TWO-DIMENSIONAL FINITE ELEMENT HEAT TRANSFER MODEL OF SOFTWOOD. PART III. EFFECT OF MOISTURE CONTENT ON THERMAL CONDUCTIVITY

*Hongmei Gu**

Postdoctoral Research Associate

and

John F. Hunt

Research General Engineer

USDA Forest Products Laboratory[1]

Madison, WI 53726-2398

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ABSTRACT

The anisotropy of wood creates a complex problem for solving heat and mass transfer problems that require analyses be based on fundamental material properties of the wood structure. Most heat transfer models for softwood use average thermal properties across either the radial or tangential direction and do not differentiate the effects of cellular alignment or differences in earlywood and latewood. A two-dimensional finite element model that considers these basic structural characteristics was developed to determine the effective thermal conductivity as a function of cell alignment, cell porosity, and moisture content. This paper extends the initial model to include moisture content effects from the oven-dry to the fully saturated condition. The model predicts thermal conductivity values as a function of density and moisture content. Comparisons are made with established empirical equations found in the literature for the thermal conductivity of wood. The models developed in this study series are useful for enhancing our understanding of fundamental heat transfer effects in various wood boards.

Keywords: Finite element modeling, thermal conductivity, moisture content, heat transfer, cellular structure, geometric model.

INTRODUCTION

Wood is an anisotropic, porous material with complicated cellular and macro-scale structural features and material properties. Structurally induced anisotropic effects on heat and mass transfer have significant implications for drying lumber, heating logs in veneer mills, and hot-pressing wood composites. Anisotropy of wood is due to the orientation of the wood fiber (radial, tangential, or longitudinal) and structural differences between the development of the earlywood and latewood bands of annual rings. Softwood cells tend to align in straight radial rows because they originate from the same cambial mother cells, but the cells are not necessarily aligned in tangential rows. In the tangential direction, cell alignment can vary from 0% to a 50% offset, which Hart (1964) defined as the maximum misalignment. Though affected by growing conditions and tree species, earlywood and latewood cells can be assumed to be made of essentially the same material within the wall substance, for the purpose of heat transfer modeling. Cell porosity (percentage of openings in wood cell) may vary from 90% to 70% in earlywood and from 30% to 10% in latewood (Gu 2001).

In addition to structure, species, and growing conditions, wood moisture content has a signifi-

* Corresponding author.

[1] The Forest Products Laboratory is maintained in cooperation with the University of Wisconsin.

cant effect on heat and mass transfer. To measure this effect is not easy. Early in the 1940s, MacLean (1941) pointed out that the conductivity of wood with certain moisture content as determined under steady-state conditions does not represent the true conductivity of the wood under the original moisture distribution conditions. The redistribution of moisture during the process of the experiment results in slight errors in moisture measurement. We believe a better value of thermal conductivity of wood could be obtained by theoretical modeling because modeling with fundamental principles does not involve the moisture redistribution errors that are unavoidable in physical testing. The theoretical model is only as good as its material property inputs and its ability to describe the process. Based on the ability to have accurately described information about the process, a model can be used to study the heat transfer issue in more detail and hopefully more accurately.

Significant research has been done to measure the thermal conductivity of different species at different moisture contents, resulting in the development of regression models (MacLean 1941; Wangaard 1943; Stamm 1960; Hendricks 1962; Kohlmann 1962). These models curve fit empirical data to develop the relationship between thermal conductivity, wood density, and moisture content. The results are average values for a combined earlywood and latewood configuration that does not include effects at the cellular level.

Hart (1964) and Siau (1995) used an electrical resistive modeling technique to describe thermal conductivity effects for a unit cell. Equivalent thermal conductivity models based on electrical resistive circuit principles have been extensively developed for many applications. These steady-state one-dimensional equations are well established and found in fundamental heat transfer texts. Hart (1964) described thermal conductivity in cell-wall material but did not include the effects of air, vapor, or free water in the cell lumen. Siau (1995) included vapor effects in the lumen, but he limited his investigation to moisture contents below the fiber saturation point and porosity greater than 25%.

Heat transfer involves complex two-dimensional flow and should be studied by a method capable of handling these complex characteristics. The model developed by Hart (1964) and Siau (1995) is one-dimensional. The finite element analysis (FEA) model developed in Part I of this series[2] and described in more detail in the following section uses two-dimensional analysis in its calculations. We believe the FEA modeling approach may be a better predictor of cellular thermal conductivity for a wide range of cellular density and moisture contents. While heat transfer is a three-dimensional phenomenon, the two-dimensional model can provide a better understanding of the wood/water relationship in the heat transfer process through a radial or transverse section of softwood. Additional modeling is needed to include the longitudinal heat transfer characteristics, but that is beyond the scope of this study.

MODEL DEVELOPMENT

In Part I, a physical model of the cellular structure of softwood was developed to simulate structural cell porosity and cell alignment/misalignment based on the microscopic structure of wood. Porosity is the fractional void volume in a wood cell. Softwood cells tend to align in straight radial rows and are much less aligned in the tangential direction. The tangential alignment or misalignment between cells varies from zero to a maximum of 50%. Fully aligned cellular structure models are shown in Fig. 1. Conversely, fully misaligned or 50% offset models between the two rows of cells are shown in Fig. 2. Both geometric models were simulated in ANSYS software (ANSYS 2004) by creating modules with the special ANSYS parametric design language (APDL).

Finite element analysis was then applied to the two geometric models to determine the effect of cell porosity (0% to 90%) on thermal conductivity. Finite element type PLANE35 (ANSYS

[2] Hunt, J. F., and H. M. Gu. Two-dimensional finite element heat transfer model of wood. Part I. Effective thermal conductivity. *Wood and Fiber Science*. 38(4):592–598.

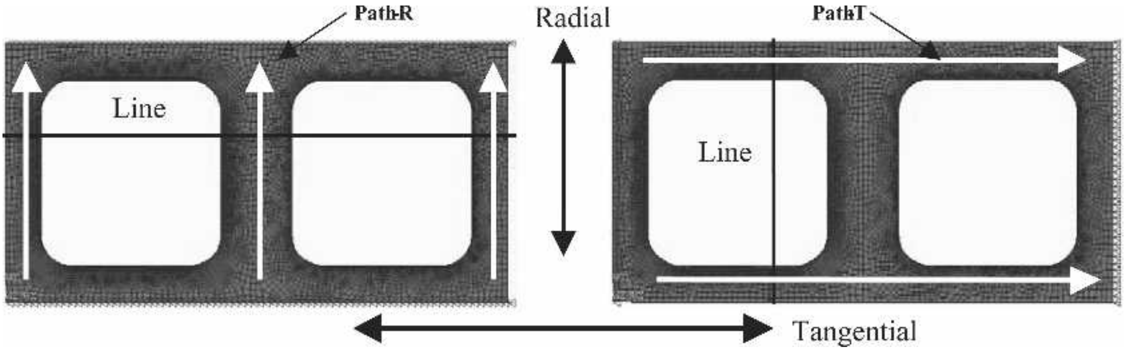

FIG. 1. Model of wood cells with 70% porosity in fully aligned case (0% offset) of cellular structure with no free water in lumen.

2004), a two-dimensional, six-node, triangular, thermal, solid element, was used to perform theoretical heat transfer analyses. The mathematical solution for conduction heat transfer of this element is based on the first law of thermodynamics, the energy conservation law (Eq. (1)):

$$\rho C_p \frac{\partial T}{\partial t} = k_{\mathrm{eff},X} \frac{\partial^2 T}{\partial x^2} + k_{\mathrm{eff},Y} \frac{\partial^2 T}{\partial y^2} \qquad (1)$$

where ρ is density of material, C_p is heat capacity, and $k_{\mathrm{eff},X}$, $k_{\mathrm{eff},Y}$ are effective thermal conductivity in x and y (radial and tangential) directions, respectively.

This model was developed for softwood species without any water in the cell. To study the effect of moisture content on thermal properties, the model can be modified by the addition of moisture in the cell wall and lumen. Six moisture conditions were studied and analyzed:

1. 0% moisture content (MC) in cell wall, dry air in lumen

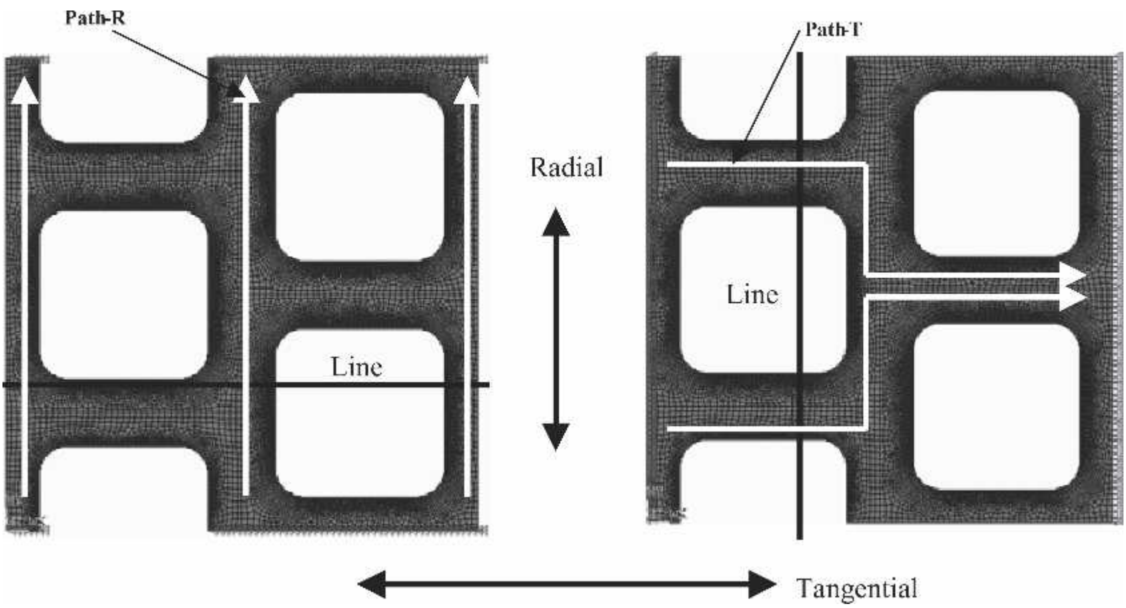

FIG. 2. Model of wood cells with 70% porosity in fully misaligned case (50% offset) of cellular structure with no free water in lumen.

TABLE 1. *Physical and thermal cellular properties used for model input variables.*

Cell component	Material properties in cellular model		
	Thermal conductivity (W/m · K)	Density (kg/m^3)	Specific heat (J/kg · K)
Cell-wall substance,[a] 0% MC	0.410	1,540	1,260
Air in lumen,[b] 0% MC	0.026	1.161	1,007
Bound water in cell wall[a,c]	0.680	1,115	4,658
Saturated cell wall,[d] FSP	0.489	1,415	2,256
Water vapor in cell lumen[e]	0.018	0.734	2,278
Free water in cell lumen[b]	0.610	1,003	4,176

[a] Source: Siau 1995.
[b] Source: Incropera and DeWitt 1981.
[c] Thermal conductivity and specific heat obtained on basis of water properties, assuming linear relationship with density.
[d] Obtained by rule of mixture.
[e] Source: Ierardi 1999.

2. Below fiber saturation point (FSP), cell wall MC is 15%
3. FSP, cell wall is fully saturated (30% MC) and only water vapor (no free water) is in lumen
4. Fully saturated cell wall and 30/70 volume ratio of water/water vapor in lumen
5. Fully saturated cell wall and 50/50 volume ratio of water/water vapor in lumen
6. Fully saturated cell wall, only free water in lumen (maximum MC at particular porosity or density)

The fiber saturation point is defined as the moisture content point where the cell wall is fully saturated with bound water and the cell lumen is full of water vapor (Siau 1995). It is assumed in this study that the fully saturated cell wall has a moisture content of 30%. Then, based on density, the cell-wall substance volume is 70.7% and bound water volume is 29.3% (Table 1) and the definition of moisture content (Eq. (2)).

$$MC_{cw} = \frac{\rho_{bw}(V_{bw}\%)V}{\rho_{cw}(V_{cw}\%)V} \qquad (2)$$

where

MC_{cw} = moisture content in cell wall only (defined as ratio of water weight over cellulose material weight or oven-dry mass weight),
ρ_{bw} = density of bound water,
ρ_{cw} = density of cell-wall substance,
V = total volume of cell (=1, assuming unit cell),
$V_{bw}\%$ = volume percentage of bound water, and
$V_{cw}\%$ = volume percentage of cell-wall substance (= 1− $V_{bw}\%$).

For 30% MC_{cw} (condition 3) and 15% MC_{cw} (condition 2) or any other moisture content below FSP, the volume percentage of bound water and cell-wall substance can be determined from Eq. (2). The overall cell moisture content can be calculated by adding vapor weight in the cell lumen to the total water weight in Eq. (2), assuming unit volume of the whole cell:

$$MC = \frac{\rho_{bw}(V_{bw}\%)(1)(1 - \text{Porosity}) + \rho_v(1)\text{Porosity}}{\rho_{cw}(1 - V_{bw}\%)(1)(1 - \text{Porosity})} \qquad (3)$$

where ρ_v is density of vapor.

Water vapor does not have a significant effect on the overall moisture content of the cell over the full range of porosity (less than 1% MC change from 0% to 90% porosity). This is because water vapor mass is very small compared to the mass of the cellulose and bound water.

The FEA model for the 30%/70% or 50%/50% volume ratio of water/vapor in the lumen assumes that 30% or 50% of the lumen volume consists of free water and 70% or 50% of the lumen consists of water vapor, respectively. Because of the surface energy tension of the water inside the lumen, the free water is assumed to spread "evenly" around the inside lumen surface, leaving a circular open volume of 70% or 50% for water vapor at the respective ratios (Fig. 3). It is assumed that the open volume is saturated water vapor only and no air.

The moisture conditions (except condition 1, 0% MC) were analyzed with ANSYS for heat flux and thermal properties in the radial and tangential directions. Effective thermal conductivity values were determined by simple conduction problems across the cellular models for 0% to 90% cell porosity at 10% increments. A temperature difference of 80 K across the two opposing boundaries was used; the other boundaries were set as adiabatic. No convective heat transfer effects between the water vapor/water and water/cell-wall interfaces were included in this model because we have no idea what values to give these interactions. A steady-state heat transfer problem with the conditions as such was solved by the finite element solver in ANSYS software. Without the convective terms, the final effective heat transfer coefficient values will be slightly conservative. The material properties used for input variables for cell-wall substance, lumen air, lumen water vapor, and lumen free water are listed in Table 1. The saturated cell-wall thermal properties were calculated by the rule of mixtures, with wall substance and bound water based on the calculated volume percentage for each component.

The total heat flux (q''_x) through the cells was determined by summation of the flux across a line (Figs. 1 and 2) and was used to determine the effective thermal conductivity (k_{eff}) using the definition (Incropera and DeWitt 1981) in Eq. (4):

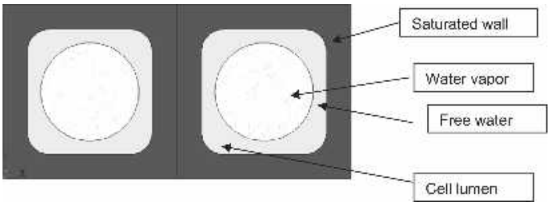

FIG. 3. Model of wood cells with 50% porosity in fully aligned case (0% offset) of cellular structure with 50% free water and 50% water vapor in lumen.

$$q''_x = -k_{\text{eff}} \frac{dT}{dx} \quad (4)$$

where

q''_x = heat flux (W/m^2),
k_{eff} = effective thermal conductivity (W/m · K),
dT = temperature change (K), and
dx = linear distance across cells (m).

RESULTS AND DISCUSSION

As found in our previous studies (Part I), the model results showed less than 1% difference between radial and tangential thermal conductivity values. Therefore, the average values of the two predicted effective thermal conductivity values from the model were plotted as a function of oven-dry density (Fig. 4). The oven-dry density for each point was calculated based on cell porosity and moisture content. For each MC line, 0% porosity (an impossible case, but shown for theoretical purpose) is at the rightmost end and 90% porosity at the leftmost end, with data points at 10% increments between these two extremes. Density at a particular moisture condition was calculated on the basis of volume percentage and density of each component in the wood cell structure, as shown in Eq. (5):

$$\rho_{\text{MC}} = (\rho_{\text{cw}} V_{\text{cw}}\% + \rho_{\text{bw}} V_{\text{bw}}\%)(1 - \text{Porosity}) + (\rho_v V_{\text{v}}\% + \rho_{\text{fw}} V_{\text{fw}}\%)\text{Porosity} \quad (5)$$

where

ρ_{MC} = wood density at certain MC,
$V_{\text{v}}\%$ = volume percentage of water vapor in lumen,
ρ_{fw} = free water density (Table 1), and
$V_{\text{fw}}\%$ = volume percentage of free water in lumen.

Figure 4 shows thermal conductivity as a function of oven-dry density for all six moisture conditions. These plots show that when wood is not fully saturated by water (i.e., certain amount of water vapor in lumen), thermal conductivity decreases significantly as density decreases. But when the cell lumen is fully filled with free water (maximum wood MC at that porosity or density), thermal conductivity increases as density

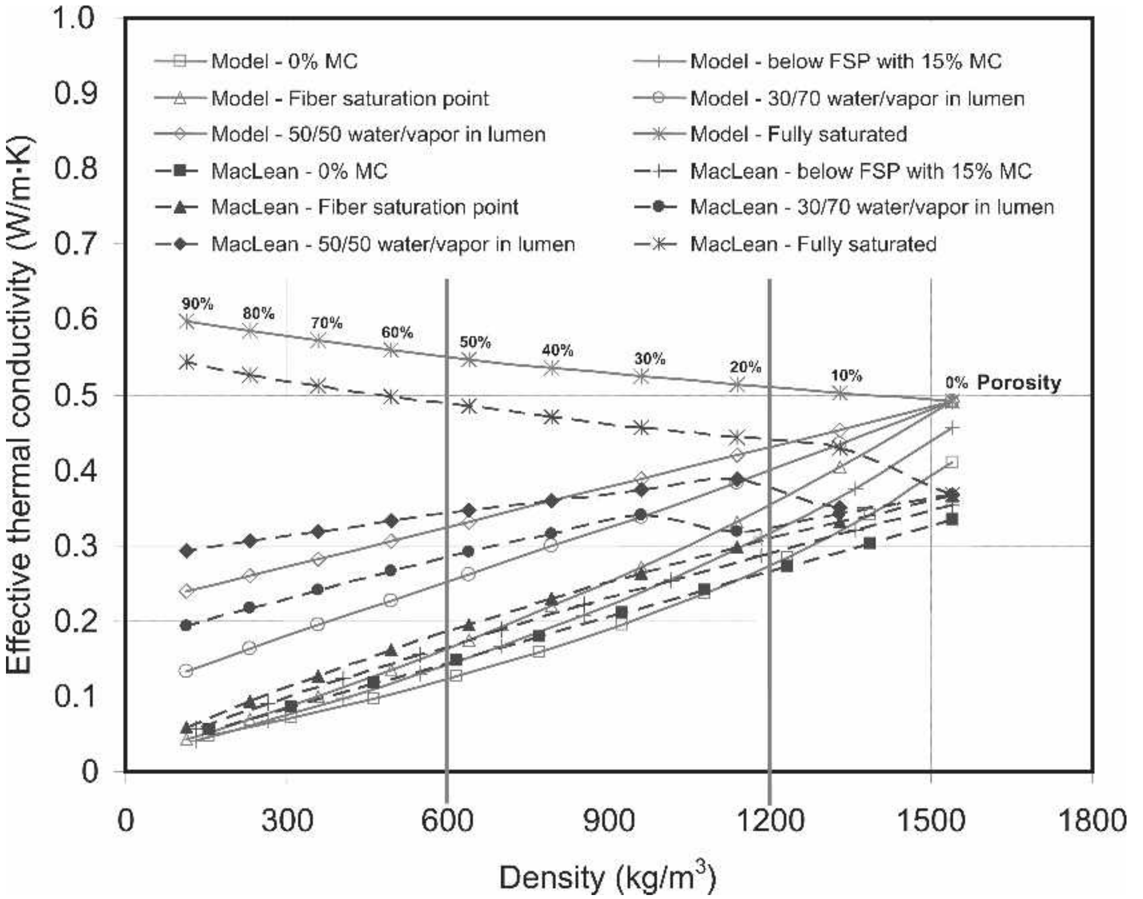

FIG. 4. Thermal conductivity values predicted from FEA model compared with empirical model estimates of MacLean (1941).

decreases. At the fully saturated condition, thermal conductivity through the water dominates the thermal conductivity effect through the wood cell structure. Thus, the lower the density (or the higher the porosity), the more the water in the wood and the higher the effective thermal conductivity for the wood. At any partially saturated condition, low thermal conductive vapor in the cell lumen has a dominant effect on the relationship between effective thermal conductivity and density. Theoretically, maximum thermal conductivity approaches that of water (0.61 W/m · K) as porosity approaches 100%.

This finite element analysis provides a complete set of thermal conductivity values as a function of oven-dry density and moisture content. To physically model wood cellulose structures under the moisture conditions, the percentage of free water and vapor in the lumen was used in generating the models and deriving thermal conductivity values. The actual overall moisture content in wood can be derived from the water/vapor percentage in the lumen, or vice versa. To make use of the model predicted thermal conductivity values as a function of wood density and moisture content, explicit and simple equations by statistical curve fitting to the model values are presented for a wide use of applications in a follow-up paper.[3]

[3] Hunt, J. F., H. M. Gu, and P. K. Lebow. Theoretical thermal conductivity equation for uniform density wood cells, in preparation.

Figure 4 also plots the results from MacLean's empirical model. Two empirical equations developed by MacLean (1941) were used in plotting, one for <40% MC and the other for >40% MC. The plots extrapolate the empirical equations beyond the data test conditions and show that the equations do not adequately describe wood cell thermal conductivity over the full range of density. However, most of MacLean's test data were taken from wood blocks in the density range of 600 to 1,200 kg/m^3. Therefore, his linear regression model shows a better agreement with our model results in that range but greater differences at the low and high ends of the range. When porosity approaches zero (high end of density), thermal conductivity should approach the cell-wall values at the specified moisture content, as shown by our theoretical model.

As discussed by MacLean (1941), the process of conducting the experiment causes moisture redistribution and results in slight interference in the true thermal conductivity measurement. As moisture content increases, the effect of moisture redistribution becomes more pronounced. The lines from MacLean's equation plotted for 50/50 water/vapor in the lumen and fully saturated conditions are significantly different from the finite element model. MacLean's test data were primarily focused around and below the fiber saturation point, with limited tests on a narrow set of species in the green or fully saturated state. The bumps in MacLean's 30/70 and 50/50 lumen water/vapor lines are due to the change of equations used for above and below 40% moisture content.

Another factor to consider is the type of samples used by MacLean. The samples were not uniform in density, but an average measurement of density was used across both earlywood and latewood portions over several rings and for various ring orientations. In contrast, the FEA model assumes no redistribution of moisture content and is based on the fundamental properties of homogeneous softwood material over the full range of porosity. The FEA model determines cell thermal conductivity with uniform porosity. Modeling thermal conductivity to include rings or earlywood/latewood bands and composition effects will be discussed in Part IV of this series of papers.

The authors believe the cellular FEA model can be used to determine effective thermal conductivity values through uniform density for earlywood and latewood material representing a more realistic characterization than the values obtained from averaged wood thermal conductivity experiments. The authors also believe the cellular finite element model can be used to determine effective thermal conductivities for a uniform density material without the problems associated with moisture redistribution. The model is only as good as are the assumptions for the inputs and the characterization heat transfer by the FEA mathematical representation. Our goal is to hopefully present a more realistic characterization than those obtained from averaged wood thermal conductivity experiments.

CONCLUSIONS

The cellular thermal conductivity finite element analysis (FEA) model provides a good theoretical approach to study heat transfer effects in a wood cell over a broad range of moisture conditions. Unlike actual tests, the model is not affected by moisture redistribution. The model is a uniform cell and thus can provide thermal conductivity values for uniform cell structure and moisture conditions for applications where earlywood and latewood differentiation is important. The empirical values in the literature were obtained from average density test samples, in which the earlywood/latewood effect was not differentiated. Knowing the thermal conductivity values of wood cells can help the study of heat transfer effects in boards where earlywood and latewood density (porosity), ring orientation, growth rate, and earlywood/latewood ratio are significantly different. The FEA model developed in this study can accommodate a more geometrical description of the cell, including the interior radius of the lumen as part of the heat transfer effects, which cannot be done at the cellular level with other models.

In Part IV of this series of papers, the thermal conductivity values from the FEA model will be used to study the effects of moisture content on transient heat transfer effects in softwood board. The effect of ring orientation, earlywood/ latewood ratio, and overall board density on heat transfer will also be addressed.

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