STATISTICAL CONSIDERATIONS FOR REAL-TIME SIZE CONTROL SYSTEMS IN WOOD PRODUCTS MANUFACTURING

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ABSTRACT

Currently, sawmill machinery companies are developing real-time size lumber size control systems using non-contact laser measuring systems. These systems rely on the application of industrial statistics to large quantities of lumber thickness and width data. Because of the sampling intensity and frequent decision making in real-time systems, there is an increased chance of committing Type I or Type II errors when drawing conclusions if statistical methods are incorrectly applied. There is confusion in the industry concerning the appropriate statistical model to use for lumber size control. This survey of the current literature discusses three distinct methods for calculating and partitioning sawing variation, and thereby calculating control limits for control charts. This paper reviews the statistical foundation and current understanding of industrial statistics for implementing real-time SPC systems and makes recommendations for improvement.

Keywords: Lumber size control, statistical process control, target sizes, control charts, real-time.

INTRODUCTION

Lumber production in a modern sawmill occurs at a high rate of speed. In practice, sawing variation is that quantity which measures the variation in width and thickness in sawn boards as a result of inaccurate sawing. This variation is usually caused by movement in the saws or the log hold down mechanisms during the cut (due to a variety of factors), or by movement in the saw or log positioning just prior to the cut. To maximize the value recovery from every log sawn, it is important that the cut be straight with as little side to side variation as possible. Many studies have shown that profits can be increased substantially by reducing the amount of the sawing variation (Maness and Lin 1995; Wang 1983; Lister 1997).

Measurement plays an important role in helping sawmills improve sawing practices and thereby reduce sawing variation. The basic size control system used by many mills today involves taking 1 or 2 samples per shift from each sawing machine. In each sample, a group of boards is pulled from the production line (usually 4 or 5), and a group of thickness and/or width measurements are taken on each board (usually 4–10 taken from fixed loca-

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tions). This results in 16–50 board measurements per sample from which the sample average and variance are calculated, as well as variance components that are used for the purpose of problem solving. In some cases, this information is plotted on statistical control charts for process averages and dispersion.

More recently, sawmill technology companies have focused on the development of real-time size control systems using non-contact laser measuring systems. Although other methods like time series modeling (Cook 1992; Young and Winistorfer 2001) have been proposed, the statistical procedures used in these systems are based largely on procedures developed in the 1970s and 1980s. Because of the sampling intensity and frequent statistical decision making in real-time systems, there is an increased chance of committing Type I or Type II[1] errors when drawing conclusions if the statistical methods are incorrectly applied.

This paper is the introductory article of a series aimed at improving the statistical foundation for implementing real-time statistical process control (SPC) systems. The paper reviews the development of the current understanding of industrial statistics for the purpose of lumber size control. In addition, it points out some assumptions made along the way that may cause problems in real-time systems and makes appropriate suggestions for improvement.

A REVIEW OF CURRENT METHODS FOR LUMBER SIZE CONTROL

A statistical approach for determining lumber target sizes

Basic methods.—Warren (1973) first suggested deriving a target board size by randomly selecting boards as they left a machine center and taking a prescribed number of thickness measurements at random locations along the length of the board. Although not explicitly stated, this technique relied on an underlying analysis of variance (ANOVA) model. Warren's design was a completely randomized, one-factor experiment with m treatments (the number of boards sampled) and n measurements per treatment (the number of places that each board is measured along its length). The observations from a one-factor completely randomized design (CRD) are described by the model:

$$y_{ij} = \mu_i + \epsilon_{ij} \qquad (1)$$

where: y_{ij} is the jth observation from the ith board (treatment), μ_i is the ith board (treatment) effect, and ϵ_{ij} is the random error component.

The overall mean of all treatments is μ. The model errors are assumed to be independent and normally distributed with a mean of zero and a variance of σ^2, i.e., $\epsilon_{ij} \sim \text{NID}(0, \sigma^2)$. The treatment means are also independent and normally distributed with a mean of μ and variance of $\sigma_\mu{}^2$, i.e., $\mu_i \sim \text{NID}(\mu, \sigma_\mu{}^2)$. The treatments and errors, μ_i and ϵ_{ij}, are independent, and therefore, the board thicknesses, y_{ij}, are normally distributed.

The total sums of squares for the model (SS_T) are partitioned into sums of squares due to treatment effects (SS_{TR}) and sums of squares due to experimental error (SS_E). This corresponds to board-to-board differences and within board differences, respectively. The elements of the analysis of variance (ANOVA) table for a one-factor CRD are shown below in Table 1 (Hicks and Turner 1999).

Warren computed the within- and between-board sample variances, S_w^2 and S_b^2, as:

$$S_w^2 = MS_E \qquad (2)$$

$$S_b^2 = \frac{MS_{TR} - MS_E}{n} \qquad (3)$$

S_w^2 and S_b^2 are the unbiased estimates of the sample variances within and between boards,

[1] In the context of statistical process control, a Type I error occurs when the manufacturer concludes (falsely) that a correctly performing process is out of control. A Type II error occurs when the manufacturer fails to detect that a poorly performing process is out of control.

TABLE 1. *Completely randomized one-factor ANOVA table.*

Source of variation	Sums of squares	Degrees of freedom	Mean squares	F
Between boards	$SS_{TR} = n\,\Sigma_i\,(\bar{y}_{i.} - \bar{y}_{..})^2$	$m - 1$	$MS_{TR} = SS_{TR}/(m - 1)$	$F = MS_{TR}/MS_E$
Error (within boards)	$SS_E = \Sigma_i\,(y_{ij} - \bar{y}_{i.})^2$	$m(n - 1)$	$MS_E = SS_E/[m(n - 1)]$	
Total	$SS_T = \Sigma_i\,\Sigma_j\,(y_{ij} - \bar{y})^2$	$mn - 1$		

where: $\bar{y}_{i.}$ is the mean thickness of the i^{th} board and $\bar{y}_{..}$ is the overall mean thickness of all boards.

$\hat{\sigma}^2$ and $\hat{\sigma}^2_{\mu}$, respectively.[2] The total variation S_t^2 is equal to the sum of the within- and between-board variances:

$$S_t^2 = S_b^2 + S_w^2 = \frac{MS_{TR} - MS_E}{n} + MS_E$$
$$= \frac{MS_{TR} + (n - 1)MS_E}{n} \qquad (4)$$

Warren gave the target board thickness, T, based on S_w and S_b, by means of a lookup table. The underlying values were calculated as:

$$T = y_F + Z_\alpha S_t = y_F + Z_\alpha\sqrt{S_b^2 + S_w^2}$$
$$= y_F + Z_\alpha\sqrt{\frac{MS_{TR} + (n - 1)MS_E}{n}} \qquad (5)$$

where: y_F is the finished lumber thickness, Z_α is from the standard normal table, and other terms are defined above.

In subsequent publications, Brown (1982, 1986) calculates S_w, S_b and the target board thickness using the same methods as Warren (1973). Neither Warren nor Brown included a system for monitoring lumber size or methods for constructing confidence limits that could be used in a statistical process control system.

This approach formalized a structure for using industrial statistics to determine the appropriate lumber target size and correctly identified the importance of partitioning the within- and between-board variance to calculate the total variation. A weakness of the method for calculating target sizes is that it assumed that the variation in lumber sizes was due solely to sawing variation. In fact, boards in a sawmill are incorrectly sized for a number of reasons in addition to sawing variation (e.g., variation in the drying and planing process). These factors should also be included in target size calculations. However, the Warren method provided a solid statistical foundation for further development in this area.

Smithies (1991) outlined a different method for computing the target board thickness. In this method, the between- and within-board variances were computed as Eqs. (6) and (7) below:

$$S^2_{b[S]} = \frac{SS_{TR}}{n(m - 1)} = \frac{MS_{TR}}{n} \qquad (6)$$

$$S^2_{w[S]} = \frac{SS_E}{m(n - 1)} = MS_E \qquad (7)$$

$$S^2_{t[S]} = \frac{(m - 1)nS^2_{b[S]} + m(n - 1)S^2_{w[S]}}{nm - 1}$$
$$= \frac{SS_{TR} + SS_E}{nm - 1} = \frac{SS_t}{nm - 1} \qquad (8)$$

The within-board variation (Eq. [6]) matches the result of Warren (1973). However, the between-board and total variation are calculated differently. In Eq. (6), the between-board variation is simply the variance of the average board thicknesses. However, the variance of the average board thicknesses is partially composed of the within-board variation, since by the central limit theorem the variance of a group of averages is σ^2/n even if there is no "between-board" effect. Total variation (Eq. [8]) is the variance of all observations, taken without regard to the different boards from which the observations were taken. This un-

[2] While the sample variance, S^2, is an unbiased estimator of the population variance, σ^2, the sample standard deviation, S, is not an unbiased estimator of the population standard deviation. Instead, S estimates $c_4\sigma$, where c_4 is a constant that depends on the sample size. See Montgomery (2001) for a discussion.

grouped or "overall variance," however, gives a biased estimate of the total variance when $\hat{\sigma}^2_\mu \neq 0$ (Hicks and Turner 1999). When $\hat{\sigma}^2_\mu > 0$, this estimate will always be less than the total variance, as computed with S_b and S_w. Unless there is no board-to-board variation, the variance computed with this method will underestimate the total variation present. This can cause problems when this estimated variance is then used either for determining target sizes or as the basis of a statistical process control system.

Substituting the mean squares into Eq. (8) yields:

$$S^2_{t|S|} = \frac{(m - 1)MS_{TR} + m(n - 1)MS_E}{nm - 1} \quad (9)$$

Comparison of (9) with (4) demonstrates the differences in the two approaches with regards to calculating total sawing variation.

Considering distinct sources of within-board variation.—Wang (1984) also conducted a study to develop equations for calculating target sizes. In addition to separating the total sawing variation, S_t, into S_w and S_b, Wang partitioned S_w into two independent components: S_{wl}, within variation along the board length, and S_{ww}, within variation across the width of the board. Thus,

$$S^2_w = S^2_{wl} + S^2_{ww} \quad (10)$$

Furthermore, the author attempted to consider the effect of sawing variation on both surfaces of the board (top and bottom). He then assumed that the total within-board variation, S^2_w, was comprised of the within-board variation on each surface.

$$S^2_{ww} = S^2_{ww_{top}} + S^2_{ww_{bot}} \quad (11)$$

Assuming that the top and bottom variation are equal yields:

$$S^2_{ww} = 2S^2_{ww_{top}} \quad (12)$$

The resulting target size equation given was:

$$T = y_F + 3(S_{ww}/\sqrt{2} + \sqrt{S^2_w - S^2_{ww}/2} + S_b) \quad (13)$$

The mathematical derivation of (13) and this result are questionable given the partitioning of variances that underlie the ANOVA model (Eq. [4]). However, the approach is of interest in that it involves several innovations. First, it accounted for two distinct sources of within-board variation. Second, it considered the impact of sawing variation on both the top and bottom faces of the board. The result would have been more successful mathematically had it started with an appropriate ANOVA model incorporating the innovations, and then derived the correct partitioning of the variances.

Summary of target size methods.—The work reviewed above has shown that the target size is a function of the sawing variation. Warren (1973) developed the basic statistical approach of Eq. (5) and used the ANOVA model to estimate the total sawing variation and correctly partition it into components. As will be seen in the next section, this foundation was a significant contribution to the understanding and development of lumber size control systems.

Extension of the target size approach to statistical process control (SPC)

Background for SPC.—The target size determination methods outlined above identified the concepts of between- and within-board variation and set the stage for applying those concepts to an SPC system. W. A. Shewhart (1931) developed the basics of SPC in which a control chart was used to signal the presence of assignable causes in the underlying process.

The basic SPC system can be described as follows. Output from a manufacturing process is sampled by selected subgroups of size m items at regular intervals. The quality characteristic (e.g., dimension) is measured on each of the m items in each subgroup, and the averages and sample standard deviations are calculated in the usual manner. A comparison over time of the sample mean and/or standard deviation against a statistical confidence interval based on a known standard determines whether the process is performing correctly. In the case of the average, the confidence interval is based on the Student's t-distribution (or a standard normal distribution if the sample size exceeds 30). In the case of the standard deviation, the confidence interval is based on the Chi-square distribution. Various constants have been developed and tabulated to simplify the method for use in practical applications and to correct for sampling bias when using the sample standard deviation instead of the sample variance in the comparisons.

To simplify use in a manufacturing environment, a graphical control chart is created. While there are many types of control charts, two in particular are used most frequently: the X-Bar chart for process averages and the S (or R) chart for process dispersion. While both must be used in combination to be effective, practitioners often use the X-Bar chart alone. A scatter plot is created in which the X-axis represents the sample number and the Y-axis represents the variable of interest. The variable of interest in the case of the X-Bar chart would be average board thickness, while in the case of the S Chart, it would be the thickness standard deviation. In both charts, the underlying process standard deviation must be known and the process should be stable (or under statistical control).

In each case, an upper and lower control limit is calculated, which is typically three standard deviations (corrected for bias) above and below the mean. For each sample, the variable of interest is plotted on the chart and, if it is outside the control limits, the presence of assignable causes is indicated and remedial action should be taken. These three sigma limits yield a probability of false rejection (α) of 0.135% in each tail. Thus, the control chart should have a false alarm rate of approximately 1 sample out of 740 on each control limit. This is generally considered acceptable in production.

From this description, it can be seen that the correct estimation of the mean and standard deviation is crucial. In most cases, the practitioner assumes that the sampled items come from a normal distribution. However, this becomes complicated in the case of lumber size control. The thickness of a piece of lumber varies all along its length. Thus, it is impossible to measure m items—instead the thickness of each item must be measured at n locations. This presents interesting possibilities. For example, the process variation is partitioned into *components* that help lead to problem resolution. However, herein lies the source of the difficulty in applying the standard SPC techniques. The practitioner must be very careful to estimate the partitioned variances correctly or the method can lead to erroneous results.

The practitioner could ignore the presence of the two distinct sources of variation (within- and between-board) and calculate only the sample average and total sawing variation, as is suggested by most standard quality control textbooks. The total sawing variation would then be the standard deviation of all the measurements taken in the sample (as was done by Smithies in [8]). This is, in fact, done in practice at many sawmills. However, as shown above, that would lead to a biased result if between-board variation is non-zero, which is likely. This is an important result in itself, and should be noted by system developers.

Development of the lumber size SPC system.—Whitehead (1978) extended the statistical methods developed by Warren to a statistical process control procedure. In the proposed procedure, a subgroups were randomly selected at time intervals. Each subgroup consisted of m boards (systematically selected), and each board was measured at n locations. In ANOVA terms, it was a nested model similar the CRD model of Equation (1).

Whitehead extended the methodology developed by Ryan (1989) for making control charts of measurements in subgroupings. In this method, the variation within subgroups is isolated and the variation between subgroups is not considered as subsequent monitoring is conducted with one subgroup at a time.

The analysis was conducted using the average range, which was converted via approximation into the standard deviation—a practice widely accepted at the time. The average range per subgroup, $\bar{R}_{w_i}$, and overall average range, $\bar{\bar{R}}_w$, were calculated as:

$$\bar{R}_{w_i} = \frac{1}{b}\sum_j R_{w_{ij}} \quad \text{and} \quad \bar{\bar{R}}_w = \frac{1}{a}\sum_i \bar{R}_{w_i} \qquad (14)$$

where: a is the number of boards in the sample subgroup, b is the number of subgroups in the overall sample, and $R_{w_{ij}}$ is the largest width measurement minus the smallest width measurement for each individual board in the subgroup.

The within-board standard deviation was calculated as:

$$S_w = \bar{\bar{R}}_w/d_2 \qquad (15)$$

where: d_2 is the statistical constant developed to convert ranges to standard deviation based on the distribution of the relative range, (R/σ). (See Montgomery [2001])

Sample ranges are converted to sample variances throughout this paper for simplicity in understanding and for ease in comparing the different statistical methods discussed. It can be shown that S_w in Eq. (15) approximately corresponds to the error variance, $\hat{\sigma} = MS_E$, within the ANOVA framework.

The average thickness of the *j*th board from the *i*th subgroup is $\bar{y}_{ij\cdot}$. The average range by subgroup, $\bar{R}_{b_i}$, and the overall average, $\bar{\bar{R}}_b$, were calculated as:

$$\bar{R}_{b_i} = \max_j(\bar{y}_{ij\cdot}) - \min_j(\bar{y}_{ij\cdot}) \quad \text{and}$$

$$\bar{\bar{R}}_b = \frac{1}{a}\sum_i \bar{R}_{b_i} \qquad (16)$$

The between-board standard deviation was then defined as:

$$S_b = \sqrt{(\bar{\bar{R}}_b/d_2)^2 - S_w^2/n} \qquad (17)$$

Using ANOVA terms and algebra, it can be shown that Eq. (17) reduces to:

$$S_b = \sqrt{\frac{MS_{B(A)} - MS_E}{n}} \qquad (18)$$

Using Eq. (18), S_b is the component of variance associated with boards within a subgroup, $\hat{\sigma}_\beta$.

As in Warren (1973), Whitehead computed the total variation as the sum of the within- and between-board variances (Eq. [4]) and the target board thickness as in Eq. (5). Whitehead used these estimates of process variation to develop a SPC system for lumber size using randomly selected groups of boards and control charts for the average board size, the within-board size variation, and the between-board size variation. The 3-sigma control limits for the Target Size, T, are:

$$T - A_2\bar{\bar{R}}_b \leq \mu \leq T + A_2\bar{\bar{R}}_b \qquad (19)$$

This corresponds to the following equation using standard deviation instead of range[3]:

$$T - 3S_b/\sqrt{m} \leq \mu \leq T + 3S_b/\sqrt{m} \qquad (20)$$

The upper control limits for the range within-boards and between-boards were calculated as $D_4\bar{\bar{R}}_w$ and $D_4\bar{\bar{R}}_b$, respectively. The lower control limit was zero because, based on the sample size, it would otherwise be negative. These control limits can be expressed in terms of standard deviation by using the estimates of S_w (Eq. [15]) and S_b (Eq. [18]) as B_4S_w and B_4S_b, respectively.

The work by Whitehead admirably extended the basic target size concepts developed by Warren into an SPC system. The basic concepts were correct; however, there is an issue of concern with respect to the estimation of statistical parameters. The standard error term in Eq. (20) does not account for all of the variation in the board thickness as it should correspond to the standard error of the mean of a subgroup of boards, $\hat{\sigma}_{\bar{y}_i\cdot}$.

SPC considering measurement location.—Brown (1979) used restricted randomization in

[3] The standard error term in (20) should use the total standard error of the mean of a subgroup, which includes the within and between components of variation. Whitehead partially corrected this mistake in a subsequent paper in which S_b was replaced by S_t in Eq. (20). All other aspects of the SPC procedures were unchanged from the earlier paper. See Neter et al. (1996) for a complete discussion of partitioning the variances.