EVIDENCE OF NONLINEAR FLOW IN SOFTWOODS FROM WOOD PERMEABILITY MEASUREMENTS

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ABSTRACT

A series of air permeability measurements of softwoods and hardwoods was conducted with flow rates from 0.015 to 32 cm^3/sec to examine nonlinear flow. The permeability of loblolly pine, Douglas-fir, and white spruce was found to decrease with flow rate. The critical Reynolds numbers obtained from the decrease in permeability and also from the increase in the applied pressure difference with flow rate, based on Tompkins' equation, were between 0.41 and 1.62. These were in good agreement with the values calculated from the length-to-radius ratio of pit openings in accordance with Siau and Petty's study of short capillaries. This indicated the presence of nonlinearity owing to kinetic energy losses at the pit openings. Nonlinear flow could not be detected for paper birch and basswood.

Keywords: Air permeability, hardwoods, softwoods, nonlinear flow, Reynolds number.

INTRODUCTION

There is a close relationship between permeability and wood structure, and many studies have been conducted to characterize structural features of wood by gas permeability measurements (Sebastian et al. 1965; Comstock 1967; Siau et al. 1981; Petty 1974, 1978). According to these studies, the permeability of softwoods depends on the interconnection of tracheid lumens by pit membranes (Stamm 1967), and that of hardwoods on the interconnection of vessel elements by perforation plates or intertissue pitting.

Darcy's and Poiseuille's laws, which are usually applied to the permeability of wood, are valid for linear flow. Turbulence would be expected at Reynolds numbers exceeding 2,000 (Scheidegger 1960), which is much higher than that which can be achieved in most woods. On the other hand, nonlinear flow due to kinetic energy loss at the entrance of a short capillary begins at much lower Reynolds numbers. The latter are approximately equal to the length-to-radius ratio of a capillary (Siau and Petty 1979). It is also suggested by Siau (1984) that nonlinear flow could occur at Reynolds numbers as low as 1 at the pit openings in softwoods because the thickness of the pit membrane is approximately 0.1 micrometers (Sebastian et al. 1965; Petty 1970), and the average pore radius for the pit opening could vary between 0.02 and 4 μm with a mean of about 0.3 μm (Siau 1984).

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Another possible place where nonlinear flow could occur is at the perforation plates in hardwoods. These also act as short capillaries.

The purpose of this study was to examine the possibility of such nonlinear flow in hardwoods and softwoods using air permeability measurements.

EXPERIMENTAL PROCEDURE

The permeability measurements were made by using the apparatus designed by Petty and Preston (1969) and described by Siau (1984). A vacuum was maintained on one side with the pressure regulated by a manostat, which permitted control of the mean pressure in the specimen from 0.01 to 0.65 atm. The flow rate was regulated by a needle valve and measured with an electronic flowmeter and by use of a soap bubble rising in a buret within the range of 0.015 to 32 cm^3/sec. Desiccant was used to dry the air entering the specimens, which were vacuum-dried before the measurement. Cylindrical specimens of *Pinus taeda* (loblolly pine), *Picea glauca* (white spruce), *Pseudotsuga menziesii* (Douglas-fir), *Betula papyrifera* (paper birch), and *Tilia americana* (basswood) were prepared. The diameters were approximately 5 cm, and the lengths varied from 1 to 12 cm to permit a wide range of flow rates. The cylindrical surfaces were coated with a silicone sealant.

THEORY

Tompkins (1974) presented an equation for liquid flow in which the total pressure drop across a short capillary is equal to the sum of viscous and nonlinear flow components.

$$\Delta P = \frac{8\eta LQ}{\pi r^4} + \frac{m\rho Q^2}{\pi^2 r^4} \tag{1}$$

where η is the viscosity; ρ, the density of the liquid; r, the radius and L, the length of the capillary; and m, the coefficient for kinetic energy and end effect losses. The first term contains Q in accordance with Poiseuille's equation. The second term with Q^2 represents the component due to kinetic energy losses at the entrance and exit of a short capillary as formerly presented by Erk (1929) and Bolton and Petty (1978).

The relationship between Q and Reynolds number (Re) is expressed as follows for a circular capillary.

$$Q = \frac{\pi r \eta Re}{2\rho} \tag{2}$$

Since one of the powers of Q in the second term of Eq. (1) may be substituted by Eq. (2), Eq. (1) may be written for gas flow as formerly presented by Siau and Petty (1979), by using Mickelson's value of 1.19 for m, by including the Couette correction for short capillaries and by the addition of corrections for slip flow and gas expansion:

$$\Delta P = \frac{8QL'\eta P}{\pi r^4 s\bar{P}}[1 + 0.074Re(r/L')k] \tag{3}$$

where $L' = L + 1.2r$; s is the slip flow factor; k, the correction for gas expansion; P is the pressure at which Q is measured; and $\bar{P}$, the average pressure in the capillary.

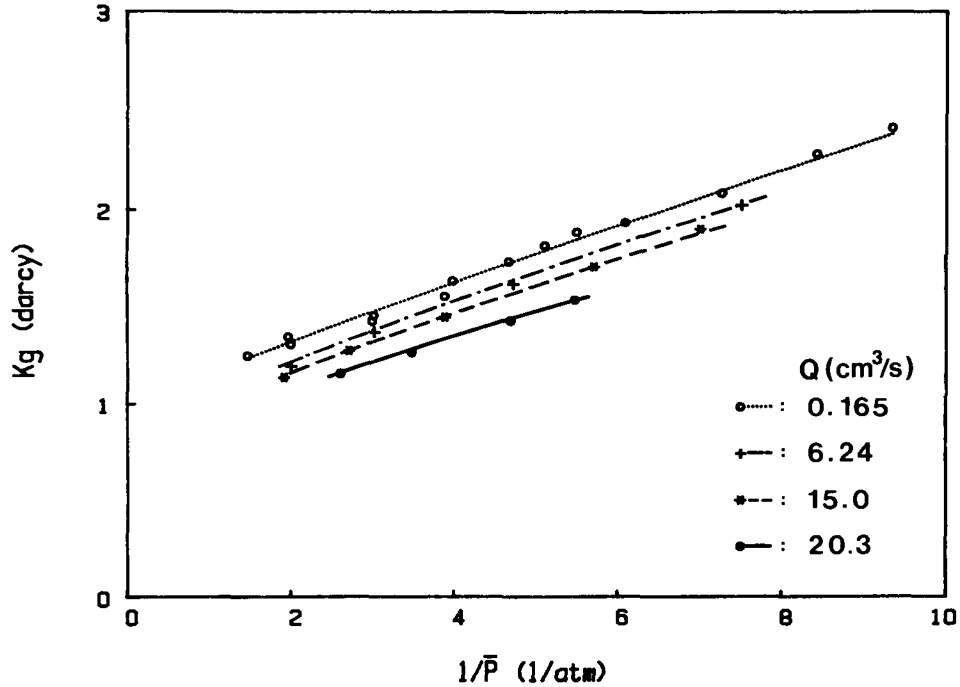

FIG. 1. Plots of permeability vs. reciprocal mean pressure of loblolly pine (L = 3.9 cm) illustrating the decrease in permeability as the flow rate is increased.

When Eq. (3) is solved for gas permeability, K_g, it assumes the form:

$$K_g = \frac{QLP}{\Delta PA\bar{P}} = \frac{\pi r^4 sL}{8\eta L'A[1 + 0.074Re(r/L')k]} \tag{4}$$

Then Eqs. (3) and (4) predict an increase in ΔP and a corresponding decrease in K_g for short capillaries. According to Siau and Petty (1979), the critical Reynolds number Re″ for a short capillary may be expressed as:

$$Re'' \approx 0.8L'/r \tag{5}$$

Therefore, Eqs. (3) and (4) also predict that, when Re becomes equal to Re″, ΔP will be increased by approximately 6% and K_g will be decreased by approximately the same amount owing to kinetic energy effects.

RESULTS AND DISCUSSION

Figure 1 shows the permeability change of loblolly pine with reciprocal mean pressure in accordance with the Klinkenberg equation. According to this relationship, the permeability is independent of flow rate, but it is clear that the permeability was reduced as the flow rate was increased from 0.165 to 20.3 cm^3/sec.

The relationship between the pressure gradient ($\Delta P/L$) and the flow rate Q at mean pressure of 0.5 atm is shown in Figs. 2 and 3. All specimens exhibited a linear relationship between gradient and flow rate with values of Q lower than 3 cm^3/sec. At higher flow rates up to 30 cm^3/sec, ΔP exhibited a curvilinear rela-

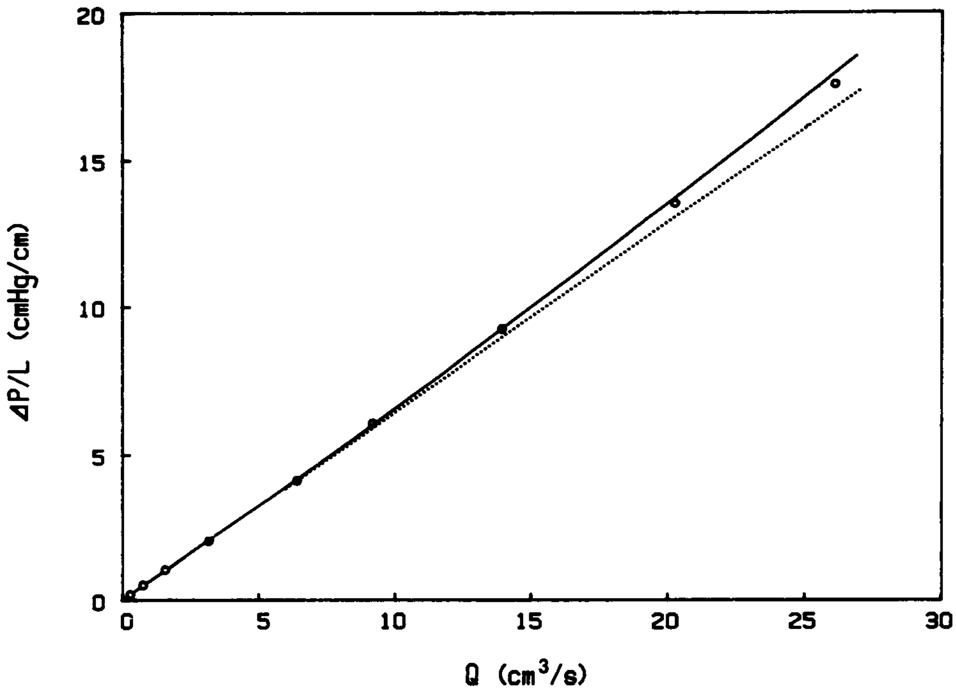

FIG. 2. The relationship between pressure gradient and flow rate for Douglas-fir (L = 1.0 cm). The linear dotted line is an extrapolation from values obtained up to Q = 3.0 cm^3/sec. $\bar{P}$ = 0.5 atm.

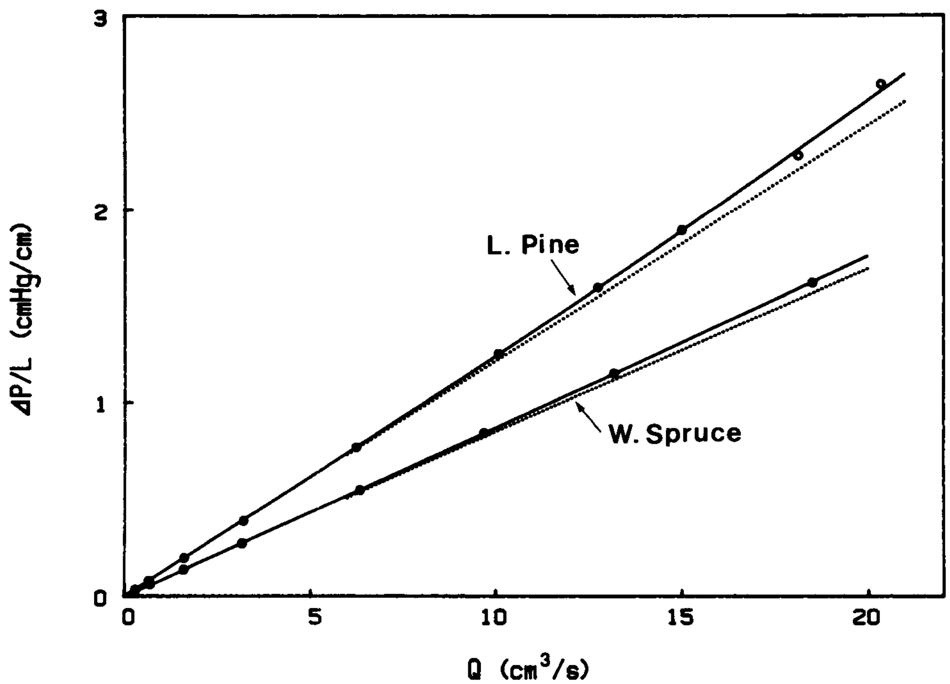

FIG. 3. The relationship between pressure gradient and flow rate for loblolly pine (L = 3.9 cm) and white spruce (L = 21.8 cm). The linear dotted lines are extrapolations of data taken up to Q = 3.0 cm^3/sec. $\bar{P}$ = 0.5 atm.

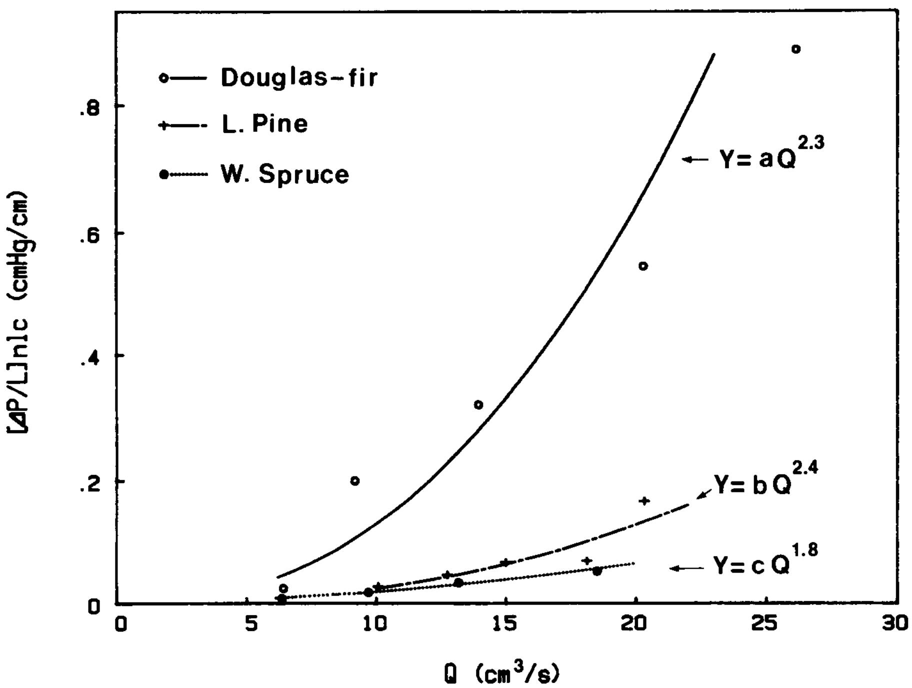

FIG. 4. The nonlinear components of the pressure gradients as a function of flow rate for one specimen each of Douglas-fir (L = 1.0 cm), loblolly pine (L = 3.9 cm), and white spruce (L = 21.8 cm).

tionship with Q as revealed by the deviations from the linear dotted lines extrapolated from the data taken at flow rates lower than 3 cm^3/sec.

Figure 4 shows the nonlinear components of $\Delta P/L$, which were obtained by subtraction of the linear components from the total pressure gradients, as indicated in Figs. 2 and 3 for one specimen of each species. When fitted to a regression equation, $\Delta P = aQ^b$, the powers of Q were 2.3 for Douglas-fir, 2.1 for loblolly pine, and 2.2 for white spruce by taking average results from a few specimens of each species. This is in good agreement with the value of 2.0 as predicted by Eq. (1). This is strong evidence of the presence of nonlinear flow due to kinetic energy losses in softwoods.

The relationship between percent increase of ΔP and flow rate is shown in Fig. 5. Similarly, the percent decrease in permeability with flow rate is revealed in Fig. 6. The intersection of the regression lines of these relationships with the 6% increase or decrease line gives the values of critical flow rate according to Eqs. (3) and (4). The critical flow rate determined by both experimental methods are relatively close to each other as revealed in Table 1. It is also interesting to note that these plots intersect close to the origin as predicted by Eqs. (3) and (4).

If the nonlinearity in softwoods is assumed to occur between the pit openings and the tracheids, and if the Comstock model (Comstock 1970; Siau 1984) is applied, the critical flow rate of one pit opening (pit-membrane pore) (Q'_p) may be expressed as:

$$Q'_p = \frac{Q'}{n_pA} = \frac{\pi\eta r_p Re_p''}{2\rho} \quad (6)$$

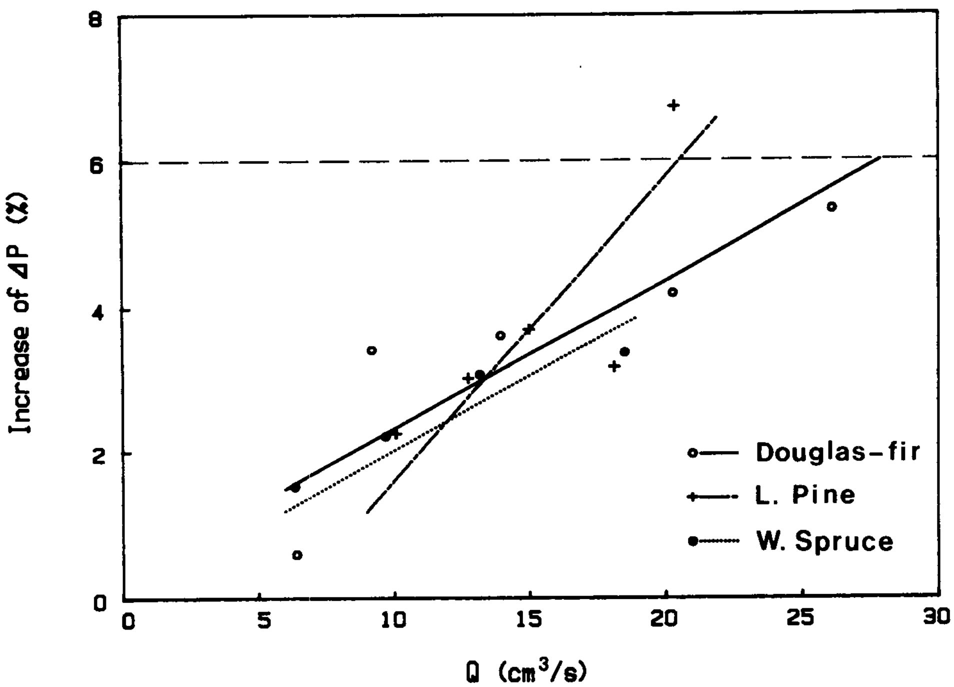

FIG. 5. The percent increase in the pressure gradient above that required at low flow rates as a function of the volumetric flow rate for Douglas-fir (L = 1.0 cm), loblolly pine (L = 3.9 cm), and white spruce (L = 21.8 cm).

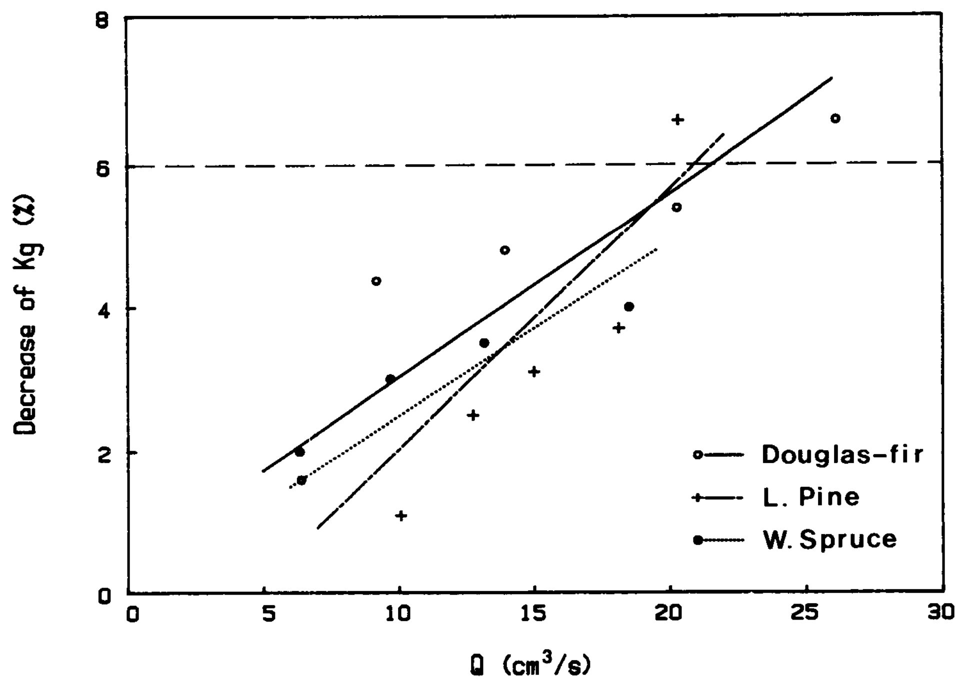

FIG. 6. The percent decrease in the gas permeability below that corresponding to low flow rate as a function of the volumetric flow rate for Douglas-fir (L = 1.0 cm), loblolly pine (L = 3.9 cm), and white spruce (L = 21.8 cm).

TABLE 1. *Comparison of critical flow rate (Q') and Reynolds number (Re) among species.*

Species	L. Pine	Douglas-fir	W. Spruce
Q' from Kg (cm^3/sec)	23.4	20.9	39.4
Q' from ΔP (cm^3/sec)	21.3	28.0	39.3
Average Q' (cm^3/sec)	22.4	24.5	39.4
Re''_p (exp)	0.41	1.62	0.63
Re''_p (theor)	1.04	1.08	1.03

where Q' is the critical flow rate of the specimen obtained from Figs. 5 and 6 based on Eqs. (3) and (4); n_p, the number of pit openings per unit area of the specimen; A, the cross-sectional area of the specimen; r_p, radius of the pit opening; and Re_p'', the critical Reynolds number of one pit opening. By rearranging Eq. (6), Re_p'' may be calculated as:

$$Re_p'' = \frac{2\rho Q'}{n_p \pi \eta r_p A} \tag{7}$$

where n_p and r_p may be calculated by analyzing the curvilinear relationship of K to $1/\bar{P}$ as described by Siau (1984) by assuming that the pore is circular.

Based on Eq. (7) and the average Q' obtained from the increase of ΔP and the decrease of K_g, the experimental critical Reynolds number for flow through a pit opening is 0.41 for loblolly pine, 1.62 for Douglas-fir, and 0.63 for white spruce. On the other hand, corresponding values of the critical Reynolds number calculated from Eq. (5) were 1.04, 1.08, and 1.03, respectively. The corresponding radii (r_p) were calculated as 0.95, 0.88, and 1.1 μm from the analysis of the plot

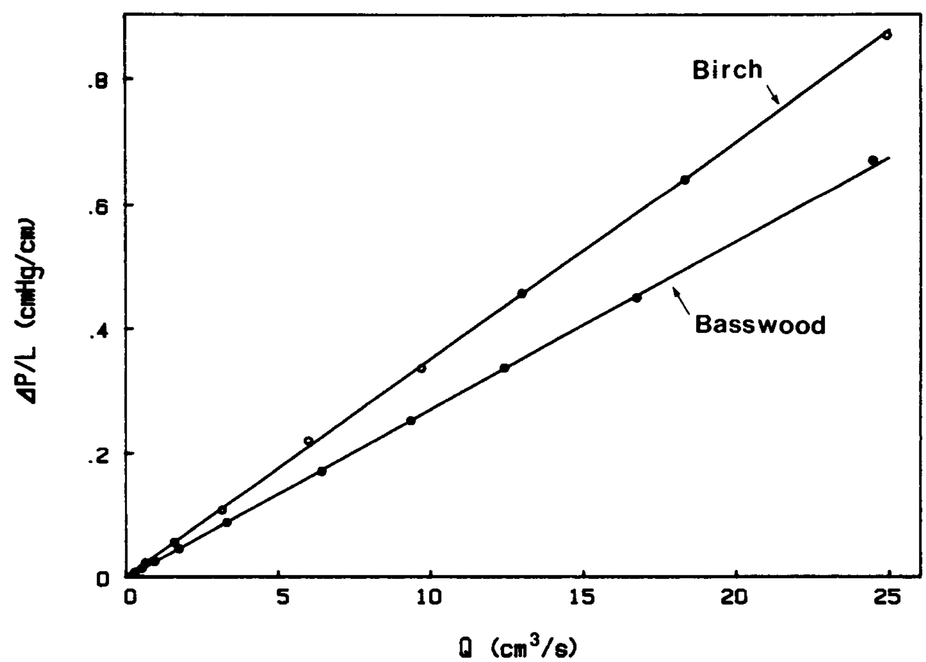

FIG. 7. The relationship between pressure gradient and flow rate for birch (L = 21.4 cm) and basswood (L = 10.7 cm). These are both linear and do not indicate the presence of nonlinear flow.

of K_g vs. 1/P using a value of 0.1 μm as the thickness of the pit membrane. Therefore, the ratios of the experimental to theoretical Reynolds numbers were 0.39, 1.50, and 0.61 for loblolly pine, Douglas-fir, and white spruce, respectively. This is considered to be a good agreement in view of the several assumptions used in their determinations. It also strengthens the evidence for the presence of nonlinear flow in these softwoods species.

On the other hand, the two hardwoods, birch and basswood, exhibited a linear relationship between $\Delta P/L$ and Q as revealed in Fig. 7. This may have been due to the relatively low resistance of the path through the perforation plates compared with that of the vessels. Therefore there is no evidence of nonlinear flow in the hardwoods up to the maximum flow rate possible under the experimental conditions.

CONCLUSIONS

A decrease in the permeability at increased flow rates may be taken as evidence of the onset of either turbulent or nonlinear flow. Calculations of the critical Reynolds numbers are much lower than the value of 2,000 which corresponds to turbulence. However, they are in reasonably good agreement with theoretical values for nonlinear flow based on the length-to-radius ratio of the pit openings. Therefore the decreased permeabilities were attributed to nonlinearity at the pit openings. In addition, the pressure gradients corresponding to the nonlinear components of flow were approximately proportional to the square of the velocity as predicted by theory. There was no evidence of nonlinear flow in the hardwoods that were investigated.

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