. (1) leads to the same conclusion about apparent joint behavior as in Case 1. Case 5 shows the result of having the deformation occur in the column and the beam where a_{ij} = 6. In Case 5, vertical deflection was increased and the connection moments decreased. The beam-to-column joint exhibited joint rotation, which is a rigid-like behavior, and simultaneously had a reduction of moment at the connections relative to the cases having columns with greater flexural rigidity values.

In these examples, the height of the column was held constant and it was assumed to be fixed at both ends, but clearly, as the boundary conditions and length parameters are changed, the column characteristics will affect the solution in a fashion that parallels the beam effects. If the column boundary conditions were modified to represent pinned connections or the column was lengthened, some additional deformation would occur as joint rotation, a result of column bending. Modification of the rotational stiffness of the beam-to-column joint will change the deformed shape of the frame, giving a more rigid-like shape if the rotational stiffness is increased (forcing beam- or column-bending deformation for the same energy input) and a more pinned-like shape if the rotational stiffness is decreased (less beam- or column-bending deformation).

TABLE 3. *Summarized results from the structural model.*

Case	Joint rotation (r)	Displacement (mm)	Moment (kNm) Joint[a]	Moment (kNm) Foundation[b]
1	0	40	10.0	5.0
2	0	70	10.0	5.0
3	0	<1	10.0	5.0
4	0.00182	44	9.1	4.6
5	0.00748	55	6.3	3.1

[a] Moment at the beam-to-column joint in the column.
[b] Moment at the column-to-foundation boundary.

Potential for rigid joints in timber structures

Inasmuch as the rigid behavior of the joint in the timber system is described by the connection rotational stiffness (k) and the beam geometry (I_b), stiffness (E_b), and beam length (L), the relationship between the k and the E_bI_b/L of the beams must be carefully balanced. Failure to assess these parameters correctly may lead to performance deficiencies at the designed connections.

In a parametric study of joint stiffness effects on moments and forces in columns and beams of a 3-story rigid frame (Leichti 1998), the joint behavior was modified from fully rigid to 50% rigid by reducing the k-value in Eq. (1) so that a_{ij} = 8. The change of rotational stiffness led to changes in numerical values for moments and forces and a reduction of required beam and column dimensions for the governing load case, while maintaining the requisite frame-like behavior. This result parallels that of Dhillon and O'Malley (1999) and suggests that timber frames may not need moment-resisting connections with levels of rigidity >50%. Additionally, analysis using the semi-rigid connection provided an optimized design solution for a frame with a known level of rigidity. The orthotropic stiffness of wood makes it very difficult to develop a connection with rigidity >50% (a_{ij} = 8). This is because the embedment deformation around the dowels (or nails) occurs at a load level that is small relative to the longitudinal stiffness of the structural member.

Our investigation used experimental data from the literature. It would have facilitated the research if the tests had been conducted by a standard methodology and reported in a standard format. Civil engineers researching connections in Europe are developing a standard protocol and report format as an activity of the European Cooperation in the Field of Scientific and Technical Research (COST), Working Group 4-Database (COST 1998). The major details to be specified in a standard test protocol would be a range of joint stiffness for evaluation of rotational stiffness; boundary conditions; and test configuration, loading practice, and measurement methods.

SUMMARY

Joint stiffness (a_{ij}) is a function of the connection rotational stiffness and beam flexural stiffness. Column characteristics also enter into the function; however, because columns in rigid timber frames are often stiffer than the beams ($I_c > I_b$), and shorter as well, the joint stiffness is largely controlled by the characteristics of the beam, rather than those of the column. A continuum of stiffness that can be used to determine the degree of rigidity in semi-rigid joints extends from pinned-like deformed shapes through rigid-like deformed shapes. The function for joint stiffness is a qualitative tool that will be useful to those seeking to investigate structural performance at a given level of rigidity. It would be useful to develop a function for joint stiffness that incorporates the beam, column, and joint characteristics.

These results show that true rigid behavior (moment transfer and deformed shape) may be neither possible nor desirable in timber frames. This is not to say that connections of sufficient rigidity cannot be designed to resist moments and forces in frames using current technology, such as nailed steel-gusset plates, glued bars, and doweled or bolted plates. The designer needs to recognize that the moments at the connections are a function of beam geometry and stiffness, as well as of rotational stiffness of the basic connection. An analysis that incorporates these variables will lead to design of moment-resisting connections that will meet performance expectations.

A standard test protocol and reporting method for assessment of moment-resisting connections would facilitate further research and possible implementation of results in design practice.

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