# APPLICATION OF A MODIFIED STATISTICAL SEGMENTATION METHOD TO TIMBER MACHINE STRENGTH GRADING

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#### ABSTRACT

In this paper, a new method for deriving grading rules is given. This method is based on the multiple regression and discrimination techniques by binary prediction trees, which are of high interest for classification purposes. A modification to the existing practice that tends to predict the bending modulus of rupture (MOR) is presented. This method consists in creating a nominal variable, the "optimal strength class value," which contains the information necessary to enter into a strength class system, i.e., the density, the bending modulus of elasticity (MOE), and the MOR. A comparison between a regression technique aimed at predicting the MOR and a discrimination technique aimed at predicting the optimal strength class assignment illustrates the innovative aspect of this method.

Keywords: Timber, machine grading, statistics, classification.

## INTRODUCTION

Most of the research carried out in the field of timber grading is based on the parametric regression method. The regressions may be simple, as for classical bending type grading machines, or multiple, as for new multiple devices grading machines. Some of the models are linear; others may be nonlinear. These parametric regressions are limited because the models apply to the whole population and only explain an average trend. Another constraint is that we need to derive grade limits that are calculated from this average trend but must insure the required safey level of a grade, given by lower fractiles (see Fewell undated). The statistical segmentation technique is innovative because it is nonparametric, i.e., there is no analytical model to predict an average trend, but a set of limits that divide the population into "segments" of distinct quality levels. The method was introduced originally by Morgan and Sonquist (1963), who developed the AID (Automatic Interaction Detection) technique. It aims at reducing the variance of the parent sample by dividing it into binary segments that

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are determined by a limited value (upper bound on the left segment and lower bound on the right segment) on a predictive variable. The method was improved by Breiman et al. (1984), who found an algorithm to "prune" the initial trees and give an optimal tree.

The CART (Classification and Regression Tree) method can be applied for predicting a continuous variable. In this case, it is named a regression method. For grading purposes, the variable to be predicted is the MOR, given for each specimen. But the CART method may also be applied for predicting a nominal variable, i.e., a variable that can take only a limited set of values. In that case, it is named a discrimination method. For grading purposes, a nominal variable is the strength class to which a given specimen "should" be assigned. It can be derived by an optimal ranking technique.

#### OPTIMAL RANKING METHOD

If we want to grade timber according to EN 338 strength classes system (AFNOR 1995), three requirements for each strength class have to be fulfilled:

- characteristic density, corresponding to 5th percentile lower fractile,
- -characteristic MOR, corresponding to 5th percentile lower fractile,
- -mean MOE.

Before describing how to rank a population of timber in an "optimal way," let us explain how to achieve a characteristic value (so-called target value) from a parent population. If a given sample of size N, ranked according to ascending values of x, is such that the target value  $x_k$  has a cumulative frequency  $f_k$  higher than 5%,  $x_k$  will have a cumulative frequency of 5% in the new sample by taking out a number of pieces n from the lower part of the distribution. To determine the cutting level *n*, let us assume that

$$f_k = \frac{i}{N} \tag{1}$$

where *i* is the rank of  $x_k$  in the original sample of size *N* (which means  $Prob(x \le x_k) = i/N$ ). We want to have

$$f'_{k} = \frac{i-n}{N-n} = 0.05$$
(2)

which gives

$$n = \frac{1}{1 - 0.05} \{ i - 0.05N \}$$
(3)

or, by using Eq. (1)

$$n = \frac{N}{1 - 0.05} \{ f_k - 0.05 \}$$
(4)

Now consider a population of size N, in which we know for each specimen the MOE, the MOR, and the density. If we want to extract from this population the specimens that should enter into a given strength class, we have to apply the algorithm given in Fig. 1. Note that for the MOE, Eq. (4) does not apply and we have to use a step by step procedure (see Fig. 2) to determine the cutting level  $n_3$ . In this algorithm, the mean value of a variable X, ranked according to ascending values  $(X_1 \le X_2 \le \ldots \le X_j \le X_{(j+1)} \le \ldots \le X_N)$ , is given by

$$\mu(X) = \frac{\sum_{i=j+1}^{N} X_i}{N-j}$$
(5)

where j is the number of pieces taken out from the lower part of the sample.

If we now want to perform an optimal ranking according to the whole EN338 system (AF-NOR 1995), we repeat the procedure by descending strength classes order. This method gives spectacular results, compared for example to a visual grading method (see Rouger et al. 1993). In Table 1, a comparison of yields between visual grading and optimal ranking is given for three species. In this table, three strength classes (C30, C22, C18) have been selected. To illustrate the computation corresponding to the algorithm given in Fig. 1, intermediate values of  $(n_1, n_2, n_3)$  for the optimal ranking of spruce and fir are reported in Table 2. It is obvious that any grading method could not reach the yields of the optimal grading method, but it gives an upper limit that we should try to work toward.

This optimal strength class is given for each specimen, and therefore can constitute a nominal variable that we wish to explain. This option is more powerful than a simple explanation of the MOR, because the strength class assignment depends on the density, the MOR, and the MOE. The following sections refer to a comparison of both methods.

#### BASIC PRINCIPLES OF THE SEGMENTATION TECHNIQUE

Let us assume that we want to explain a variable Y by a set of predicting variables  $X_1$ ,  $X_2, \ldots, X_p$ . The purpose of classification is to reduce the initial variance of the parent sample by giving limiting values on the predictive variables. When dividing a segment into two subsegments, the best division has to minimize the weighted mean of the variances of Y in the subsegments (so-called "intravariance"). The algorithm that is used to divide a segment into two "subsegments" is illustrated in Fig. 3.

The "largest tree" is a tree in which each

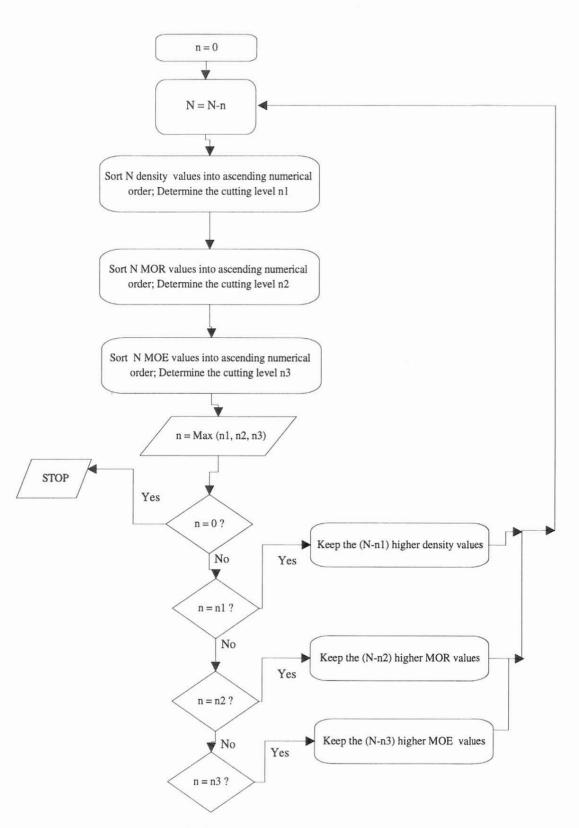


FIG. 1. Algorithm for optimal ranking (for one strength class).

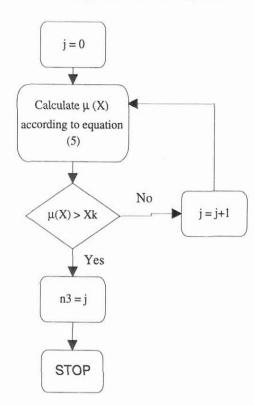


FIG. 2. Algorithm for MOE cutting level.

"terminal segment" contains only one specimen. The intravariance of this tree equals zero. Two mathematical functions are used to measure the performance of a given tree:

- the apparent error of prediction (EAP), which is used to measure the capacity of prediction of the tree on the learning sample,
- --the theoretical error of prediction (ETP), which is used to measure the capacity of prediction of the tree on the test sample.

For the largest tree, the EAP equals zero, but

 TABLE 2. Optimal ranking of spruce and fir: Illustration of algorithm given in Fig. 1.

Strength class	Iter- ation n.	N	n <sub>1</sub>	n <sub>2</sub>	n3	N – max(n <sub>i</sub> )
C30	1	2,967	0	399	498	2,469
	2	2,469	0	120	0	2,349
	3	2,349	0	0	0	2,349
C22	1	618	0	123	488	130
	2	130	0	22	0	108
	3	108	0	0	3	105
	4	105	0	0	0	105
C18	1	513	0	40	377	136
	2	136	0	3	0	133
	3	133	0	0	4	129
	4	129	0	1	0	128
	5	128	0	0	1	127
	6	127	0	0	0	127

there is very little chance that this tree is applicable to another sample (the ETP is high). For the smallest tree, both the EAP and the ETP are high (this tree does not predict anything). These functions (EAP and ETP) are also expressed in relative terms (relative errors of prediction). The algorithm of "pruning" aims to optimize the combination of the EAP and the ETP.

To calculate the EAP, let us consider a given tree (A). For each terminal segment (t), the weighted variance is calculated

$$R(t) = p(t)s^2(t) \tag{6}$$

where R(t) is the weighted variance of segment t, p(t) is the yield of segment t,  $s^2(t)$  is the variance of segment t. p(t) and  $s^2(t)$  are calculated according to

$$p(t) = \frac{N(t)}{N} \qquad s^2(t) = \frac{\sum_{i \in t} \{y_i - \mu(y(t))\}^2}{N(t)} \quad (7)$$

TABLE 1.	comparison of	yields between	visual grauing	unu optimui	runking

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Species Strength	Spruce and fir 2967 specimens		Douglas-fir 1750 specimens		Maritime pine 1400 specimens	
	Visual	Optimal	Visual	Optimal	Visual	Optimal
C30	10%	79%	9%	78%	0%	61%
C22	49%	3.5%	11%	13.5%	0%	13%
C18	12%	4.5%	22%	5%	54%	7.5%
Reject	29%	13%	58%	3.5%	46%	18.5%

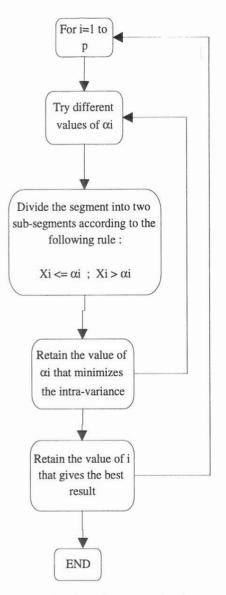


FIG. 3. Algorithm for constructing the trees.

where N(t) is the number of specimens in segment (t), N is the total number of specimens,  $y_i$  is the individual value of Y for a specimen i in segment t,  $\mu(y(t))$  is the mean of Y in segment t. The EAP of the tree (A) is given by

$$EAP(A) = \sum_{t \in A} R(t).$$
 (8)

### ALGORITHM OF "PRUNING"

The tree constructed by the method described earlier is symmetrical and not "optimal," in the sense that it is not applicable to another sample. The algorithm of pruning is necessary to extend the prediction capability of the tree. It can be summarized according to the following rules:

- (a) Separate randomly the parent sample into two subsamples: the "LEARNING SAM-PLE," which will be further used for creating the trees, and the "TEST SAMPLE," which will be used to select among these trees the optimal one. Work is currently being done to check the effect of the random selection on the results. A further publication will focus on this topic.
- (b) With the LEARNING SAMPLE construct a largest tree A<sub>max</sub> with a minimum number of specimens in each terminal segment (e.g., n<sub>min</sub> = 5).
- (c) Construct a series S\* of smaller and smaller trees (up to a tree of one segment only, which is the LEARNING SAMPLE), by suppressing pairs of segments.
- (d) Each tree A<sub>i</sub> of S\* has a given number of terminal segments (i). Among the possibilities of trees with the same number of terminal segments, choose the one that minimizes the EAP.
- (e) Among the trees A<sub>i</sub> of S\* which satisfy rule (d), select the optimal tree by using the TEST SAMPLE. The optimal tree A\* is the smallest one that gives the smallest ETP. The ETP of a tree (A) is given by

$$ETP(A) = \sum_{t \in A} R^{test}(t)$$
(9)

with

$$R^{test}(t) = p^{test}(t)(s^{test})^2(t)$$
(10)

and

$$p^{test}(t) = \frac{N^{test}(t)}{N^{test}}$$
$$(s^{test})^2(t) = \frac{\sum_{i \in t} \{y^{test}_i - \mu(y^{test}(t))\}^2}{N^{test}(t)} \quad (11)$$

where  $N^{test}(t)$  is the number of specimens in segment (t),  $N^{test}$  is the total number of specimens of the TEST SAMPLE,  $y^{test}_i$  is the individual value of Y for a specimen i in segment t,  $\mu(y^{test}(t))$  is the mean of Y in segment t.

(f) Apply the "One standard-deviation rule": If A\* is the optimal tree, i.e.,

$$ETP(A^*) = Min\{ETP(A_k)\}$$
(12)

where k is the number of terminal segments of the tree  $A_k$ , then choose the tree with  $k_o$  segments, where  $k_o$  is the maximum value of k which verifies

$$ETP(A_k) \le ETP(A^*) + + std-dev(ETP(A^*)) \quad (13)$$

where std- $dev(ETP(A^*))$  is the standard deviation of the theoretical errors of prediction (see Eq. (9)) corresponding to the trees having the same number of terminal segments as the tree A\*.

For each tree  $A_i$  of S\*, the relative errors of prediction are given by

$$RET(A_i) = \frac{ETP(A_i)}{s^2(test)}$$
(14)

where  $s^2(test)$  is the initial variance of the TEST SAMPLE.

$$REA(A_i) = \frac{EAP(A_i)}{s^2(basic)}$$
(15)

where  $s^2(basic)$  is the initial variance of the LEARNING SAMPLE.

#### DISCRIMINATION METHOD

A similar technique is called the "Discrimination Technique by binary trees." In this case, the variable Y to be predicted is a nominal variable, i.e., it can take only discrete values. The errors of prediction are subsequently calculated by evaluating the percentage of "badly classified specimens."

On the whole sample, an initial assignment to ONE value of  $Y(Y_o)$  is given, which corresponds to the most frequent value of Y in the whole sample. An initial ratio of badly classified specimens is calculated according to

$$TEI = \frac{\sum_{i=1}^{N} \delta_{i0}}{N} \quad \text{with } \delta_{i0} = \begin{cases} 1 \text{ if } Y_i \neq Y_0 \\ 0 \text{ if } Y_i = Y_0 \\ (16) \end{cases}$$

where TEI is the initial ratio of badly classified specimens, N is the number of specimens in the whole sample,  $Y_i$  is the value of Y for specimen *i*,  $Y_o$  is the initial assignment of the sample.

For a given tree, each terminal segment (t) is assigned to a value of  $Y(Y_t)$ , which corresponds to the most frequent value of Y in the segment t. For each terminal segment, a ratio of badly classified specimens is calculated according to

$$RBC(t) = \frac{\sum_{i=1}^{N(t)} \delta_{it}}{N(t)} \quad \text{with } \delta_{it} = \begin{cases} 1 \text{ if } Y_i \neq Y_t \\ 0 \text{ if } Y_i = Y_t \end{cases}$$
(17)

where RBC(t) is the ratio of badly classified specimens of segment t, N(t) is the number of specimens of segment t,  $Y_i$  is the value of Y for specimen i,  $Y_t$  is the assignment of segment t.

For each tree, the ratio of badly classified specimens is calculated by a weighted mean of the ratios of badly classified specimens for each terminal segment. This calculation is performed for both the LEARNING SAMPLE (*TEA*) and the TEST SAMPLE (*TET*).

$$TEA = \sum_{t \in A} p(t) \cdot RBC(t)$$
$$TET = \sum_{t \in A} p^{test}(t) \cdot RBC^{test}(t) \quad (18)$$

where p(t) and  $p^{test}(t)$  are given by Eqs. (7) and (11). These ratios are divided by the initial ratio (*TEI*) to give the relative errors (*REA*, *RET*). Therefore, the ratios of badly classified specimens are given by

$$\begin{aligned} TEA &= REA \cdot TEI \\ TET &= RET \cdot TEI \end{aligned} \tag{19}$$

Therefore, the ratios of "well classified specimens" are given by

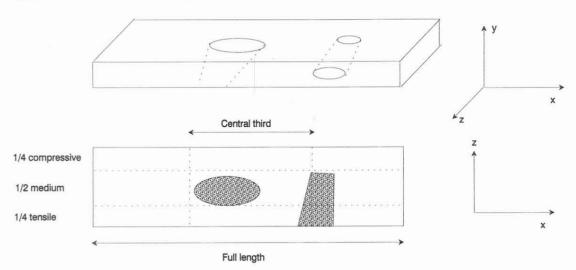


FIG. 4. Pseudo-KAR calculations.

$$\begin{cases} WCSA = 1 - TEA \\ WCST = 1 - TET \end{cases}$$
(20)

### APPLICATION OF BOTH METHODS TO STRENGTH GRADING

The regression technique may be applied to strength grading, by predicting the MOR values from nondestructive variables. The discrimination technique may also be applied by predicting the optimal strength class. The optimal trees are then treated to get the correct strength classes:

- (a) Apply the tree on the whole sample (learning + test).
- (b) For each segment, calculate the 5% MOR, mean MOE, 5% density.
- (c) Classify the terminal segments according to the 5% MOR.
- (d) Select a set of strength classes.
- (e) Aggregate the terminal segments (and the corresponding criteria) if necessary to meet the requirements of the strength classes, i.e., 5% MOR, mean MOE, 5% density.

The following example illustrates the efficiency of the method: A sample of 166 specimens of Scots pine ( $50 \times 150 \times 3000 \text{ mm}$ ) have been tested by using different nondestructive devices, as well as tested in destructive bending to get the MOE and MOR values. For each specimen, 32 nondestructive variables have been measured by using 4 devices:

- -slope of grain
- -gamma-radiation device
- -Cook-Bolinder flatwise bending machine
- -SCANWOOD vision system.

The list of nondestructive variables is reported below. The gamma-radiation device is used to get the density, but also to calculate pseudo-KAR values (knots area ratios), as illustrated in Fig. 4. Whereas the usual knots area ratios are calculated on the (y, z) plane, the pseudo-KAR values are calculated in the (x, z) plane. In the bending machine, two runs are analyzed, due to a different influence of a knot at the beginning of the board and at the end of the board. A combination of both runs is subsequently derived.

### \*) SLOPE OF GRAIN

Average face slope of grain	SOG1
Minimum face slope of grain	SOG2
Maximum face slope of grain	SOG3

SOG4 Average edge slope of grain Minimum edge slope of grain SOG5 Maximum edge slope of grain SOG6 GAMMA-RADIATION DEVICE Average density GAM1 Pseudo-KAR-tensile zonefull length GAM3 Pseudo-KAR-medium zone-GAM5 full length Pseudo-KAR-compressive zone-full length GAM7 Pseudo-KAR-full widthfull length GAM9 Pseudo-KAR-tensile zone-GAM12 central third Pseudo-KAR-medium zonecentral third GAM14 Pseudo-KAR-compressive zone-central third GAM16 Pseudo-KAR-full widthcentral third GAM18 \*) COOK-BOLINDER FLATWISE **BENDING MACHINE** Mean E<sub>flat</sub>-first run STR1 Standard-deviation Effatfirst run STR2 Minimum E<sub>flat</sub>-first run STR3 Maximum E<sub>flat</sub>-first run STR4 Mean E<sub>flat</sub>-second run STR5 Standard-deviation Effat-STR6 second run Minimum E<sub>flat</sub>-second run STR7 Maximum Effat-second run STR8 Mean E<sub>flat</sub>-combination STR9 Standard-deviation Effatcombination STR10 Minimum E<sub>flat</sub>-combination STR11 Maximum Effat-combination STR12

 SCANWOOD SYSTEM (VIDEO SCANNING OF KNOTS)

SCAMMING OF KINOIS)	
Face knots	NOD1
Edge knots	NOD2
Tensile KAR	NOD3
Comprehensive KAR	NOD4
Total KAR	NOD5

TABLE 3. Visual grading of the sample.

	Visual grading		
Strength class	Face knots	Edge knots	Yield
C24	≤1/3	≤2/3	39%
C18	$\leq 1/2$	$\leq 2/3$	23%
Reject			38%

## Ungraded material

The ungraded material has the following characteristics, which justify an assignment to C18.

MOR 5% = 20.8 MPa mean MOE = 10.1 GPa density 5% = 463 kg/m<sup>3</sup>

#### Visual grading

The visual grading method, as proposed in a draft version of NFB52001-4 (French Visual Grading Rule) (AFNOR 1996) gives the yields shown in Table 3.

#### Optimal ranking

When applying the algorithm given in previous sections, one gets the optimal yields given in Table 4. The strength classes requirements are given in Table 5. It can be clearly noticed that the MOE is the limiting variable that determines the optimal strength class. Therefore, one can suspect that the discrimination should give a better grading than a regression aiming at predicting only the MOR.

#### Regression method

The initial variance of the whole sample is equal to 274.3. According to the algorithm described earlier, a list of trees is given in Table 6.

TABLE 4. Optimal ranking of the initia	l sample.
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Strength class	Number of spec. (yield)	Mean MOE (in GPa)	MOR 5% (in MPa)	Density 5% (in kg/m <sup>3</sup> )
C40	23 (14%)	14.0	53.5	550
C24	55 (33%)	11.0	27.8	473
C18	66 (40%)	9.0	19.4	460
Reject	22 (13%)	7.0	16.1	453

 TABLE 5. Strength classes requirements according to EN338.

Strength class	Mean MOE (in (GPa)	MOR 5% (in MPa)	Density 5% (in kg/m <sup>3</sup> )
C40	14	40	420
C35	13	35	400
C30	12	30	380
C27	12	27	370
C24	11	24	350
C22	10	22	340
C18	9	18	320
C16	8	16	310
C14	7	14	290

TABLE 6.	List of trees	(S*) given by	the regression method.
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		Relative errors of prediction (REP)		
Tree n.	Number of terminal segments	TEST SAMPLE (RET)	LEARNING SAMPLE (REA)	
A1	41	0.66	0.0115	
A38	3	0.51	0.362	
A40	1	1.0	1.0	

minimum number of segments (N = 3). This tree is illustrated in Fig. 5.

Only two variables are used to construct the tree:

-GAM3: Pseudo-KAR-tensile zone-full length

-NOD3: Tensile KAR

segments. It predicts very well the LEARN-ING SAMPLE (REA = 0.0115) but does not fit to the TEST SAMPLE (RET = 0.66). The tree A40 corresponds to the initial sample. It does not predict anything (REA = RET = 1). The tree A38 has been selected as the optimal one (REA = 0.36, RET = 0.51), because it gives the best compromise between the LEARNING and the TEST SAMPLE for a

The tree A1 has the maximum number of

For each division level of the tree, the means of each segment are separated and the variances are reduced as much as possible. This is illustrated in Fig. 6. When applying the grading

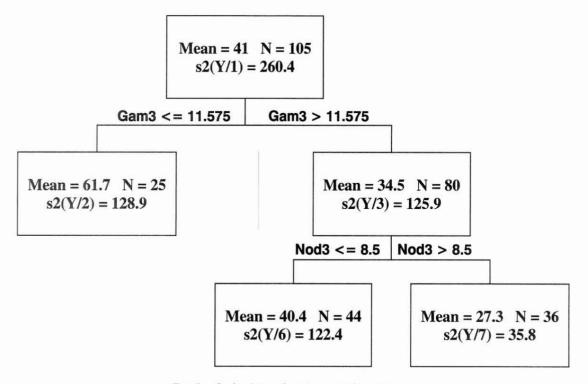


FIG. 5. Optimal tree given by regression method.

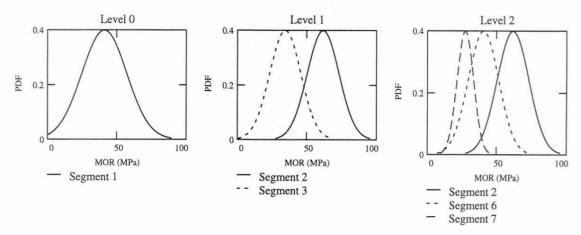


FIG. 6. Reduction of the initial sample according to the tree level.

rules to the whole sample, one can derive strength classes, as illustrated in Table 7.

### Discrimination method

The discrimination technique gives a series of trees, illustrated in Table 8. The tree A1 has the maximum number of segments. It predicts very well the LEARNING SAMPLE (REA = 0.09) but does not fit to the TEST SAMPLE (RET = 0.51). The tree A8 corresponds to the initial sample. It does not predict anything (REA = RET = 1).

The tree A5 has been selected as the optimal one (REA = 0.27, RET = 0.51), because it gives the best compromise between the LEARNING and the TEST SAMPLE for a minimum number of segments (N = 4). This tree is illustrated in Fig. 7. The percentage values correspond to the ratios of well-classified specimens.

Only three variables are used to construct the tree:

-STR11: Minimum E<sub>flat</sub>-combination of two runs

-STR1: Mean E<sub>flat</sub>-first run

# -STR5: Mean E<sub>flat</sub>-second run

One can also see that, as the division level increases, the minimum ratio of well-classified specimens increases. This is illustrated in Fig. 8. When applying the grading rules to the whole sample, one can derive strength classes, as illustrated in Table 9.

### Comparisons of the results

It is difficult to compare different grading results, since the strength classes are different and the yields are different. One can suggest a performance factor that is influenced by higher yields in higher strength classes:

TABLE 8. List of trees (S\*) given by the discrimination

Relative errors LEARNING SAMPLE Number of TEST SAMPLE TABLE 7. Strength class assignment. terminal Tree n. (RET) (REA) MOR Density Mean 0.51 0.09 15 MOE 5% 5% (in MPa) (in kg/m<sup>3</sup>) A1 Segment Number of Strength n. spec. (yield) (in GPa) class . . . 4 0.51 0.27 2 31 (19%) 12.5 42.4 473 C30 A5 . . . 76 (46%) 10.0 23.6 468 C22 . . . . . . . . . 6 7 A8 1 1.0 1.0 59 (35%) 8.9 17.5 451 Reject

technique.

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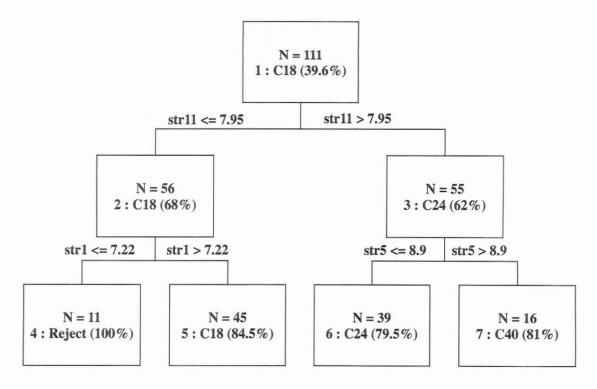


FIG. 7. Optimal tree given by the discrimination method.

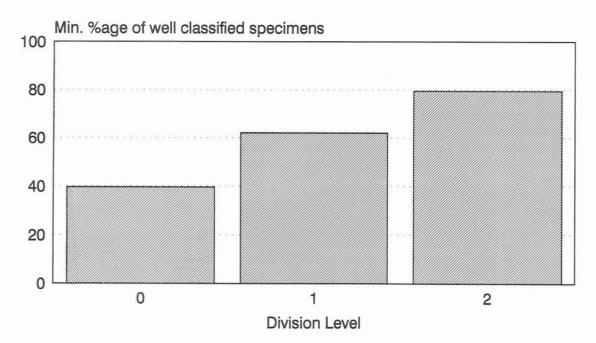


FIG. 8. Illustration of the effect of segmentation.

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Mean Density 5% MOE (in GPa) Segment Number of MOR 5% Strength spec. (yield) (in MPa) (in kg/m<sup>3</sup>) n. class 7 22 (13%) 14.0 42.6 539 C40 6 62 (37%) 10.8 24.8 476 C22 5 64 (39%) 460 9.0 19.2 C18 4 18 (11%) 7.0 12.4 408 Reject

TABLE 9. Strength class assignment.

erjorma	ince ja	ctor =	
=	Σ	$(5\% MOR)^2 \cdot yield$	(21)
str	ength class	ses	

The performance factors for the different analysis described earlier are given in Table 10.

### CONCLUSIONS

The nonparametric regression and discrimination methods are very powerful and give simple rules for machine strength grading, even with multiple sensing devices. An alternative to regression methods has been presented, by using an optimal ranking algorithm before performing a discrimination analysis. One should prefer the regression method when the MOR is determining the strength class assignment and the discrimination when either the MOE or the density is determining the strength class assignment. When looking at the relative performance of these methods, it is clear that one should expect very efficient grading rules, to be confirmed by further analysis. It also shows that one should focus not only on different measuring technologies, but also on new classification methods, which can highly improve

 TABLE 10. Performance factor for different grading systems.

Grading system	Performance factor	Re-scaled value
Visually graded	17.3	0.0
Ungraded	18.0	0.12
Regression	19.7	0.40
Discrimination	22.8	0.92
Optimal ranking	23.3	1.0

the existing grading machines. Beside the method presented in this paper, one could mention neural networks or object-oriented classification systems, which seem very promising.

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