INVESTIGATION OF THE PROCEDURE FOR ESTIMATING CONCOMITANCE OF LUMBER STRENGTH PROPERTIES[1]

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ABSTRACT

An analytical method that utilizes information generated through proofloading to estimate the correlation between lumber strength properties is reviewed. A computer simulation was conducted to determine the effects of sample size and proofload level and also the degree of true correlation between lumber properties on the estimates of the correlation coefficient. Results indicate that reasonable estimates of the correlation coefficient can be obtained using sample sizes of 100 or more with a minimum proofload that would be expected to break 40% (proofload level = 0.40) of the specimens tested. Additional studies are suggested, however, before the technique is used.

Keywords: Correlation coefficient, concomitance, bending strength, tensile strength, computer simulation.

INTRODUCTION

Studies by Suddarth et al. (1978) have shown that the degree of correlation between tension and bending strength may have a significant effect on the load-carrying capacity and reliability of structural systems such as the metal plate wood truss. This cofunctioning, or concomitance, of properties in a piece of lumber has been the subject of a number of investigations concerning the strength of wood subject to combined stresses (Newlin and Trayer 1956; Norris 1962; Senft and Suddarth 1970; Zahn 1982).

Galligan et al. (1979) suggested an approach using a novel use of proofloading to estimate the correlation or degree of concomitance between two lumber strength properties. This approach has already been shown to be useful but not without

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TABLE 1. *Summary of estimates of the correlation of residuals, $\hat{\rho}$, in the regression model based on the work of Galligan et al. (1979).*

Data	Proofload failure mode	Survivor failure mode	Proof-load level	Estimated residual correlation ($\hat{\rho}$)	Approximate 95% confidence interval for $\hat{\rho}$
			Psi		
Southern pine No. 2 KD	Tension	Bending	1,657	0.4	(−0.2, 0.7)
Southern Pine No. 2 KD	Compression	Bending	3,544	−0.3	(−0.5, 0.1)
Hem-Fir 1.5E MSR	Tension	Bending	2,857	0.9 or above	Above 0.75
Hem-Fir 1.5E MSR	Compression	Bending	4,038	0.6	(0.3, 0.8)

some limitations (Galligan et al. 1979; Tichy 1981; Johnson and Galligan 1982). Applying this procedure in an experimental design has sometimes resulted in correlation estimates with extremely wide confidence limits (Table 1). However, before the technique can be used efficiently in the design or analysis of concomitance studies, we need to know how the experimental parameters influence the accuracy of the estimates of the correlation coefficient.

This paper reports the results of computer simulations of the effect of sample size, proofload level, and degree of true correlation on the accuracy of the proofload level, and degree of true correlation on the accuracy of the prooflading procedure in estimating the true correlation between two lumber strength properties. The simulations were limited to an unconditional form of the concomitant equation in which correlation coefficient estimates depend only on the strength estimates and not on estimates of nondestructive properties such as knot size and modulus of elasticity (MOE).

BACKGROUND

Suppose we want to estimate the degree of correlation between two strength properties such as bending and tensile strength. Also suppose that predictor variables such as knot size or MOE are not available. First, we obtain a sample of N specimens, and proofload each specimen in bending to a stress level L. If a specimen does not fail in bending at this level, then it is tested to failure in tension. Under this scheme we observe either

$$b_j = \text{bending strength if } b_j \leq L$$

or

$$t_j = \text{tensile strength if } b_j > L$$

for each of N specimens.

Let F denote j values where b_j is observed. Assuming a bivariate normal distribution for (b_j, t_j), we obtain the likelihood

$$L = \prod_{j \in F} (1/\sqrt{2\pi}\sigma_b)\exp\{-\tfrac{1}{2}[(b_j - \mu_b)/\sigma_b]^2\} \cdot \prod_{j \notin F} (1/\sqrt{2\pi}\sigma_t)\exp\{-\tfrac{1}{2}[(t_j - \mu_t)/\sigma_t]^2\} \cdot \int_{a_j}^{\infty} (1/\sqrt{2\pi})\exp\{-z^2/2\}\, dz \quad (1)$$

where

a_j = $[L - \mu_b - \rho(\sigma_b/\sigma_t)(t_j - \mu_t)]/\sigma_b\sqrt{1-\rho^2}$
μ_b and σ_b^2 = the mean and variance of the samples failed in bending
μ_t and σ_t^2 = the mean and variance of the samples failed in tension
ρ = the correlation between b and t

Given values of μ_b, σ_b, μ_t, and σ_t, the maximum likelihood estimate, $\hat{\rho}$, can be obtained by minimizing minus 2 times the natural logarithm of the likelihood given in Eq. (1).

$$-2\ln(L) = c_1 + \sum_{j \in F} [(b_j - \mu_b)/\sigma_b]^2 + \sum_{j \notin F} [(t_j - \mu_t)/\sigma_t]^2 - 2\sum_{j \notin F} \ln\left\{\int_{a_j}^{\infty} (1/\sqrt{2\pi})\exp[-z^2/2]\, dz\right\} \quad (2)$$

where c_1 is a constant. This minimization involves using a search procedure. Given values of μ_b, μ_t, σ_b, and σ_t, only the last term of Eq. (2) changes for different ρ values.

Note from Eq. (2) that the estimate $\hat{\rho}$ of ρ depends upon where our proofloads and observations are in a standard normal distribution and not on particular values of μ_b, μ_t, σ_b, and σ_t. Even a_j is expressible as a standardized variable and standardized load.

Theoretically, estimates for all the parameters could be obtained by maximizing the likelihood over all unknown parameters. However, in practice, this proved impractical. For our simulations, we treated all parameters except ρ as known. This is quite realistic in the present application because the results of a relatively large sample of specimens broken in tension or in bending are available (Galligan et al. 1979). The sample means and standard deviations then provide accurate estimated values for μ_b, μ_t, σ_b, and σ_t.

THE SIMULATION

To study the proofload procedure under ideal conditions, we conducted the simulation using a normal distribution where ρ was the only unknown parameter, and the true values of the sample means and standard deviations were used.

We started by generating trivariate normal observations[3]

[3] Recall that statistical notation for a normal distribution of p dimension is $N_p\{\mu, \Sigma\}$ when p is the number of dimensions, μ is a vector of means, and Σ is the variance-covariance matrix. The symbol ~ means "is distributed as."

$$\begin{bmatrix} b_j \\ t_j \end{bmatrix} \sim N_2\left(\begin{bmatrix} \mu_b \\ \mu_t \end{bmatrix}, \begin{pmatrix} \sigma_b^2 & \rho_{bt}\sigma_b\sigma_t \\ \rho_{bt}\sigma_b\sigma_t & \sigma_t^2 \end{pmatrix}\right) \tag{3}$$

where the parameter values (see Appendix) except ρ_{bt} were based on the results of other rather extensive studies. As noted above, if the values of μ_b, μ_t, σ_b, σ_t are known, the distribution of the estimate of correlation, $\hat{\rho}$, does not depend on their particular values.

Data sets with either 20, 40, 60, 80, 100, 200, or 300 samples were generated according to Eq. (3) using the GGNSM subroutine in the IMSL package (IMSL 1979). In many laboratory situations a sample size of N = 300 would probably represent a practical extreme for estimating lumber properties. Small sample sizes were included in the analysis as a reference base for those who want to use the procedure with a limited number of samples.

The proofload level and true bending-tension correlation, ρ, were varied in a factorial arrangement of levels. The proofloads were selected percentiles of the bending strength distribution corresponding to the expected proportions 0.2, 0.4, 0.5, or 0.6 of specimens broken in bending. A range of true correlation, $\rho = 0.2$, 0.6, 0.8, and 0.9, was selected because, *a priori,* we expected a high positive correlation between strength properties. For N = 100 and N = 300, additional proofloads of 0.7, 0.8, and 0.9 were considered to better understand the procedures at high proofloads.

For each set of N observations generated, the value of ρ which minimized Eq. (2) was obtained. A search procedure, based on the algorithm of Davies, Swann, and Campey (Adby and Dempster 1974) was used for this purpose. Convergence of the search was defined as having occurred when successive iterations of the procedure produced estimated minimums of Eq. (2) occurring at values of ρ that varied by less than 0.00001. Calculations were performed in double precision. For larger sample sizes no problem arose with the search technique because numerous examples showed $-2 \ln(L)$ to be a smooth function and to have no local minima. For small samples, however, there were some cases in which the value of $-2 \ln(L)$ had two minima in the region -1 to 1 for ρ. Thus, the starting point of the search could determine to which minimum the procedure converged. Usually one point of convergence was for a value of $\hat{\rho}$ less than zero and the other for $\hat{\rho}$ greater than zero, with one of these local systems having a much smaller value of $-2 \ln(L)$ than the other. Because the values of $-2 \ln(L)$ descend slowly for different L values to each minimum, convergence to the wrong minimum was easily eliminated by using a grid search for ρ and using the resulting best $\hat{\rho}$ value in a starting point for the more extensive search described earlier.

We did restrict $\hat{\rho}$ to the range -0.99 to 0.99 to avoid numerical problems when evaluating the normal integral. The value of a_j in the last term in Eq. (2) was sometimes greater than 10^8 without this restriction. We do not believe that this minor truncation of the range of $\hat{\rho}$ appreciably affects the properties reported.

For each sample size and for each of the $4 \times 4 = 16$ combinations of proofload level and ρ, a total of 500 data sets were generated. Thus, 500 $\hat{\rho}$ values were calculated for each situation. Several summary statistics were calculated from each collection of 500 $\hat{\rho}$ values.

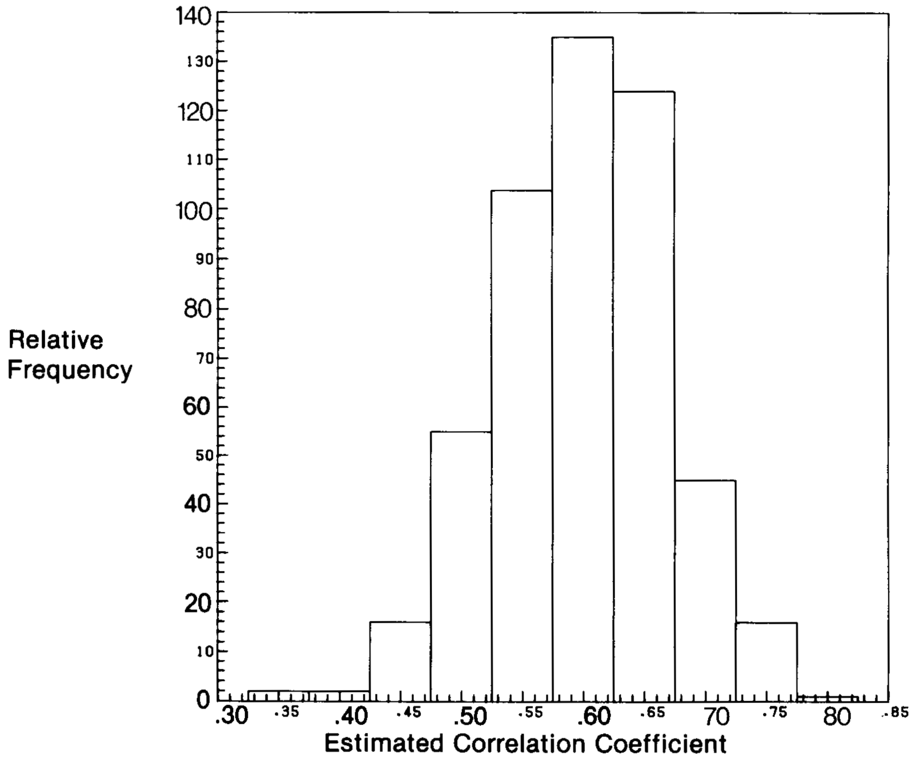

FIG. 1. Typical frequency distribution (sample size = 300, true correlation = 0.6, proofload level = 0.5).

sample mean: $\bar{\hat{\rho}} = \sum_{i=1}^{500} \frac{\hat{\rho}_i}{500}$

sample standard deviation: $\left[\sum_{i=1}^{500} (\hat{\rho}_i - \bar{\hat{\rho}})^2/500\right]^{1/2}$

bias: $\bar{\hat{\rho}} - \rho$

mean square error (MSE): $\sum_{i=1}^{500} \frac{(\hat{\rho}_i - \rho)^2}{500}$

average absolute error (AAE): $\sum_{i=1}^{500} \frac{|\hat{\rho}_i - \rho|}{500}$

coefficient of variation (COV): $\frac{\text{sample standard deviation}}{\text{sample mean}}$

For each set of 500 estimates, a frequency distribution was also produced. A typical example is given in Fig. 1. Each cell extends a distance 0.025 from the printed class mark. From this and the other frequency distributions of the 500 $\hat{\rho}$ values, it appears that the estimator is nearly normally distributed for the cases studied where N was at least 100.

TABLE 2. *Estimated correlation between destructive tests.*[1] *Sample size = 20. Based on 500 trials.*

True correlation	Measurement	Proofload level (proportion of population)			
		0.2	0.4	0.5	0.6
0.2	Mean	0.1796	0.1976	0.1785	0.2198
	SD[2]	0.5831	0.4406	0.4197	0.4051
	Bias	−0.0204	−0.0024	−0.0215	0.0198
	MSE[3]	0.3405	0.1942	0.1766	0.1645
	AAE[4]	0.4958	0.3602	0.3391	0.3186
0.6	Mean	0.3902	0.5742	0.6010	0.5984
	SD	0.5814	0.3602	0.3104	0.2911
	Bias	−0.2098	−0.0258	0.0010	−0.0016
	MSE	0.3821	0.1304	0.0963	0.0847
	AAE	0.4538	0.2679	0.2432	0.2288
0.8	Mean	0.7079	0.7928	0.8015	0.8060
	SD	0.4685	0.2033	0.1951	0.1668
	Bias	−0.0921	−0.0072	0.0015	0.0060
	MSE	0.2280	0.0414	0.0381	0.0279
	AAE	0.2676	0.1470	0.1410	0.1342
0.9	Mean	0.8239	0.8934	0.9071	0.9043
	SD	0.3714	0.1518	0.0982	0.0988
	Bias	−0.0761	−0.0066	0.0071	0.0043
	MSE	0.1438	0.0231	0.0097	0.0098
	AAE	0.1688	0.0900	0.0757	0.0767

[1] Convergence to a boundary point occurred frequently.
[2] SD = standard deviation.
[3] MSE = mean squared error.
[4] AAE = average absolute error.

RESULTS

Tables 2–8 summarize the results of the simulation in terms of the five summary statistics: sample mean, standard deviation, bias, MSE, and AAE. Of the five statistics, MSE is especially useful in spotting trends in the tables. From these tables and the computer runs on which they are based, a number of general patterns emerge.

1. Convergence problems existed for the small sample sizes of N = 20 and N = 40. Convergence to a boundary point occurred with great regularity. This problem rapidly disappeared as the sample size increased. For N = 60 there was little problem, and by N = 80 the occurrences had disappeared.
2. The bias, if any, of the maximum likelihood estimator, $\hat{\rho}$, has no discernible pattern of overestimating or underestimating ρ. The bias is usually larger for the extreme proofload levels (0.2 and 0.9). This difference in the effect of proofload level decreases as N increases so that by N = 100 it has virtually disappeared.
3. As can be seen from the values of MSE and AAE, no one proofload level is best. This is because the minimum value does not occur at the same proofload level for all correlations.
4. A proofload that is expected to break only 0.2 (or 20%) of the samples is never best, and the variability of $\hat{\rho}$ is much larger at this proofload. For small

TABLE 3. *Estimated correlation between destructive tests.*[1] *Sample size = 40. Based on 500 trials.*

True correlation	Measurement	Proofload level (proportion of population)			
		0.2	0.4	0.5	0.6
0.2	Mean	0.1787	0.1935	0.2031	0.1945
	SD[2]	0.4668	0.3286	0.2989	0.2752
	Bias	−0.0213	−0.0065	0.0031	−0.0055
	MSE[3]	0.2184	0.1080	0.0894	0.0758
	AAE[4]	0.3792	0.2657	0.2361	0.2198
0.6	Mean	0.5303	0.6174	0.5918	0.6146
	SD	0.4012	0.2045	0.2090	0.1844
	Bias	−0.0697	0.0174	−0.0082	0.0146
	MSE	0.1658	0.0421	0.0437	0.0342
	AAE	0.2860	0.1608	0.1596	0.1474
0.8	Mean	0.7485	0.7979	0.8070	0.8055
	SD	0.3117	0.1393	0.1250	0.1165
	Bias	−0.0515	−0.0021	0.0070	0.0055
	MSE	0.0998	0.0194	0.0157	0.0136
	AAE	0.1749	0.1063	0.0950	0.0906
0.9	Mean	0.8912	0.9017	0.9080	0.9024
	SD	0.1516	0.0761	0.0719	0.0717
	Bias	−0.0088	0.0017	0.0080	0.0024
	MSE	0.0231	0.0058	0.0052	0.0051
	AAE	0.0825	0.0601	0.0583	0.0568

[1] Convergence to a boundary point occurred frequently.
[2] SD = standard deviation.
[3] MSE = mean squared error.
[4] AAE = average absolute error.

TABLE 4. *Estimated correlation between destructive tests. Sample size = 60. Based on 500 trials.*

True correlation	Measurement	Proofload level (proportion of population)			
		0.2	0.4	0.5	0.6
0.2	Mean	0.1430	0.1998	0.1959	0.2028
	SD[1]	0.3986	0.2616	0.2229	0.2206
	Bias	−0.0570	−0.0002	−0.0041	0.0028
	MSE[2]	0.1621	0.0684	0.0497	0.0487
	AAE[3]	0.3237	0.2146	0.1776	0.1771
0.6	Mean	0.5582	0.5939	0.5993	0.6033
	SD	0.2986	0.1789	0.1666	0.1563
	Bias	−0.0418	−0.0061	−0.0007	0.0033
	MSE	0.0909	0.0321	0.0278	0.0244
	AAE	0.2138	0.1380	0.1318	0.1220
0.8	Mean	0.7793	0.7976	0.7997	0.8041
	SD	0.1974	0.1042	0.0960	0.0944
	Bias	−0.0207	−0.0024	−0.0003	0.0041
	MSE	0.0394	0.0109	0.0092	0.0089
	AAE	0.1197	0.0822	0.0755	0.0748
0.9	Mean	0.9013	0.9004	0.9048	0.9007
	SD	0.0811	0.0622	0.0555	0.0599
	Bias	0.0013	0.0004	0.0048	0.0007
	MSE	0.0066	0.0039	0.0031	0.0035
	AAE	0.0617	0.0498	0.0449	0.0472

[1] SD = standard deviation.
[2] MSE = mean squared error.
[3] AAE = average absolute error.

TABLE 5. *Estimated correlation between destructive tests. Sample size = 80. Based on 500 trials.*

True correlation	Measurement	Proofload level (proportion of population)			
		0.2	0.4	0.5	0.6
0.2	Mean	0.1951	0.2025	0.2046	0.1989
	SD[1]	0.3278	0.2182	0.1905	0.1899
	Bias	−0.0049	0.0025	0.0046	−0.0011
	MSE[2]	0.1075	0.0476	0.0363	0.0361
	AAE[3]	0.2621	0.1764	0.1484	0.1525
0.6	Mean	0.5768	0.5952	0.6038	0.6096
	SD	0.2499	0.1484	0.1395	0.1289
	Bias	−0.0232	−0.0048	0.0038	0.0096
	MSE	0.0630	0.0220	0.0195	0.0167
	AAE	0.1819	0.1155	0.1129	0.1052
0.8	Mean	0.7941	0.7977	0.8082	0.8023
	SD	0.1281	0.0895	0.0767	0.0856
	Bias	−0.0059	−0.0023	0.0082	0.0023
	MSE	0.0164	0.0080	0.0060	0.0073
	AAE	0.0944	0.0709	0.0613	0.0690
0.9	Mean	0.9034	0.9013	0.9029	0.9023
	SD	0.0667	0.0521	0.0485	0.0498
	Bias	0.0034	0.0013	0.0029	0.0023
	MSE	0.0045	0.0027	0.0024	0.0025
	AAE	0.0529	0.0416	0.0390	0.0399

[1] SD = standard deviation.
[2] MSE = mean squared error.
[3] AAE = average absolute error.

TABLE 6. *Estimated correlation between destructive tests. Sample size = 100. Based on 500 trials.*

True correlation	Measurement	Proofload level (proportion of population)						
		0.2	0.4	0.5	0.6	0.7	0.8	0.9
0.2	Mean	0.1946	0.1912	0.0283	0.2015	0.2000	0.2002	0.1994
	SD[1]	0.3079	0.1875	0.1723	0.1709	0.1593	0.1566	0.1931
	Bias	−0.0054	0.0088	0.0083	0.0015	0.0000	0.0002	−0.0006
	MSE[2]	0.0948	0.0353	0.0298	0.0292	0.0254	0.0245	0.0373
	AAE[3]	0.2465	0.1506	0.1380	0.1399	0.1296	0.1236	0.1508
0.6	Mean	0.5655	0.5967	0.6018	0.6004	0.5969	0.6001	0.5981
	SD	0.2145	0.1366	0.1214	0.1109	0.1093	0.1194	0.1622
	Bias	−0.0345	−0.0033	0.0018	0.0004	−0.0031	0.0001	0.0019
	MSE	0.0472	0.0187	0.0147	0.0123	0.0120	0.0143	0.0263
	AAE	0.1560	0.1066	0.0962	0.0886	0.0869	0.0947	0.1200
0.8	Mean	0.7901	0.7994	0.8039	0.8025	0.8036	0.8028	0.8094
	SD	0.1303	0.0788	0.0752	0.0737	0.0718	0.0786	0.0956
	Bias	−0.0099	−0.0006	0.0039	0.0025	0.0036	0.0028	0.0094
	MSE	0.0171	0.0062	0.0057	0.0054	0.0052	0.0062	0.0092
	AAE	0.0938	0.0617	0.0585	0.0580	0.0565	0.0619	—
0.9	Mean	0.9010	0.9050	0.9042	0.9038	0.9037	0.9019	0.9022
	SD	0.0587	0.0451	0.0408	0.0471	0.0448	0.0497	0.0528
	Bias	0.0010	0.0050	0.0042	0.0038	0.0037	0.0019	0.0022
	MSE	0.0035	0.0021	0.0017	0.0022	0.0020	0.0025	0.0028
	AAE	0.0463	0.0363	0.0323	0.0370	0.0356	0.0401	—

[1] SD = standard deviation.
[2] MSE = mean squared error.
[3] AAE = average absolute error.

TABLE 7. *Estimated correlation between destructive tests. Sample size = 200. Based on 500 trials.*

True correlation	Measurement	Proofload level (proportion of population) 0.2	0.4	0.5	0.6
0.2	Mean	0.1774	0.1941	0.1992	0.1927
	SD[1]	0.2106	0.1377	0.1234	0.1110
	Bias	−0.0226	−0.0059	−0.0008	−0.0073
	MSE[2]	0.0449	0.0190	0.0152	0.0124
	AAE[3]	0.1680	0.1100	0.0976	0.0881
0.6	Mean	0.5950	0.6077	0.5944	0.6042
	SD	0.1379	0.0904	0.0865	0.0764
	Bias	−0.0050	0.0077	−0.0056	0.0042
	MSE	0.0190	0.0082	0.0075	0.0059
	AAE	0.1034	0.0728	0.0693	0.0626
0.8	Mean	0.7971	0.8030	0.7979	0.7992
	SD	0.0782	0.0529	0.0518	0.0512
	Bias	−0.0028	0.0030	−0.0021	−0.0008
	MSE	0.0061	0.0028	0.0027	0.0026
	AAE	0.0622	0.0434	0.0414	0.0408
0.9	Mean	0.9009	0.8996	0.9019	0.9004
	SD	0.0437	0.0347	0.0302	0.0311
	Bias	0.0009	−0.0004	0.0019	0.0004
	MSE	0.0019	0.0012	0.0009	0.0010
	AAE	0.0349	0.0267	0.0246	0.0245

[1] SD = standard deviation.
[2] MSE = mean squared error.
[3] AAE = average absolute error.

TABLE 8. *Estimated correlation between destructive tests. Sample size = 300. Based on 500 trials.*

True correlation	Measurement	Proofload level (proportion of population) 0.2	0.4	0.5	0.6	0.7	0.8	0.9
0.2	Mean	0.1856	0.2020	0.2004	0.2017	0.1950	0.2057	0.1961
	SD[1]	0.1817	0.1069	0.0981	0.0921	0.0895	0.0903	0.1041
	Bias	−0.0144	0.0020	0.0004	0.0017	−0.0050	0.0057	−0.0039
	MSE[2]	0.0332	0.0114	0.0096	0.0085	0.0080	0.0082	0.0108
	AAE[3]	0.1431	0.0853	0.0785	0.0711	0.0728	0.0725	0.0819
0.6	Mean	0.5999	0.5998	0.5985	0.5994	0.5980	0.5971	0.6036
	SD	0.1037	0.0762	0.0694	0.0679	0.0649	0.0695	0.0779
	Bias	−0.0001	−0.0002	−0.0015	−0.0006	−0.0020	−0.0029	0.0036
	MSE	0.0107	0.0058	0.0048	0.0046	0.0042	0.0048	0.0061
	AAE	0.0809	0.0607	0.0550	0.0537	0.0520	0.0548	0.0625
0.8	Mean	0.8006	0.8035	0.7968	0.8005	0.8006	0.8056	0.8070
	SD	0.0579	0.0453	0.0434	0.0401	0.0400	0.0457	0.0530
	Bias	0.0006	0.0035	−0.0032	0.0005	0.0006	0.0056	0.0070
	MSE	0.0033	0.0021	0.0019	0.0016	0.0016	0.0021	0.0029
	AAE	0.0456	0.0362	0.0346	0.0316	0.0319	0.0366	0.0427
0.9	Mean	0.9020	0.9002	0.8997	0.9007	0.9008	0.9011	0.9025
	SD	0.0329	0.0263	0.0257	0.0264	0.0264	0.0294	0.0312
	Bias	0.0020	0.0002	−0.0003	0.0007	0.0008	0.0011	0.0025
	MSE	0.0011	0.0007	0.0007	0.0007	0.0007	0.0009	0.0010
	AAE	0.0263	0.0210	0.0209	0.0211	0.0207	0.0229	0.0252

[1] SD = standard deviation.
[2] MSE = mean squared error.
[3] AAE = average absolute error.

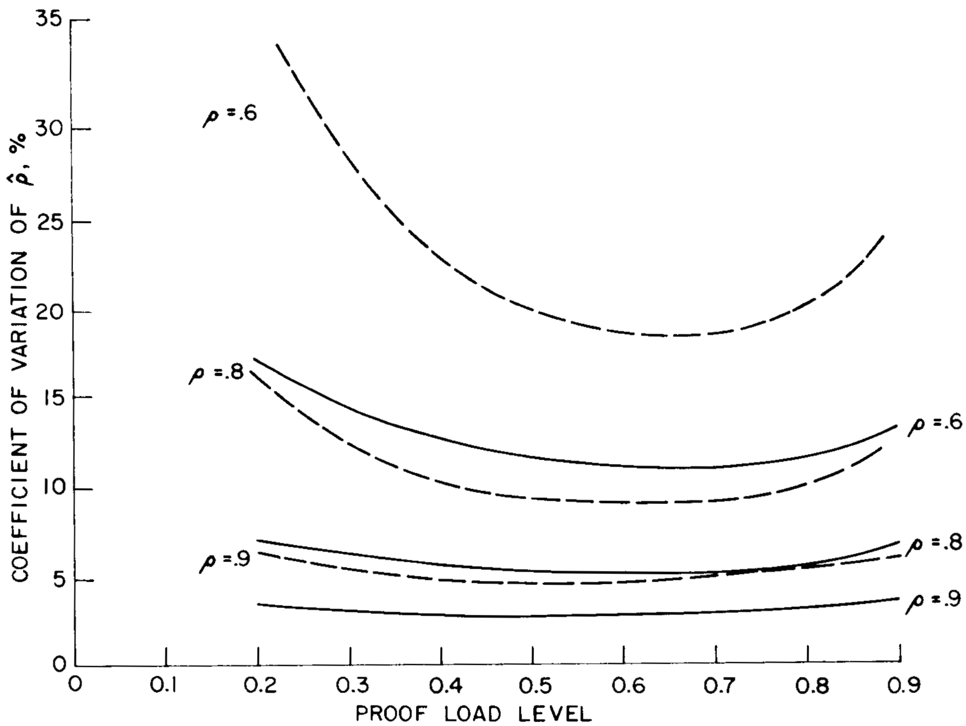

FIG. 2. Effect of sample size, proofload level, and true correlation (ρ) on the precision of the estimate of the correlation coefficient ($\hat{\rho}$) for sample sizes of 100 (– – –) and 300 (——).

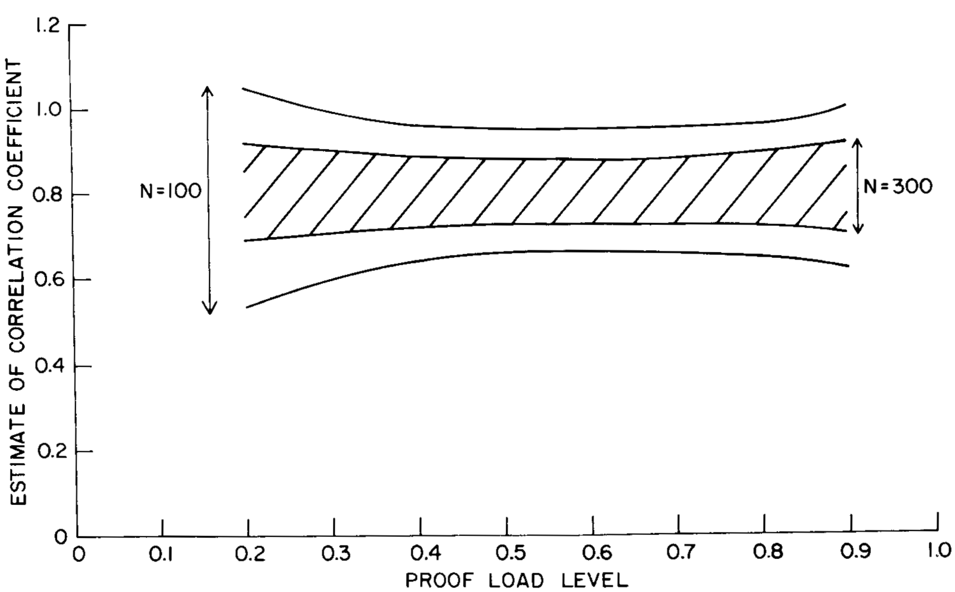

FIG. 3. Effect of sample size (N) and proofload level on the 95% confidence interval of the correlation estimate for a sample having a true correlation coefficient of 0.80.

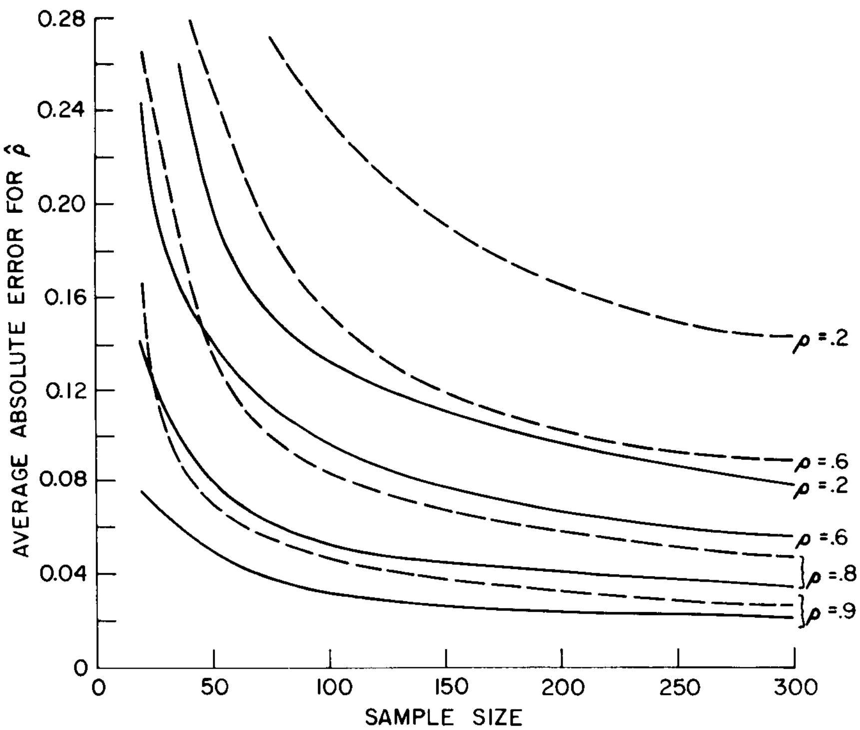

FIG. 4. Effect of sample size and true correlation coefficient on the average absolute error in estimating the correlation coefficient for proofload levels of 0.20 (– – –) and 0.50 (——).

samples, a proofload of 0.2 is relatively poor. For N = 100 and larger, the difference in a proofload of 0.2 and 0.4 has decreased substantially.

5. A proofload that is expected to break 0.6 (or 60%) of the specimens is usually close to the best. Proofloads as small as 0.4 appear reasonably good, particularly for sample sizes of 60 or more.
6. It appears that the optimal proofload decreases as the true correlation coefficient, ρ, increases. (See contour plots, Fig. 5.)
7. Using ±2 times the square root of the MSE gives a range in which approximately 95% of the estimates of ρ fell. The tables show that except for proofload 0.2 we should be within 0.38 of the true correlation 95% of the time for N = 100 and within 0.21 of ρ for N = 300.

The interaction of sample size, proofload level, and true correlation coefficient on our ability to estimate the correlation is shown in Figs. 2, 3, and 4. Figure 2 shows the coefficient of variation of the estimates plotted against the proofload level. Separate lines for N = 100 and N = 300 are produced for true ρ values of 0.6, 0.8, and 0.9. The estimates for $\rho = 0.2$ were considerably more variable and are not included in Fig. 2. For the higher correlations, the desirability of using a proofload level of 0.4 to 0.8 is evident.

Figure 3 shows only the $\rho = 0.80$ data and plots estimated regions containing 95% of the estimate (i.e., $\bar{x} \pm 2\sqrt{\text{MSE}}$) versus the proofload level. Separate curves

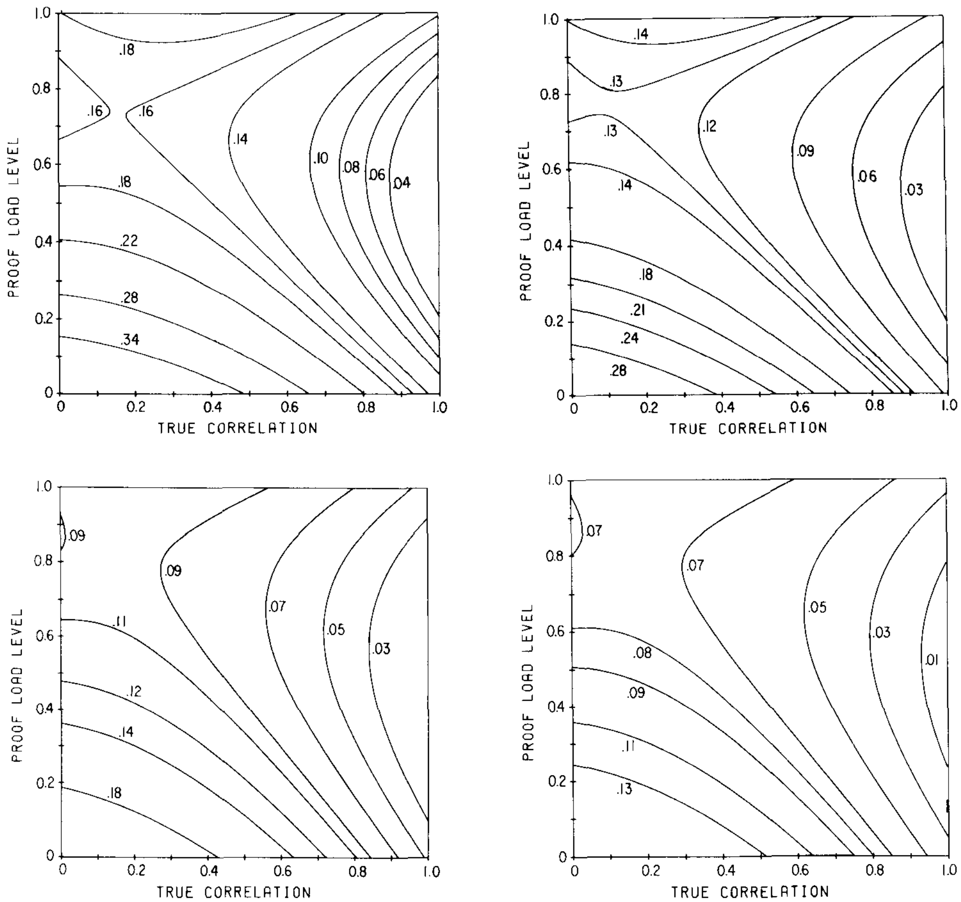

FIG. 5. Contours of fitted relationships between dependent variables measuring the quality of estimation and the independent variables measuring the proofload level and true correlation. a. Standard deviation (coefficient of determination (R^2) = 0.97, sample size (N) = 100); b. Average absolute error (R^2 = 0.97, N = 100); c. Standard deviation (R^2 = 0.96, N = 300); and d. Average absolute error (R^2 = 0.96, N = 300).

for N = 100 and N = 300 are produced. Result (7) above is graphically displayed here for this ρ value.

Figure 4 shows the effect of sample size and true correlation on the AAE of the estimates for proofload levels of 0.2 and 0.5. As expected, the values of AAE decrease as the sample size increases. However, the decrease from N = 100 to N = 300 is small compared to earlier decreases and indicates that with respect to absolute error N = 100 might be a very practical sample size.

To summarize recommendations on the proofload level for different values of the true correlation coefficient, a two-dimensional quadratic model was fit using ρ and proofload level as the independent variables and the values of $\hat{\rho}$ in Tables 6 and 8 as the dependent variable. For example, the plot for AAE was obtained by fitting the values in Table 8 where proofload level and true correction were considered independent variables. The quadratic model fit to the data was

$$Y = 0.35 - 0.13x_1 - 0.55x_2 - 0.13x_1^2 + 0.34x_2^2 + 0.21x_1x_2$$

where

Y = value of the AAE
x_1 = true correlation
x_2 = proofload level

The coefficient of determination, R^2, for this model was 0.97. Contours of equal response of some of the resulting curves are given in Fig. 5. The R^2 values indicated a reasonable fit for all curves. These curves can be used to help decide an appropriate proofload level based on a prior guess of the true correlation. As mentioned previously, it appears from the contours that the optimal proofload decreases somewhat as ρ increases.

CONCLUSIONS

The estimates of concomitance appear reasonable using sample sizes of 100 or more and a proofload level of 0.40 or higher if the true correlation coefficient is at least 0.60.

It is theoretically possible that this type of procedure might be extended to obtain pairwise estimates of the correlation between three or more strength properties using a series of loads. For instance, a sample could be proofloaded in bending, survivors proofloaded in tension, and then these last survivors failed in compression. Although this procedure may work with other materials, its applicability to wood is questionable because of possible damage to the specimens inflicted by multiple loadings and higher proofload levels.

In conducting simulation studies of truss performance, it might be desirable to use the conditional form of Eq. (2) in which estimates of ρ depend on some parameters measured nondestructively such as MOE and knot size. These results and those of earlier studies suggest that a greater understanding of the unconditional form is needed before the more complex case can be investigated. In particular, the effect of using the mean and standard deviation estimates for the samples that failed in bending and tension (μ_b, μ_t, σ_b, and σ_t) proofloading should be studied. Also, the optimal allocation of resources given a fixed number of pieces available for testing should be addressed.

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APPENDIX

The actual parameter values used in Eq. (3) to generate the observations used in the simulation are the results of large-scale studies of the strength properties of lumber. The distribution of the estimate of correlation, $\hat{\rho}$, does not depend upon their particular values. The values used were as follows:

$$\mu_b = 6{,}400 \text{ psi} \qquad \sigma_b^2 = 3{,}240{,}000$$
$$\mu_t = 4{,}900 \text{ psi} \qquad \sigma_t^2 = 2{,}722{,}500$$