TWO-DIMENSIONAL HEAT FLOW ANALYSIS APPLIED TO HEAT STERILIZATION OF PONDEROSA PINE AND DOUGLAS-FIR SQUARE TIMBERS

William T. Simpson†

USDA Forest Service
Forest Products Laboratory[1]
One Gifford Pinchot Drive
Madison, WI 53726-2398

(Received December 2003)

ABSTRACT

Equations for a two-dimensional finite difference heat flow analysis were developed and applied to ponderosa pine and Douglas-fir square timbers to calculate the time required to heat the center of the squares to target temperature. The squares were solid piled, which made their surfaces inaccessible to the heating air, and thus surface temperatures failed to attain the temperature of the heating air. The surface temperatures were monitored during heating and related to time by an empirical equation. When this equation was used as the boundary condition in the finite difference solution, calculated time estimates required to heat the center to target temperature agreed favorably with experimentally observed heating times.

Keywords: Invasive species, heat sterilization, lumber, timber.

INTRODUCTION

Heat treatment is an increasingly common way to sterilize lumber, timbers, and pallets against invasive species such as insects and fungi. In a previous study (Simpson 2003), a one-dimensional finite difference solution to heat flow equations was used to calculate estimates of heating time for heat sterilization of slash pine (*Pinus ellióttii*) boards. The solution also included boundary conditions that allowed a continuously variable surface temperature. The one-dimensional solution works well for boards that are considerably wider than they are thick, but it cannot be applied to large cross-sectional dimension timbers and squares. In another study (Simpson et al. 2003), heating time data were collected for ponderosa pine (*Pinus ponderosa*) and Douglas-fir (*Pseudotsuga menziesii*) squares.

The objectives of the study reported here were to extend the one-dimensional finite difference solution to two dimensions so that it could be applied to large cross-sectional dimension timbers and squares and to test this solution on previously collected data for ponderosa pine and Douglas-fir.

EXPERIMENTAL METHODS

The experimental methods are described in detail in Simpson et al. (2003). Ponderosa pine specimens were 102-, 152-, and 305-mm (4-, 6-, and 12-in) square. Douglas-fir specimens were 89, 146, and 298 mm (nominal 4, 6, and 12 in; actual 3.5-, 5.75-, and 11.75-in) square. (Hereafter, squares will be referred to as 4×4, 6×6, and 12×12.) The squares were freshly sawn. The ponderosa pine was all sapwood, whereas the Douglas-fir had considerable heartwood (Fig. 1).

The previous study (Simpson et al. 2003) included two stacking configurations, stickered and solid piled. It also included measurement of surface temperatures as well as center temperatures, which are necessary to determine when the

† Member of SWST.

[1]The Forest Products Laboratory is maintained in cooperation with the University of Wisconsin. This article was written and prepared by U.S. Government employees on official time, and it is therefore in the public domain and not subject to copyright.

FIG. 1. Stack of ponderosa pine boards and square timbers ready for the kiln. This figure shows stickers between layers of boards/squares, but the data for this study were taken from unstickered stacks. Pieces with white tags have thermocouples embedded in their center.

target heat sterilization temperature has been achieved. In the experiments with solid piled squares reported here, surface temperatures were monitored on two adjacent surfaces of selected squares; these surface temperature data were used to test the two-dimensional finite difference solution. Thermocouples were used to measure both center and surface temperatures. The experimental method was given in Simpson (2003) and Simpson et al. (2003).

All heating was done in a 3.5-m^3 (1,500 board foot) experimental dry kiln at a constant dry-bulb temperature of 71°C (160°F) and a constant wet-bulb depression of approximately 0.8°C to 1.1°C (1.5°F to 2°F). Target center temperature was 56°C (133°F). Air velocity was approximately 3.1 m/s (600 ft/min).

Five replicates of the 4- × 4- and 6- × 6-in. squares and four replicates of the 12 × 12 squares were heated together. Dummy boards and squares were included in each stack so that all test squares were surrounded by other boards or squares. Thus, all test squares responded as squares within the stacks and not as edge squares. Figure 1 shows a stack of stickered test material ready for the kiln. Except for stickering, the unstickered stacks were similar to the stickered stacks.

ANALYTICAL METHODS

The equation that governs two-dimensional heat flow is

$$\frac{\partial T(x,y,t)}{\partial t} = \alpha_x \left(\frac{\partial^2 T(x,y,t)}{\partial x^2} \right) + \alpha_y \left(\frac{\partial^2 T(x,y,t)}{\partial y^2} \right) \quad (1)$$

where T is temperature, t is time, x and y are spatial coordinates, and α_x and α_y are diffusivities in the x and y directions, respectively, and are assumed to be the same in this analysis. MacLean (1941) has shown that diffusivities in the radial and tangential directions do not differ by much.

The two-dimensional finite difference equations that approximate the solution to Eq. (1) and the finite difference grid are shown in Appendix A; one equation is for the interior temperatures and the other for the center temperatures. As a first test of the two-dimensional finite difference analysis, heating times were compared with the two-dimensional equation developed by MacLean (1932) (also discussed in Kollmann and Côté 1968) and described and applied by Simpson (2001). MacLean's equation (Appendix B) can be applied only under the boundary conditions where the surface immediately attains and maintains the temperature of the heating medium. The finite difference analysis can be applied to these boundary conditions as well as boundary conditions where surface temperature varies with time. Comparisons of the calculated times required for the center of various sizes of lumber and timber to reach a target center temperature of 56°C (133°F) are shown in Table 1. The times calculated by the two methods are in close agreement.

The main purpose for the two-dimensional finite difference analysis was to accommodate boundary conditions of time-dependent surface temperature. In this study, surface temperatures were derived from fitting the following empirical equation to the surface temperature–time data collected for each experimental heating run:

$$T_s = T_h + \sum_{i-1}^{n} a_i \exp\left(b_i t^{i/2}\right) \quad (2)$$

where T_s is surface temperature,

TABLE 1. *Comparison of heating times of two-dimensional wood configurations calculated by MacLean's equation and by a finite difference method.*[a]

Wood configuration (inch (mm))	MacLean (min)	Finite difference (min)
2 × 2 (51 × 51)	23.1	21.2
6 × 6 (152 × 152)	207	204
8 × 8 (203 × 203)	369	361
10 × 10 (254 × 254)	576	567
12 × 12 (305 × 305)	830	864
16 × 16 (406 × 406)	1,475	1,444

[a]MacLean 1932, Kollmann and Côté 1968, and Appendixes.
Heating times calculated from center of specimen to 56°C (133°F). Conditions: heating temperature, 71°C (160°F); heating temperature, 71°C (160°F); initial temperature, 21°C (70°F); specific gravity, 0.5; moisture content, 90%; diffusivity, 1.60 × 10^{-3} cm²/s.

T_h is temperature of heating medium = 71°C (160°F) in this study,

t is time (minutes in this analysis), and

a and b are coefficients determined by nonlinear regression.

Equation (2) was developed from the basic requirement that the surface temperature will eventually attain the temperature of the heating medium. Thus, the equation was developed to be forced to converge to T_h at large times. For this analysis, n was taken up to a value of 3.

RESULTS AND DISCUSSION

Solid stacking the squares prevented the heating medium access to the surfaces. If the square timbers had been stickered, the surfaces would have immediately attained and maintained the approximate temperature of the heating medium because, with the small wet-bulb depression, the heating medium was nearly saturated steam. However, by solid stacking the squares (which is sometimes more practical than breaking down solid piled stacks), stickering them for heat treatment, and then restacking them in solid piled configuration, access to the surfaces was denied. Consequently, the surfaces did not immediately attain and maintain the temperature of the heating medium. This, in turn, reduced the temperature that the squares would have attained had they not been solid stacked. Similarly, even if the squares had been stickered but the heating medium had been air drier than saturated steam, the surfaces would not have immediately attained the air temperature because the drier air would have caused water to evaporate and thus cool the surfaces below the air temperature.

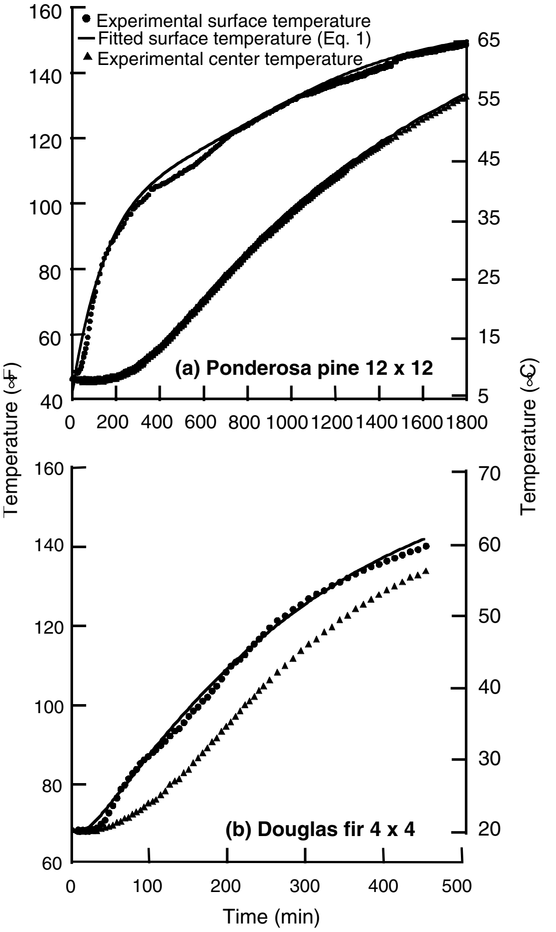

FIG. 2. Increase in surface and center temperatures with time for square timbers: (a) 12 × 12 ponderosa pine, (b) 4 × 4 Douglas-fir.

Figure 2 shows examples of the variation of surface and center temperatures with time for two different cases. The coefficients of Eq. (2) are shown in Table 2. Equation (2) is effective in characterizing the variation of surface temperature with time, with coefficients of determination (R^2) in excess of 0.99.

Table 3 compares the time for the centers of solid piled ponderosa pine and Douglas-fir square timbers to reach 56°C (133°F) when

TABLE 2. *Coefficients of Eq. (2) relating surface temperature to time (min) for solid piled ponderosa pine and Douglas-fir squares heated at 71°C (160°F).*

Species, config.	a_1	b_1	a_2	b_2	a_3	b_3	R^2
Ponderosa pine							
4 × 4	49.6	–0.202	–52.8	–0.0220	–91.8	–0.0000763	0.9981
6 × 6	–23.9	–0.0248	–3.09	–0.0393	–86.1	–0.0000486	0.9993
12 × 12	–2.37	-5.41×10^{-9}	–237	–0.00194	121	–0.0000885	0.9952
Douglas-fir							
4 × 4	123	–0.0285	–158	–0.00338	–52.4	-1.58×10^{-13}	0.9988
6 × 6	144	–0.0104	–179	–0.000341	–54.5	–0.000167	0.9987
12 × 12	–6.89	–0.193	–62.6	–0.000754	–28.1	–0.0000694	0.9987

TABLE 3. *Comparison of experimental and calculated (MacLean equation) heating times for timbers heated at 71°C (160°F) dry-bulb temperature and 1.1°C (2°F) wet bulb depression*[a]

	Heating time (min)			
Species, config.	Experimental[b]	MacLean	Finite difference in heating time (min)	Deviation (%)
Ponderosa pine				
4 × 4	831 (14.0)	101	730	13.8
6 × 6	1,201 (30.1)	217	1,214	–1.1
12 × 12	1,736 (26.4)	871	1,724	0.7
Douglas-fir				
4 × 4	432 (27.2)	70	427	1.2
6 × 6	977 (9.3)	197	1,038	–5.9
12 × 12	1,931 (13.5)	817	1,903	1.5

[a]Calculations of diffusivity (Simpson 2001) take ponderosa pine and Douglas-fir specific gravity as 0.38 and 0.45 and moisture content as 112% and 97%, respectively, giving diffusivities of 1.68×10^{-3} and 1.60×10^{-3} cm^2/s, respectively.
[b]Values in parentheses are coefficients of variation.

heated in saturated steam at 71°C (160°F). The comparisons are between experimental times, times calculated by MacLean's equation assuming that surface temperature immediately attains the heating medium temperature, and times calculated by the two-dimensional finite difference equations utilizing Eq. (2) to characterize the change in surface temperature with time. Times calculated by MacLean's equation were grossly shorter than experimental times, as expected. The finite difference equations were effective in estimating heating times, with the average of the absolute value of deviation from experimental times of less than 5%.

The practical usefulness of this two-dimensional approach requires knowledge of how surface temperature varies with time under any heating conditions (where the target center temperature is above the wet-bulb temperature of the heating medium (Simpson 2003)). Knowledge of how coefficients *a* and *b* of Eq. (2) vary with heating conditions or any other factors could lead to practical applications, and further studies could clarify this variation.

CONCLUSIONS

The two-dimensional finite difference heat flow analysis accurately estimates the time required to heat the center of ponderosa pine and Douglas-fir square timbers to target temperature when the heating medium is saturated steam and when the boundary conditions of a time-varying surface temperature are included in the solution.

REFERENCES

KOLLMANN, F. F. P., AND W. A. CÔTÉ, JR. 1968. Principles of wood science and technology. Springer-Verlag, New York, NY. 592 pp.

MacLean, J. D. 1932. Studies of heat conduction in wood. Part II. Results of steaming green sawed southern pine timber. *In* Proc. American Wood Preservers' Association. 28:303–329.

———. 1941. Thermal conductivity of wood. Heating, Piping, and Air Conditioning 13:380–391.

Simpson, W. T. 2001. Heating times for round and rectangular cross sections of wood in steam. Gen. Tech. Rep. FPL–GTR–130. USDA, Forest Serv., Forest Prod. Laboratory, Madison, WI.

———. 2003. Mechanism responsible for the effect of wet bulb depression on heat sterilization of slash pine lumber. Wood Fiber Sci. 35(2):175–186.

———, X. Wang, and S. Verrill. 2003. Heat sterilization time of ponderosa pine and Douglas-fir boards and square timbers. Res. Paper FPL–RP–607. USDA, Forest Serv., Forest Prod. Lab., Madison, WI.

APPENDIX A. FINITE DIFFERENCE TWO-DIMENSIONAL HEAT FLOW ANALYSIS

Schematic of two-dimensional quadrant (Fig. A) of wood square for finite difference calculations of temperature during heating.

	i direction (*x*)	*i* direction (*x*)	*i* direction (*x*)	*i* direction (*x*)	*i* direction (*x*)	*i* direction (*x*)
j direction (*y*)	T(1,1)	T(2,1)	—	—	T(n–1,1)	*T(n,1)*
j direction (*y*)	T(1,2)	**T(2,2)**	—	—	**T(n–1,2)**	*T(n,2)*
j direction (*y*)	—	—	—	—	—	—
j direction (*y*)	—	—	—	—	—	—
j direction (*y*)	T(1,n–1)	**T(2,n–1)**	—	—	**T(n–1,n–1)**	*T(n,n–1)*
j direction (*y*)	*T(1,n)*	*T(2,n)*	—	—	*T(n–1,n)*	*T(n,n)*

Finite Difference Equations

Surface temperatures (*i* or *j* = 1 in grid; Eq. (2) in text):

$$T_s = T_h + a_1 \exp(b_1\, t^{1/2}) + a_2 \exp(b_2\, t^{2/2}) + a_3 \exp(b_3\, t^{3/2}) + \ldots + a_n \exp(b_n\, t^{n/2})$$

Interior temperatures (bold-faced print in grid):

$$T_{i,j,t+1} = T_{i,j,t} + \alpha\, \Delta t/(\Delta x)^2 [(T_{i+1,j,t} - T_{i,j,t}) - (T_{i,j,t} - T_{i-1,j,t})] + \alpha\, \Delta t/(\Delta y)^2 [(T_{i,j+1,t} - T_{i,j,t}) - (T_{i,j,t} - T_{i,j-1,t})]$$

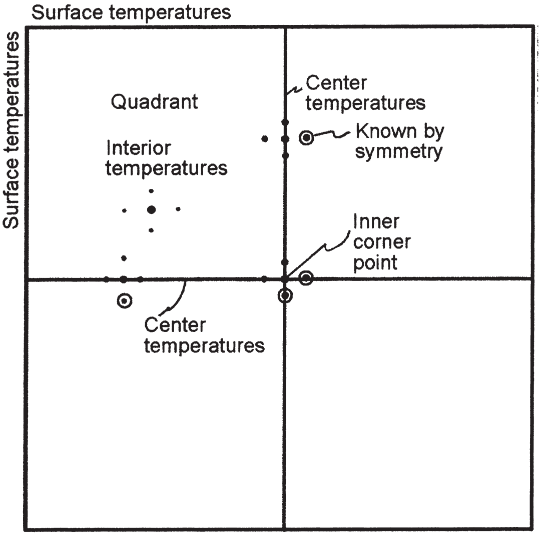

Fig. A. Schematic of cross section with quadrant and typical finite difference grid points.

Center temperatures (italics in grid):

x direction:

$$T_{n,j,t+1} = T_{n,j,t} + 2\alpha\ \Delta t/(\Delta x)^2\ (T_{n-1,j,t} - T_{n,j,t}) + \alpha\Delta t/(\Delta y)^2\ (T_{n,j+1,t} - 2T_{n,j,t} + T_{n,j-1,t})$$

y direction:

$$T_{i,n,t+1} = T_{i,n,t} + 2\alpha\ \Delta t/(\Delta y)^2\ (T_{i,n-1,t} - T_{i,n,t}) + \alpha\ \Delta t/(\Delta x)^2\ (T_{i+1,n,t} - 2T_{i,n,t} + T_{i-1,n,t}) \quad (A.3)$$

Inner corner point ($T_{n,n}$ in grid):

$$T_{n,n,t+1} = T_{n,n,t} + 2\ \alpha\ \Delta t/(\Delta x^2)\ (T_{n-1,n,t} - T_{n,n,t}) + 2\alpha\ \Delta t/(\Delta y)^2\ (T_{n,n-1,t} - T_{n,n,t})$$

APPENDIX B. MACLEAN TWO-DIMENSIONAL HEAT FLOW EQUATION FOR RECTANGULAR CROSS SECTIONS

The equation for rectangular cross sections is taken from MacLean (1932) and is the solution to the differential equation of heat conduction in the two dimensions of a rectangular cross section. The temperature *T* at any point *x* and *y* is given by

$$\begin{aligned} T = T_s &+ (T_0 - T_s)(16/\pi^2) \\ &\times\{\sin(\pi x/a)\ \sin(\pi y/b)\ \exp[-\pi^2 t(\alpha_x/a^2+\alpha_y/b^2)] \\ &+(1/3)\ \sin(3\pi x/a)\ \sin(\pi y/b)\ \exp[-\pi^2 t(9\alpha_x/a^2+\alpha_y/b^2] \\ &+(1/3)\ \sin(\pi x/a)\ \sin(3\pi y/b) \\ &\quad \exp[-\pi^2 t(\alpha_x/a^2+9\alpha_y/b^2] \\ &+(1/5)\ \sin(5\pi x/a)\ \sin(\pi y/b)\ \exp[-\pi^2 t(25\alpha_x/a^2+\alpha_y/b^2] \quad (A.4) \\ &+(1/5)\ \sin(\pi x/a)\ \sin(5\pi y/b)\ \exp[-\pi^2 t(\alpha_x/a^2+25\alpha_y/b^2] \\ &+(1/7)\ \sin(7\pi x/a)\ \sin(\pi y/b\ \exp[-\pi^2 t(49\alpha_x/a^2+\alpha_y/b^2] \\ &+(1/7)\ \sin(\pi x/a)\ \sin(7\pi y/b)\ \exp[-\pi^2 t(\alpha_x/a^2+49\alpha_y/b^2] \\ &+ \ldots\} \end{aligned}$$

where

T_s is surface temperature (which must be attained immediately),

T_0 initial temperature,

a one cross-sectional dimension,

b other cross-sectional dimension,

α_x thermal diffusivity in the *x* direction (dimension^2/time),

α_y thermal diffusivity in the *y* direction, and

t time.

Equation (A.4) converges quickly, so only the first few terms are necessary. In this report, seven terms were used. Because thermal conductivity and thermal diffusivity do not differ much in the radial and tangential directions of wood, in Eq. (A.4) we can set $\alpha_x = \alpha_y$ (MacLean (1941). Equation (4) can easily be converted to calculate the temperature at the center of the cross section by setting $x = a/2$ and $y = b/2$.