USING DYNAMIC PROGRAMMING TO OBTAIN EFFICIENT KILN-DRYING SCHEDULES[1]

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ABSTRACT

This paper outlines a method for determining kiln-drying schedules that is based on the optimization technique of dynamic programming. The method is described with reference to the kiln drying of cedar pencil slats, but could be extended, with appropriate adjustments, to the kiln drying of other wood materials.

Keywords: Kiln drying, optimization, dynamic programming.

1. INTRODUCTION

The purpose of this paper is to propose the use of dynamic programming (DP), a mathematical optimization technique to improve the handling of wood drying in a kiln. Dynamic programming is a method applicable to processes with sequential decisions along time. No limitations are imposed on the form of the process, but knowledge is required on how it evolves as decisions are taken and on the costs involved. The method is suitable for problems in which only a small number of decision variables and states define the process. Typically, the procedure is based on the following notion: at any given stage t, the state of the system is defined. A decision is taken that a) leads to an immediate cost or return, and b) defines a new state to start the next stage, t + 1. If we could evaluate the optimal cost from each state in stage (t + 1) to the end of the process, this would lead us to choose a best decision at stage t. Using DP permits precisely obtaining such values in an efficient way. The DP procedure can be defined forwards (starting from a present stage and advancing toward the end of the process) or backwards. The choice will depend on the form of the problem. The definition of a DP procedure is very much dependent on the problem to be solved. In our case, we will present it in the context of a real case of drying cedar pencil stats.

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We will introduce the characteristics of the kiln-drying process of the pencil slats, and show how these decisions can be made more efficiently through the use of DP. [An initial effort in this direction was proposed by Claxton (1968).] It should be noted that the application to pencil slats can be viewed as an illustration of how a DP procedure is applied to kiln drying; we will discuss later how the implementation must be adjusted for applications to other types of wood. For pencil slats, drying time, cost, and quality of the dried material are the three main determining factors of drying efficiency. Drying costs are mainly dependent on fuel consumption. The quality of the dried wood can be measured by the average final moisture content and by other quality dimensions such as shrinkage, collapse, and so on. The change in quality incurred through drying can be called "drying degrade."

A drying schedule is a predetermined time-sequence of temperature, humidity, and velocity of air circulation inside the drying kiln. In our analysis the decision variables of the drying process are: the length of the drying period, the temperature, and the wet-bulb depression at each time during the period. Air velocity and the stacking pattern will be assumed as constant.

The state variables of the process—the characteristics of the drying wood at each moment—are the moisture content and a measure of the drying degrade. We assume that the species and anatomical characteristics of the wood are homogeneous for a given drying run. Under these conditions, the defined state variables can represent the process with sufficient precision.

The problem consists of determining an efficient drying schedule, defined as one that will bring the wood to an acceptable level of moisture content at least cost, while maintaining predetermined standards of quality. In the process considered here, the controls for temperature and depression can be set at intervals of any desired length (for example, each 24 h). For the purpose of illustration, it will be assumed that controls are set at the beginning of each defined time interval and maintained constant through it.

Determination of an efficient drying schedule is based upon the following factors:

1. Time cost: A fixed cost per day for overhead, maintenance, inventory holding, depreciation, etc.
2. Energy cost: The variable cost of setting and maintaining temperature and humidity at predetermined levels for a given time interval. This cost depends not only on the levels of temperature and humidity in the current time interval but also on their levels in the previous time interval. It also depends on the moisture content at the start of the current time interval.
3. Quality costs: At the termination of the process, a fraction of the pieces of wood will have to be rejected or reprocessed because their moisture content falls outside the acceptable range of moisture content.
4. Constraints: The choice of control settings in each period is restricted by considerations of quality and drying degrade.

2. QUANTITATIVE CHARACTERIZATION OF THE DRYING PROCESS

Notation:

N = Number of drying periods or intervals.
t = Drying interval; t = 1, 2, 3, . . . , n.

M_t = Moisture content in the wood at the beginning of drying interval t.
T_t = Temperature of kiln atmosphere during time interval t.
D_t = Wet-bulb depression during time interval t.
TD_t = Pair of values (T, D), denoting a combination of kiln temperature and wet bulb depression.

Note that the subscript t indicates both the beginning of a time interval in the case of a state variable—and a whole time interval in the case of a control variable.

The quantitative relations describing the drying process were determined empirically, applying regression analysis to experimental observations.

Since we are concerned mainly with showing how a DP procedure can be developed for the case of pencil slats, we will present only the basic notions of the drying transformations. Details of these aspects are given in Rensi and Weintraub (1984).

2.1. The drying-rate equation

The drying-rate equation expresses the reduction of moisture content in the wood per unit of time. In terms of the analytical factors affecting the drying process, the rate of drying is directly related to temperature and inversely related to humidity in the kiln atmosphere. Wet-bulb depression was used as a proxy variable for humidity (the higher the wet-bulb depression, the lower the humidity).

The parameters of the drying rate equation were estimated by regression analysis. The drying rate equation is based on the following relationship between Bramhall's resistance to drying and the average moisture content during the unitary time interval Δt

$$\frac{P(TD_t)}{M_{t+1} - M_t} \cdot \Delta t = Ae^{-M_t} \tag{2.1}$$

where TD_t is the temperature-depression setting for time interval t, and $P(TD_t)$ is the corresponding vapor pressure differential (see Bramhall 1976).

The variables in the regression equation were obtained from observations of moisture content and temperature-depression settings in a series of drying experiments on cedar pencil slats (Rensi and Weintraub 1976). On the basis of the estimated parameters, the moisture content at t + 1, M_{t+1}, can be calculated as

$$M_{t+1} = -22.73 \ln[e^{-0.044M_t} + 0.000393P(TD_t)] \qquad (2.2.)$$

where M_t is the moisture content at the beginning of interval t.

To measure vapor pressure differential, given a pair of temperature-depression controls (T, D), the values of the vapor pressures v(T) and v(TD) are obtained from the vapor pressure table. The difference v(T) − v(TD) is the vapor pressure differential P(TD).

2.2 Loss from variability in final moisture content

Even though the average final moisture content of the whole kiln load may be well within the range of acceptable moisture contents, the moisture content in some individual wood pieces may fall outside such range. Let B_M be the set of acceptable final moisture content values and let M_* be the upper bound of such set. Because of the variability of the final moisture content in the wood pieces, a fraction Q_M of the pieces may have moisture content higher than M_*. Let N represent the final interval of the drying period. Q_M can be computed as a function of M_N and M_{N+1}, the moisture contents at the beginning and at the end of that interval, respectively

$$Q_M = Q_M(M_N, M_{N+1}) \quad \text{or} \quad Q_M = Q_M(M_{N+1}, TD_N) \tag{2.3}$$

Given Q_M, one can compute the loss attributed to excess moisture content, as

$$L(Q_M) = V(1 - s)Q_M \tag{2.4}$$

where V is the value of a defect-free kiln load and s is the ratio of the salvage or reprocessing value over the normal value of a unit of dried wood. Details on the computation of Q_M are found in Rensi and Weintraub (1984).

2.3 Determination of bounds on acceptable levels of defects

Sustained high wet-bulb depressions may bring out large differences between surface and core moisture contents resulting in shrinkage-based defects. The wood of some species is prone to collapse-based defects when heated excessively at moisture content levels near complete saturation. Shrinkage-based defects were considered in our analysis. A particular measure for shrinkage-based defects was used. It is called "bow-tie," a major defect caused by residual stress in dried pencil-slats (Rensi and Rhemrev 1976). An equation relating S, bow-tie and wet-bulb depression was estimated:

$$S_{t+1} = S(D_t) = 1.45e^{0.033D_t} \tag{2.5}$$

Note that because of their form, pencil slats are more vulnerable to drying degrade than other types of wood. The normal distribution was found adequately to describe the variability of "bow-tie." The above equation can be used to estimate the average final bow-tie $S(D_N)$. Given an estimate of the standard deviation, $\sigma(S)$, and an upper bound on tolerable bow-tie levels, one can calculate Q_S, the estimated fraction of defective pieces. Conversely, given Q_S*, the maximum acceptable fraction of defective items, a corresponding upper bound on D_N, say D* can be calculated. D* represents the maximum allowable value of the depression control compatible with a defect incidence less than Q_S*. For another formulation of slat drying degrade, see Gorvard and Arganbright (1979).

2.4 Drying cost

The drying cost can be calculated on a per interval basis. Let C represent the cost of drying the wood during the time interval t. C can be expressed as the sum of two components, i.e., fuel and nonfuel costs.

$$C = F \cdot C_F + C_H$$

F is the amount of fuel in BTUs required to heat the kiln. C_F is the unit dollar cost per BTU of fuel. C_H is the fixed dollar cost per interval. C_H includes overhead cost, depreciation, maintenance and repair cost (fixed), inventory in process cost, and electricity cost. These costs can be assumed to be constant per unit of time, independently of the type of kiln schedule adopted.

The fuel consumption, F, can be calculated as the sum of four components, namely,

- F_1 the BTU equivalent required to evaporate the water out of the wood,
- F_2 the BTU equivalent required to increase the temperature inside the wood,
- F_3 the BTU equivalent required to increase the temperature in the kiln atmosphere, including the loss of heat in walls and cracks,
- F_4 the BTU equivalent required to heat the air taken into the kiln when reducing the humidity in the kiln (through air exchange).

F_1 is a function of ΔM_t the change of wood moisture content in time interval t, and T_t, the temperature in the kiln in the same time interval. F_2 and F_4 are functions of temperature-depression in the kiln both in interval t and in interval t−1. F_3 is a function of temperature-depression in the preceeding time interval t−1, as well as of the temperature in interval t.

The cost equation can be formulated as

$$C_t(M_t, TD_{t-1}, TD_t) = C_H + C_F \cdot (M_t, TD_{t-1}, TD_t) \tag{2.6}$$

Further details for the calculation of the costs are given in Rensi and Weintraub (1984).

3. DYNAMIC PROGRAMMING MODEL

3.1 Problem formulation

As was mentioned in the introduction, the DP formulation depends strongly on the problem to be solved. Thus, the DP formulation we present corresponds to this particular case of drying cedar pencil slats. Other drying processes could perfectly well lead to different model formulations, while preserving the basic philosophy of the DP approach.

In this case, there are two decision or control variables: temperature (dry-bulb) and depression (dry-bulb − wet-bulb temperatures). These two control variables will modify, among other things, the moisture content of the slats in the kiln. Since DP works with discrete values only, time duration as well as temperature or humidity will be assigned discrete values. This should cause no major problems, even in continuous process kilns; for example, intervals of 1 or 2 h between decisions could adequately simulate a continuous setting of the controls given the rather smooth form of variations in these controls. As for values of temperatures and humidity, mechanisms are suited to discrete settings, since the sensing instruments produce measurements no more precise than one or two degrees.

3.2 The process

For each time interval t, given a starting moisture content M_t, two controls are modified: temperature (T_t) and depression (D_t). We define TD_t as a joint measure of temperature and depression. These decisions lead to three immediate consequences: a) a cost associated to M_t, TD_t, and TD_{t-1}, b) a resulting moisture content

M_{t+1} at the end of period t (start of period t + 1), and c) a level of residual stress that may cause deformation in the wood pieces.

If at the start of any interval t, the moisture content M_t is within acceptable bounds, the process can terminate. A loss, depending on the distribution of moisture among slats (function of M_t, D_{t-1}, T_{t-1}), must also be considered.

We can describe the problem as that of determining a schedule of controls TD_t for each period t, that leads from a starting moisture content M_1 to an acceptable moisture content M_{N+1} at the end of an undetermined number of periods at total minimum cost, while maintaining acceptable wood deformations.

3.3 Solution method

The solution method, which is developed in (Rensi and Weintraub 1976) is based on: 1) the use of DP recurrence relations, which relate the state of a system in one period, the decision made, and the state of the system in the following period, when an optimal strategy in used (Dreyfus and Law 1977); and 2) the fact that in any period t, for the described process, the parameters D_{t-1}, T_{t-1}, M_t and the controls D_t, T_t define completely the cost incurred in period t, and the state of the system for period (t + 1).

For any moisture content M_t^j at the start of interval t, let $A(M_t^j)$ be the set of controls TD_t in period t, which lead to acceptable average deformations, as defined in section 2.

Let $I_{t-1,j}$ be the set of control decisions which, starting from any moisture content at the beginning of interval (t − 1), lead to a moisture content M_t^j at the start of interval t [see Eq. (2.1)], within acceptable levels of deformation.

Let C_t^{ij} be the optimal total cost of the process from the beginning of the drying procedure up to the start of interval t, with moisture content M_t^j when control decision i in $I_{t-1,j}$ was taken in period (t − 1).

Remark.— We note that if at the start of period t the slats have a moisture content M_t^j, and a control setting TD_t^r is chosen for that period, then there exists a control variable i_o in $I_{t-1,j}$ that minimizes the cost of the process up to the end of period t. Thus i_o is determined as the control i in $I_{t-1,j}$ that minimizes the value:

$$C_{t+1}^{rs} = \min_{i \in I_{t-1,j}} [C_t^{ij} + C_t(TD_{t-1}{}^i, M_j{}^t, TD_t{}^r)] \tag{3.1}$$

where control TD_t^r leads to a moisture $M^s{}_{t+1}$ at the start of period (t + 1). Then for the given pair of starting moisture content M_t^j and control setting TD_t^r in period t, $TD^{i_o}{}_{t-1}$ is an optimal control variable for period (t − 1).

We note that in this case, the DP procedure is in the forward direction, as we move from each defined point in period t to several alternatives for period (t + 1). Control decisions taken in period (t − 1) affect only period t, but not directly periods (t + 1), (t + 2), ...

Figure 1 illustrates the relationship (3.1).

3.4 Flow chart of the dynamic programming algorithm

Initialization.—Start period 1 with the initial moisture content (MC) M_1 and initial kiln conditions (control values) DT_0. Consider each control setting $DT_1{}^i$ in

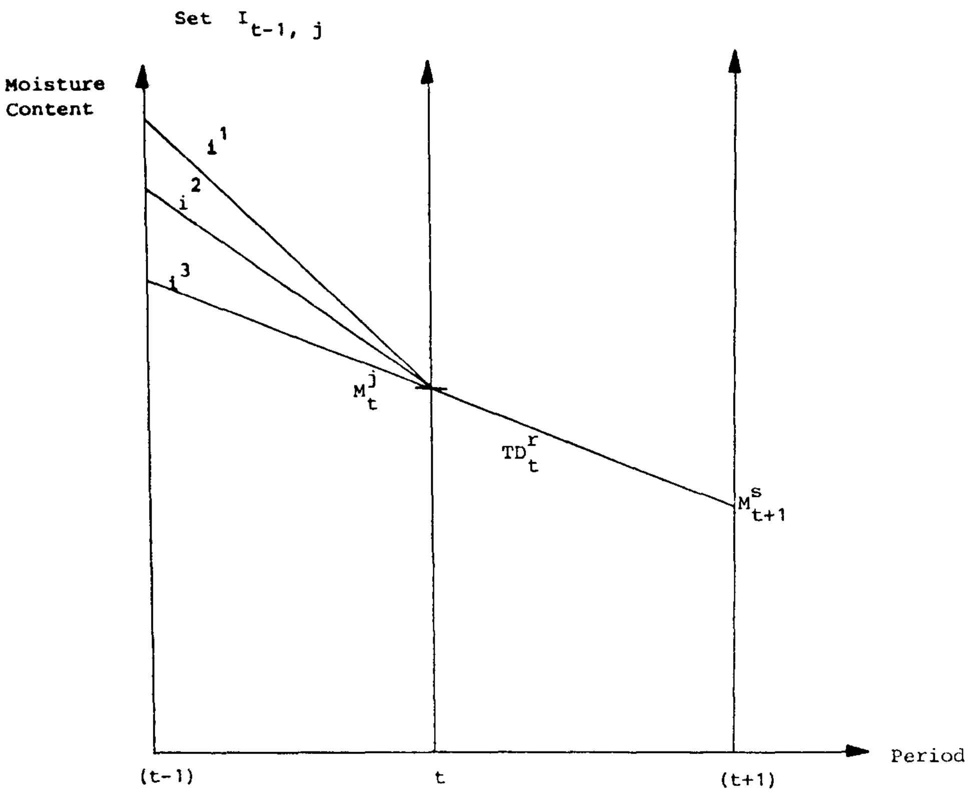

FIG. 1. Illustration of relation between $I_{t-1,j}$, M_t^j and TD_t^r. i^1, i^2, i^3 are feasible control variables in period (t − 1) that all lead to a moisture content M_t^j at the start of period t. If in period t, control TD_t^r is chosen, then we choose among i^1, i^2, i^3 the one that minimizes the total cost to reach M^s_{t+1} [as defined in expression (3.1)].

$A(M_1)$ (leading to an acceptable average deformation). For this set evaluate the resulting moisture content M_2^j and the costs associated to each control, C_2^{ij}. Start with a defined optimal cost $TC_o = \infty$ (as no solution exists yet).

1. For any interval t we have, for every defined moisture content M_t^j, a set of controls that led to that MC and an optimal associated cost: C_t^{ij}. Thus, to each defined moisture content there corresponds a set of pairs (TD^i_{t-1}, C_t^{ij}). The first value in each pair refers to the control decision i in interval (t − 1) that led to the moisture content M_t^j, while the second value refers to the optimal cost associated to reaching the MC M_t^j when that control setting i was used in interval (t − 1).
   If some MC has not yet been reached in interval t, i.e., it is not possible to reach that level of moisture content in t intervals within the constraints of the system, the set of pairs (TD^i_{t-1}, C_t^{ij}) is empty and M_t^j is said to be not defined. For each defined MC M_t^j, consider all acceptable control settings TD_t^s in $A(M_t^j)$, and determine:
   a) the optimal control variable i_0 in $I_{t-1,j}$ for interval (t − 1) corresponding to the pair (M_t^j, TD_t^s), through the recursive relation (3.1).

b) The MC M_{t+1} resulting from the pair (M_t^j, TD_t^s). Let it be M^k_{t+1}.
From the determination of i_0 in a), we can evaluate C^{sk}_{t+1} the optimal cost of reaching moisture content k at the start of period (t + 1) when control TD_t^s is used in period t. This defines one entry for the moisture content M^k_{t+1} in the table of period (t + 1) as (TD_t^s, C^{sk}_{t+1}).

2. Determine if termination is possible
   a) At the start of a period (t + 1), consider all MC M_{t+1} in the interval of acceptable final moisture content B_M. If no defined M_{t+1} exists in B_M, take $t = t + 1$ and go to 1. Otherwise, for every MC M^j_{t+1} defined in B_M, consider all controls TD_t^r in interval t that led to that MC M^j_{t+1} and evaluate the total cost involved if the process were terminated at that point. The total cost consists of:
      i) The optimal cost C^{rj}_{t+1} associated to reaching the MC M^j_{t+1} through control TD_t^r.
      ii) The loss due to the variance in the distribution of MC in the slats. This loss was defined in section 2 as the function $L^{jr}_{t+1} = L(M^j_{t+1}, DT_t^r)$. Let $TC(M^j_{t+1}) = \text{Min}(C^{rj}_{t+1} + L^{jr}_{t+1})$ be the optimal total cost associated with the final MC M^j_{t+1} among all control variables DT_t^r that led to the MC M^j_{t+1}. If $TC(M^j_{t+1} < TC_o$, take $TC_o = TC(M_{t+1})$ and the solution just found becomes the best available so far, the incumbent.
   b) Determine if through any moisture content M^j_{t+1} not in B_M there can exist a drying path superior to the incumbent. Since for any such MC at least one more drying period will be needed, the present cost is a lower bound on the final optimal cost of such drying paths. A simple way of determining if this is the case is to check

$$\underset{i \in I_{t,j}}{\text{Min}}\ C_{t+1}{}^{rj} \geq TC_o \tag{3.2}$$

If Eq. (3.2) is true for MC M^j_{t+1} not in B_M, no better solution can be found going through it, as the cost that path can only increase above the value obtained so far through the cost of operating at least one additional period, plus the termination loss function L, due to the variance in the distribution of MC in the slats. If Eq. (3.2.) is valid for all MC M_{t+1} not in B_M, the current solution constitutes an optimal schedule and the process is terminated. Otherwise, eliminate from further considertion all M_{t+1} not in B_M such that Eq. 3.2.) is satisfied, take $t = t + 1$ and go to 1.

3.5 Consideration of a cumulative degrade process

To consider percent collapse Gorvad and Arganbright (1977), which is a cumulative deterioration process along time, the straightforward procedure would be to include it in the recurrence relation as an additional state variable (in addition to moisture content and the control decision of the previous period). This will substantially increase the computational requirements.

A more efficient approach is through use of Lagrange Multipliers (Dreyfus and Law 1976), where the percent collapse is considered in the objective function, with a cost (in BTUs) assigned to it. In this form, we ensure that the program, in minimizing the total cost, will reduce collapse as much as possible.

Thus, the procedure to be followed is to run the program with a trial value of this price, or Lagrangian. If the resulting value of collapse is close enough to the target value, we have a solution that is close to optimal. If the resulting value is too high, we should recompute the solution, with a higher value for the Lagrangian, and we will obtain a new solution, with a lower value of collapse, although at the expense of increased costs. If the value of the collapse is smaller than the target value, we should recompute, with a smaller Lagrangian. Although this will allow the collapse to increase, costs will be reduced. The procedure generally converges (always under convexity conditions), and usually quickly. In addition, in initial trial phases, less precise and thus smaller problems can be run with less CPU expenditure until a reasonable guess of the value of the Lagrangian is obtained.

4. A NUMERICAL EXAMPLE

A small numerical example is given to illustrate how the Dynamic Programming approach is used.

Data:

Initial moisture content	$M_1 = 0.75$ (75%)
Initial dry-bulb temperature	$T_1 = 60$ F
Initial wet-bulb temperature	$V_1 = 50$ F
Initial depression	$D_1 = T_1 - V_1 = 10$ F
Weight of wood: W = 4,000 lb	A = 6540.0
Dry-bulb temperature allowable range:	135–165
Wet-bulb temperature allowable range:	125–150
Depression allowable range:	5–15

These constraints are motivated by possible degrade of the wood.

For the purposes of the example, temperature and depression are taken only in values that are multiples of 5 degrees, moisture content in multiples of 2%, and periods are of one day.

There exist 17 possible drying policy combinations, each day, of dry-bulb and wet-bulb temperature. Table 1 indicates the resulting moisture content and cost for all policies for day 1. The third defined component of cost F_3 is excluded (energy required to increase the temperature in the kiln atmosphere) as its effect is negligible in the total costs. Moisture content is calculated using Eq. (2.1), costs are based on Eq. (2.6). To simply the example, costs will be attributed directly to BTUs and no consideration will be given to fixed costs per day.

Moisture contents at the end of day 1 (start of day 2) will be approximated to the nearest even integer.

From Table 1 we have the moisture contents that are feasible at the start of period 2. Along each value of moisture, the policies that led to that moisture content are given.

The DP procedure continues by considering, for each moisture content reached at the start of period 2, all feasible drying policies. By the recurrence relationship, for each such policy for day 2, applied to an initial moisture M_2, the best policy that led to that moisture M_2 is determined. For example, two policies in day 1 led to a (approximated) moisture of 62%: X_6, X_{16}. For each drying policy for day 2, the minimal total cost will be calculated between X_6 and X_{16}. Analogous

TABLE 1. *Resulting moisture contents and costs for day 1.*

Policy	T (°F)	TD (°F)	M_2 (%)	F_1 (BTU × 10^4)	F_2 (BTU × 10^4)	F_4 (BTU × 10^4)	Total cost (BTU × 10^4)
X_1	135	125	66.8	33.3	22.5	7.4	63.2
X_2	135	130	70.2	19.5	24.0	3.7	47.2
X_3	140	125	63.0	48.3	22.5	11.6	82.4
X_4	140	130	65.9	36.9	24.0	7.4	68.3
X_5	140	135	69.8	22.5	25.5	3.7	51.7
X_6	145	130	61.9	52.9	24.0	11.4	88.3
X_7	145	135	66.7	33.5	25.5	6.4	65.4
X_8	145	140	69.2	23.4	27.0	3.9	54.3
X_9	150	135	60.7	57.6	25.5	11.6	94.7
X_{10}	150	140	64.0	44.3	27.0	7.8	79.1
X_{11}	150	145	68.5	26.2	28.5	4.1	58.8
X_{12}	155	140	59.6	61.9	27.0	11.6	100.5
X_{13}	155	145	63.1	47.8	28.5	7.8	84.1
X_{14}	155	150	68.1	27.7	30.0	4.0	61.7
X_{15}	160	145	58.4	66.5	28.5	11.5	106.5
X_{16}	160	150	62.3	50.9	30.0	7.8	88.7
X_{17}	165	150	57.2	71.1	30.0	11.5	112.6

evaluations can be carried out for the other moisture contents M_2 = 58, 60, 64, 66, 70. This would complete the first iteration.

In Table 2, the evaluation of all feasible control decisions for period 2, starting with MC 62% is shown, indicating for each case the corresponding optimal choice in period 1.

Obviously at this early stage of the example no stopping of the process is yet possible, as the lowest MC obtained so far is still above 50%.

The procedure would continue in the same form, determining in any period t for each defined moisture all feasible drying policies, and using the recursive relation (3.1) to evaluate the best corresponding policy in period (t − 1). The process goes from one period to the next until the termination rule can be applied. For reasons of space, it is not feasible to develop the example further.

5. CONCLUSION

The approach presented constitutes a feasible way of efficiently planning wood drying in kilns. Limited computational experience was carried out successfully (Gorvad et al. 1978).

We note that the DP procedure presented is tailored to the particular problem of drying cedar pencil slats. For other cases, potential users may consider the same basic methodological approach. However, special care must be taken to consider the particular characteristics of each problem. This leads to a major effort on the part of the user. On the one hand, if a different wood species, kind of equipment, or technology is used, this obviously implies that equations and estimates for drying rates, costs, loss functions, and degrade must be determined for those particular characteristics, with all the experimentation and computer processing this entails. Even though it may be less obvious, the DP model will differ according to the problem. We have already described one modification, when the degrade process is considered in a cumulative form. As a general rule,

TABLE 2. *Cost evaluation for all feasible control decisions for day 2 with starting moisture content 62%.*

Policy	T (°F)	TD (°F)	M_3 (%)	F_1 (BTU × 10^4) X_6 and X_{16}	F_2 (BTU × 10^4)		F_4 (BTU × 10^4) X_6 and X_{16}	Total cost (BTU × 10^4)		Best policy for period 1
					X_6	X_{16}		X_6	X_{16}	
X_1	135	125	57.0	20.3	—	—	4.5	113.1	113.5	X_6
X_2	135	130	59.2	11.4	—	—	2.2	101.9	102.3	X_6
X_3	140	125	54.5	30.4	—	—	7.2	125.9	126.3	X_6
X_4	140	130	56.5	22.3	—	—	4.5	115.1	115.5	X_6
X_5	140	135	59.0	12.2	1.2	—	2.2	103.9	103.1	X_{16}
X_6	145	130	53.7	33.6	—	—	7.2	129.1	129.5	X_6
X_7	145	135	57.0	20.2	1.2	—	3.8	113.5	112.7	X_{16}
X_8	145	140	58.6	13.7	2.4	—	2.3	106.7	104.7	X_{16}
X_9	150	135	52.9	36.7	1.2	—	7.4	133.6	132.8	X_{16}
X_{10}	150	140	55.2	27.4	2.4	—	4.8	122.9	120.9	X_{16}
X_{11}	150	145	58.1	15.7	3.7	—	2.4	110.1	106.8	X_{16}
X_{12}	155	140	52.1	39.8	2.4	—	7.4	137.9	135.9	X_{16}
X_{13}	155	145	54.6	29.8	5.0	—	4.9	128.0	123.4	X_{16}
X_{14}	155	150	57.9	16.5	6.2	—	2.4	113.4	107.6	X_{16}
X_{15}	160	145	51.2	43.3	5.0	—	7.5	144.1	139.5	X_{16}
X_{16}	160	150	54.0	32.1	6.2	—	4.9	131.5	125.7	X_{16}
X_{17}	165	150	50.4	46.4	6.2	—	7.5	148.4	142.6	X_{16}

Cost for initial day: X_6 = 88.3; X_{16} = 88.7.

the user must be aware, that while DP is a valid general approach for kiln drying, and in many cases the model to use will not differ much from the one presented here, he will have to study carefully the characteristics of his problem, to develop the adequate DP model.

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