

LUMBER PROPERTY RELATIONSHIPS FOR ENGINEERING DESIGN STANDARDS

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ABSTRACT

Data from the In-Grade Testing Program for visually graded dimension lumber are used to identify lumber property relationships for engineering design standards. The properties studied are modulus of rupture (MOR), modulus of elasticity (MOE), ultimate tensile stress parallel to grain (UTS), and ultimate compression stress parallel to grain (UCS). The relationships identified between UCS and either MOR or UTS vary little with species, lumber grade, and lumber width. The data show a closer relationship between UCS and MOR than between UTS and MOR. The historical basis for assigning allowable properties to machine stress-rated (MSR) lumber is reviewed. The UCS-MOR and UTS-MOR relationships presented in the paper are shown to be consistent with historical data obtained with visually graded lumber but different from relationships currently assumed for MSR lumber. The effects of species, moisture content, test span, lumber width, and presence or absence of pith on strength property relationships are discussed. Strength property relationships are shown to be a result of inherent growth characteristics of the lumber. Finally, a procedure is provided for estimating coefficient of variation as a function of grade that may be useful in reliability-based engineering design standards.

Keywords: Modulus of rupture, modulus of elasticity, ultimate tensile stress, ultimate compression stress, coefficient of variation.

INTRODUCTION

The relationships between lumber properties have been used extensively in deriving allowable properties for lumber. The relationship between modulus of elasticity (MOE) and modulus of rupture (MOR) forms the basis for sorting most mechanically stress-rated (MSR) lumber sold in the United States (Galligan et al. 1979). Because of the difficulty in assessing the strength of wood in tension parallel to the grain, the ratio of ultimate tensile stress (UTS) to MOR has historically been used to estimate allowable tensile strength for both visually and mechanically graded lumber. The ultimate compression stress parallel to the grain (UCS) of MSR lumber is also estimated from MOR. Further, the use of UTS/MOR and UCS/MOR ratios greatly simplifies quality control requirements for MSR lumber.

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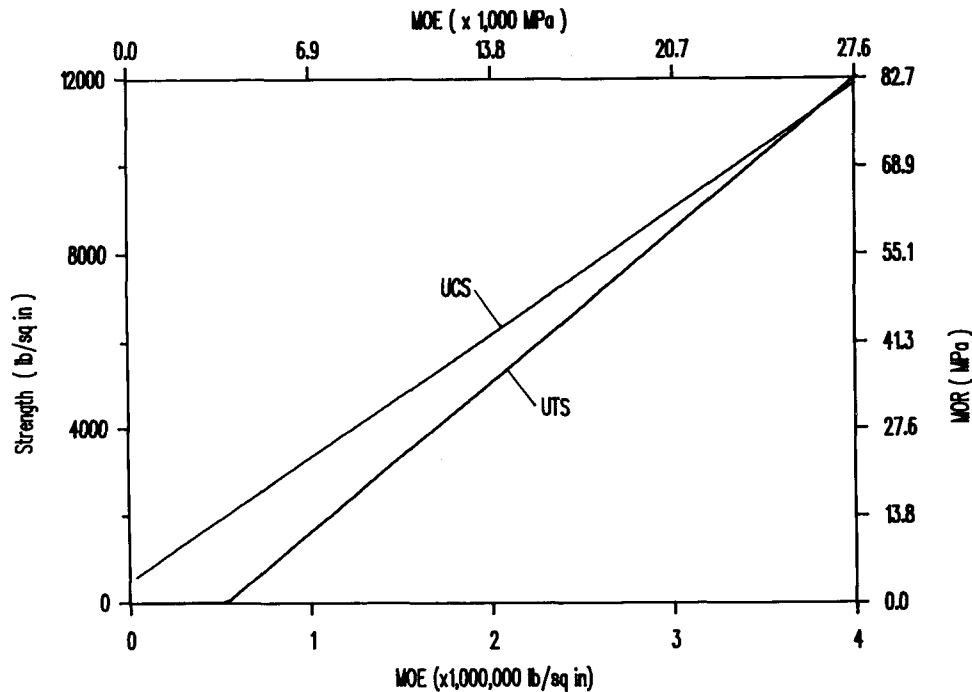


FIG. 1. Relationship of ultimate compression stress (UCS) and ultimate tensile stress (UTS) to modulus of elasticity (MOE) (Hoyle 1968).

Property relationships have also been used to reduce the cost associated with large lumber-testing programs (Green and Evans 1988). Moreover, property ratios are used in international standards as a basis for standardized property classification (stress class) systems (Fewell 1989; Green and Kretschmann 1990).

A better understanding of lumber property relationships is therefore essential for improved property assignment in engineering design standards. Until recently, studies of lumber property relationships have tended to be based on a limited number of specimens or specimens collected over a limited geographic range. The availability of large data sets collected over a wide geographic range offers the opportunity to establish a better basis for lumber property relationships used in engineering design standards (Green and Evans 1987; Canadian Wood Council 1988).

For grading purposes, an investigation of lumber property relationships may be focused on identifying species-dependent differences in the relationships. For standards, however, the development of lumber property relationships requires a search for common trends across species. The objective of this paper is to develop relationships between lumber properties using In-Grade data and to discuss their application in engineering design standards. Thus, we will identify common trends between species. This paper concentrates on relationships between MOR, MOE, UTS, and UCS. The effect of lumber species, moisture content, test span, and lumber width on these relationships will be discussed. The reader is referred to Green and Kretschmann (1989) and Barrett and Griffin (1989) for additional information on property relationships relative to those assumed in Eurocode 5,

TABLE 1. Relationship between allowable tensile stress parallel to the grain and modulus of rupture (MOR) adopted for machine stress-rated lumber.^{a,b}

Allowable stress in bending (lb/in. ² (MPa))	F_t/F_b
900 (6.2)	0.39
1,200 (8.3)	0.50
1,500 (10.3)	0.60
1,800 (12.4)	0.65
2,100 (14.5)	0.75
2,400 (16.5)	0.80
2,700 (18.6)	0.80
3,000 (20.7)	0.80
3,300 (22.8)	0.80

^a Adopted in 1969. See Galligan et al. 1979.

^b F_t is stress in tension parallel to the grain.

and to Green and Kretschmann (1990) for information on the use of these relationships in establishing property classification systems.

BACKGROUND

Property relationships for machine stress-rated lumber

The allowable design stress is the 5th percentile ultimate stress value divided by the appropriate general adjustment factor (ASTM D245 1989). In the early 1960s, the allowable design stress for visually graded lumber stressed in tension parallel to the grain, F_t , was assumed to be equal to the allowable design stress in bending, F_b (Galligan et al. 1979). However, research on the UTS of MSR lumber indicated that UTS was only about 80% of MOR (McKean and Hoyle 1964). Thus, early grading rules for MSR lumber specified that $F_t = 0.8F_b$ (WWPA 1965). Research also indicated that regressions of UCS on MOE for the lower grades produced higher values of UCS than did regressions of UTS on MOE (Fig. 1) (Hoyle 1968). For simplicity, initial grading rules assumed that allowable compression stress parallel to the grain, F_c , was equal to F_t ; therefore, $F_c \cong 0.8F_b$.

Allowable properties for mechanically graded lumber still assume that $F_c = 0.8F_b$. In 1969, however, F_t values were linked to F_b values using a sliding scale (Table 1) (Galligan et al. 1979). The basis for this change was data suggesting that $F_t \cong 0.39F_b$ for 900f lumber and data confirming that $F_t = 0.8F_b$ for 2400f lumber.² A linear relationship was established to estimate the UCS/MOR ratio for grades between 900f and 2400f. Above 2400f, the historic F_t/F_b relationship was maintained.

In-Grade data

Data developed in the In-Grade Testing Programs in the United States and Canada provide a comprehensive data base for the evaluation of lumber property relationships. The sampling, testing, and analysis procedures used for the In-Grade data are discussed in the report by Green et al. (1989). The data collected in the United States are summarized by Green and Evans (1987), and the data

² Neil Pinson, retired Director of Technical Services, Western Wood Products Association, personal communication, 1990.

TABLE 2. *Species tested in the United States In-Grade Testing Program.*

Species group	Number of specimens			
	Bending	Tension	Compression	Total
Douglas fir-larch	6,067	2,817	2,618	11,502
Southern pine	4,944	4,068	2,719	11,731
Hem-fir	3,605	2,743	2,468	8,816
Douglas fir (South)	564	548	395	1,507
(Minor) southern pines	870	924	1,042	2,836
U.S. spruce-pine fir				
Engelmann spruce	471	—	—	471
Eastern spruce group	360	—	—	360
Lodgepole pine	439	—	—	439
Jack pine	240	—	—	240
Subalpine fir	524	—	—	524
Balsam fir	61	—	—	61
Mixed species				
Eastern hemlock	361	—	—	361
Tamarack	369	—	—	369
Sitka spruce	203	—	—	203
Red pine	358	—	—	358
Eastern white pine	362	—	—	362
Idaho white pine	240	—	—	240
Ponderosa pine	539	—	—	539
Sugar pine	299	—	—	299
Aspen-cottonwood	329	—	—	329
Yellow-poplar	365	100	—	465
(Total)	21,870	11,200	9,242	43,312

for Canadian spruce-pine-fir (SPF) are summarized by the Canadian Wood Council (1988).

Data on Douglas fir-larch, hem-fir, and southern pine dimension lumber were obtained in bending, and in tension and compression parallel to the grain using test procedures given in ASTM D4761 (ASTM 1989) (Table 2). All bending specimens were tested using a third-point load with a span to depth ratio of 17 to 1. Test spans for Douglas fir-larch, hem-fir, and southern pine tested in the United States and spruce-pine-fir tested in Canada are shown in Table 3. Data were generally limited to Select Structural and No. 2 grades for nominal widths of 4, 8, and 10 inches (100, 200, and 250 mm). Some data were obtained for Construction, Standard, and Utility grades of Douglas fir-larch and hem-fir 2 by 4s tested in bending.

Historical data on southern pine

In addition to the In-Grade data, data collected on southern pine dimension lumber were also used for our analysis (Table 4) (Doyle and Markwardt 1966, 1967). These historical data were obtained from lumber equilibrated to 12% moisture content. Testing was conducted at the USDA Forest Service, Forest Products Laboratory (FPL) using procedures identical to those specified in ASTM D198 (ASTM 1989).

TABLE 3. *Test spans for various species and lumber sizes in United States and Canada.*^a

Test mode	Country	Test span [in. (m)]		
		2 by 4	2 by 8	2 by 10
Bending	United States & Canada	60 (1.5)	123 (3.1)	156 (4.0)
Tension	United States & Canada	96 (2.4) ^b	144 (3.7)	144 (3.7)
Compression	United States	9 (0.2)	18 (0.5)	23 (0.6)
	Canada	96 (2.4)	144 (3.7)	168 (3.7)

^a United States: Douglas fir-larch, hem-fir, and southern pine. Canada: spruce-pine-fir.

^b Southern pine 2 by 4s tested at 144 in. (3.7 m); spruce-pine-fir 2 by 4s tested at 104 in. (2.6 m).

Volume adjustments

Development of recent draft design standards in Europe and the United States has focused on lumber adjusted to a nominal width of 7.9 in. (200 mm) (Glos and Fewell 1989). Analysis of lumber property relationships in our paper is based on either data "as tested" or data adjusted to a nominal 2 by 8 width³ using one of two procedures. The first procedure is a width adjustment developed in the In-Grade Program (Johnson et al. 1989). This In-Grade adjustment does not separate the effects of length and width.

$$P_2 = P_1(W_1/7.25)^N \quad (1)$$

where P_1 is the property measured at width W_1 , P_2 is the property adjusted to a nominal 2 by 8 width, and $N = 0.357$ for MOR or UTS and 0.133 for UCS. The second procedure for adjusting data is the procedure that is being discussed in a draft ASTM standard for deriving allowable properties from In-Grade data. In this procedure, MOR and UTS data are adjusted to a nominal 2 by 8 width and a length of 144 in. (3.7 m).

$$P_2 = P_1[(W_1/7.25)^{0.296}][(L_1/144)^{0.143}] \quad (2)$$

where P_1 is the property measured at width W_1 and length L_1 , and P_2 the property adjusted to a nominal 2 by 8 width at a length of 12 ft (3.7 m). The second procedure assumes Eq. (1) is applicable to UCS.

RELATIONSHIPS BETWEEN MODULUS OF ELASTICITY AND STRENGTH

This section presents information on the relationship of MOE to MOR and UTS. Information is not presented on the relationship between MOE and UCS. In the In-Grade Testing Program in the United States, MOE was not determined for lumber tested in compression. The reader is referred to FPL Research Paper 64 for MOE-UCS relationships for southern pine (Doyle and Markwardt 1966).

Modulus of rupture

Mean trends in the MOE-MOR relationship for In-Grade data at 12% moisture content vary little by species (Fig. 2). These trends are also virtually identical to

³ Lumber was tested on edge. Therefore the "width" of the piece is the "depth" of the member if expressed in common engineering terms.

TABLE 4. *Historical data on southern pine used in study.*^a

Lumber	Lumber	Number of samples tested		
		Bending	Compression	Tension
2 by 4	No. 1 KD	100	100	99
	No. 1 Dense KD	100	—	—
	No. 2 KD	100	100	99
	No. 2 Dense KD	99	—	—
	No. 3 mg KD	98	97	98
	Special KD	79	—	—
2 by 6	No. 2 KD	99	50	50
2 by 8	No. 1 KD	99	50	50
	No. 1 Dense KD	99	—	—
	No. 2 KD	99	48	50
	No. 2 Dense KD	100	—	—
	No. 3 mg KD	98	50	50
	Special KD	80	—	—
2 by 10	No. 2 KD	99	—	—

^a From Doyle and Markwardt (1966, 1967).

those obtained for southern pine using FPL-64 data (Doyle and Markwardt 1966). The slopes of the MOE-MOR relationship decrease with increasing moisture content, especially when comparing 15% and green moisture content values (Table 5).

Traditional property assignments for MSR lumber in the United States as well as property assignments in the draft Eurocode 5 stress class system are not based on mean trends. Rather, the property assignments are based on 5th percentile level MOR and mean MOE. In-Grade results for Select Structural and No. 2 grades compare favorably with traditional MSR assumptions and with relationships assumed in Eurocode 5 (Fig. 2).

Ultimate tensile stress

Mean trends in the relationship between MOE and UTS for the In-Grade data are shown in Fig. 3. Again, the slopes of the equations vary little between species (Table 5), and they parallel the slopes determined using FPL-84 data. There is little variation in the MOE-UTS relationship at various moisture content levels.

RELATIONSHIPS BETWEEN LUMBER STRENGTH PROPERTIES

A single piece of lumber cannot be broken in more than one failure mode. Although procedures are available for evaluating the degree of correlation between two or more strength properties (Evans et al. 1984), any comparison of the properties themselves must necessarily involve certain assumptions about the relationships between property distributions. In this section, an equal rank assumption is used as the basis for expressing lumber property relationships. Thus, for the percentile levels of 0.01, 0.05, 0.10, 0.25, 0.50, 0.75, 0.90, 0.95, and 0.99, property estimates are first obtained using ASTM D2915 nonparametric procedures (ASTM 1989). A ratio of one property to another is then determined by equating strength values at percentiles of equivalent rank. These property ratios are then plotted against the property used in the denominator of the property ratio.

TABLE 5. Relationships between modulus of elasticity and strength for adjusted In-Grade data.^{a,b}

Ultimate stress	Species group	Moisture content (percent)	A(MOE) + B		R ²
			A	B	
Bending	Southern pine	12	4.655	-0.699	0.522
		15	4.249	+0.012	0.521
		Green	3.397	+0.307	0.518
	Douglas fir-larch	12	4.764	-1.139	0.541
		15	4.341	-0.394	0.538
		Green	3.473	+0.030	0.541
	Hem-fir	12	4.744	-0.832	0.522
		15	4.299	-0.175	0.520
		Green	3.478	+0.123	0.525
Tensile	Southern pine	12	3.349	-1.493	0.442
		15	3.420	-1.258	0.442
		Green	3.285	-0.853	0.440
	Douglas fir-larch	12	2.820	-0.743	0.404
		15	2.878	-0.515	0.405
		Green	2.772	-0.232	0.406
	Hem-fir	12	3.295	-1.082	0.420
		15	3.363	-0.867	0.421
		Green	3.237	-0.547	0.423

^a Data adjusted to that from nominal 2 by 8 lumber using Equation (1).

^b The MOR values are calculated as 10^3 lb/in.^2 ($1 \times 10^3 \text{ lb/in.}^2 = 6.985 \text{ MPa}$) using MOE values input as 10^6 lb/in.^2 ($1 \times 10^6 \text{ lb/in.}^2 = 6.985 \text{ MPa}$).

Ratio of ultimate compression stress to modulus of rupture

Figure 4 shows the UCS/MOR ratio as a function of MOR for 2 by 4, 2 by 8, and 2 by 10 lumber as tested in the In-Grade Program and adjusted to a moisture content of 15% (Table 3). This relationship is a generalization of that reported by Curry and Fewell (1977) for 1st and 5th percentile values. Although slight differences may occur between species, for standards the UCS/MOR ratio can be assumed to be species independent. The UCS/MOR ratio also tends to be independent of grade, and it increases only slightly as lumber width increases.

For the purpose of standards, the ratio is affected by moisture content and volume adjustments. The ratio decreases as moisture content increases (Fig. 5). The ratio for 2 by 4s also increases slightly if the MOR data are adjusted to a 12-ft (3.7-m) length using Eq. (1) (Fig. 6). Because there is little change in test length, adjusting lumber length to 12 ft (3.7 m) has little effect on the ratio for 2 by 8s and 2 by 10s. The mean trend for the data shown in Fig. 4 for 2 by 8s is

$$\begin{aligned} \text{UCS/MOR} &= 1.745 - 0.3201(\text{MOR}) \\ &\quad + 0.0223(\text{MOR})^2 \quad [\text{MOR} < 7.2 \times 10^3 \text{ lb/in.}^2 (49.6 \text{ MPa})] \\ &= 0.596 \quad [\text{MOR} \geq 7.2 \times 10^3 \text{ lb/in.}^2 (49.6 \text{ MPa})] \quad (3) \end{aligned}$$

where MOR is entered in thousands of pounds per square inch (49.6 MPa). A conservative relationship could be established by fitting a curve to the data shown in Fig. 4 and lowering the curve to the lower edge of the data.

Figure 7 shows the UCS/MOR relationship for 2 by 4s at 12% moisture content. Data collected on southern pine visually graded lumber in the FPL-64 series (Doyle and Markwardt 1966) show the same UCS-MOR trend as the In-Grade

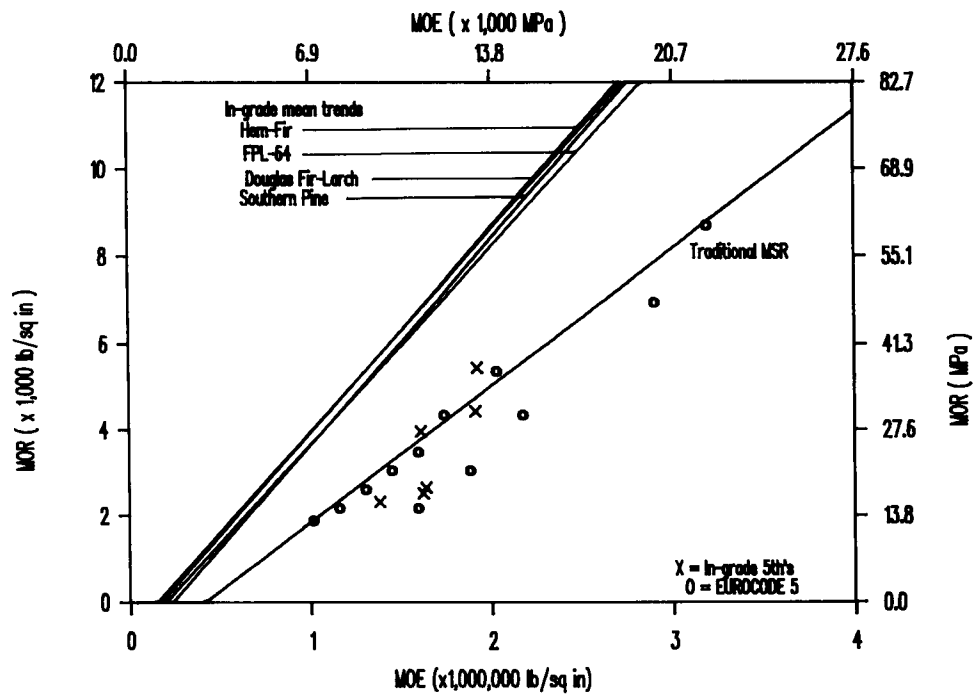


FIG. 2. Relationship of modulus of elasticity (MOE) to modulus of rupture (MOR) at 12% moisture content.

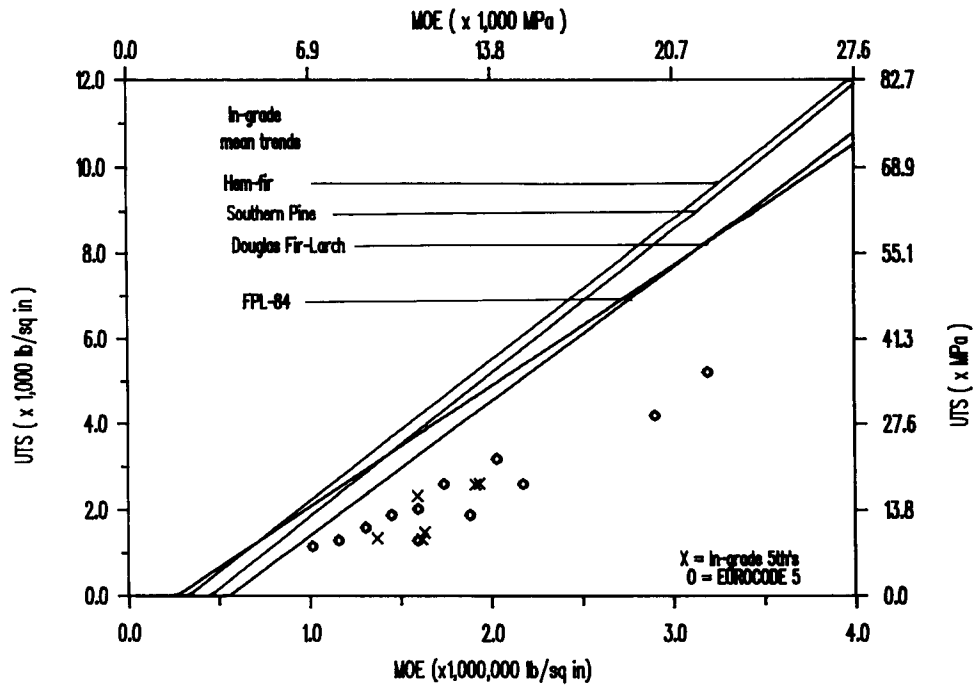


FIG. 3. Relationship of modulus of elasticity (MOE) to ultimate tensile stress (UTS) at 12% moisture content.

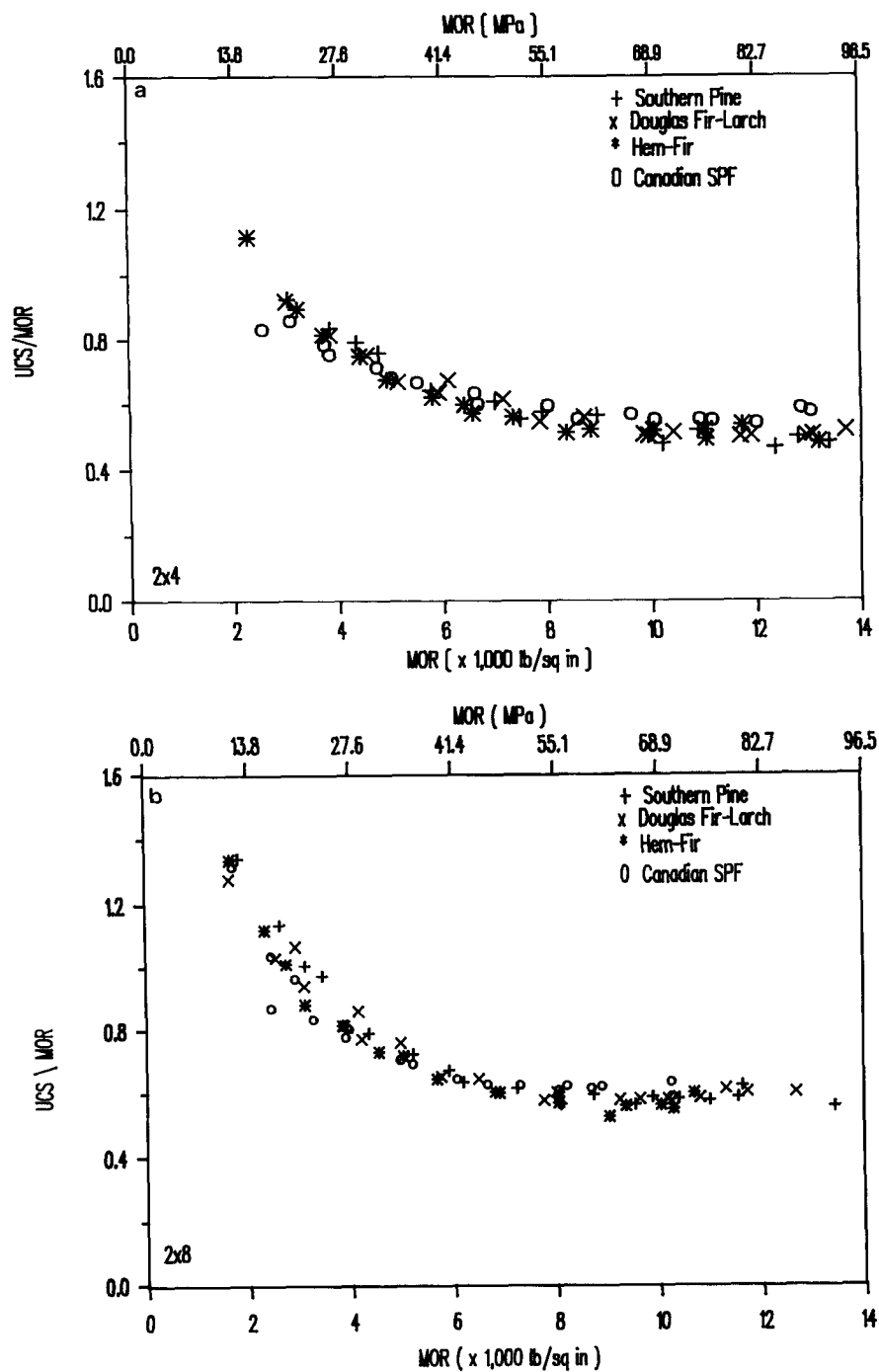


FIG. 4. Relationship of modulus of rupture (MOR) to ultimate compression stress (UCS) at 15% moisture content for lumber of varying width.

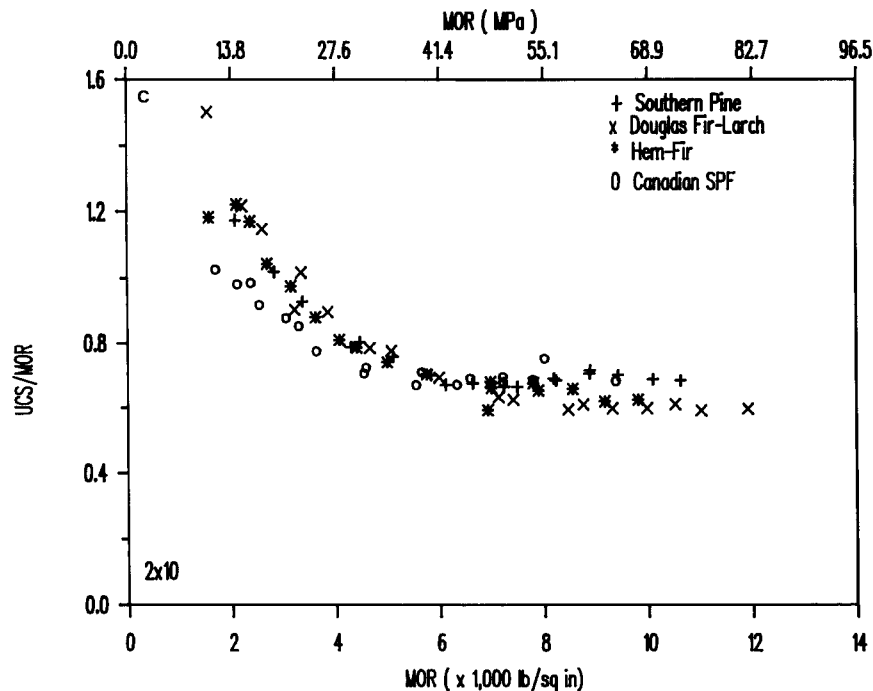


FIG. 4. Continued.

data. Mean clear wood values for both hardwoods and softwoods (USDA 1987) also follow this trend. Unpublished FPL data for 1650f-1.5E and 2100f-1.8E hem-fir 2 by 4s follow the trend of the In-Grade data. The In-Grade trend differs, however, from the linear relationships [$\text{UCS} = 0.72(\text{MOR})$ or $F_c = 0.80F_b$] currently assumed for MSR lumber.

The In-Grade data were also used to estimate the effect of juvenile wood on the UCS/MOR ratio. Exclusion of data pertaining to southern pine 2 by 4s containing pith and having six or fewer rings per inch produced UCS/MOR ratios virtually identical to those of lumber without the pith and having nine or more rings per inch (Fig. 8). Thus, the results presented in this section could be used to assign allowable compression strength values to lumber from plantation-grown trees.

Ratio of ultimate tensile stress to modulus of rupture

Figure 9 shows the UTS/MOR ratio for 2 by 4, 2 by 8, and 2 by 10 lumber as a function of MOR at 15% moisture content using lengths as tested in the In-Grade Program. In general, the property ratio tends to remain constant at lower MOR levels and then increases with increasing MOR. At the highest MOR levels, the ratio for 2 by 8s and 2 by 10s decreases with increasing MOR values. However, this drop at the highest MOR levels is probably due to slippage in the grips during tensile testing rather than a true indication of material behavior (Green and Evans 1987).

Allowable properties for solid sawn lumber would not be expected to exceed

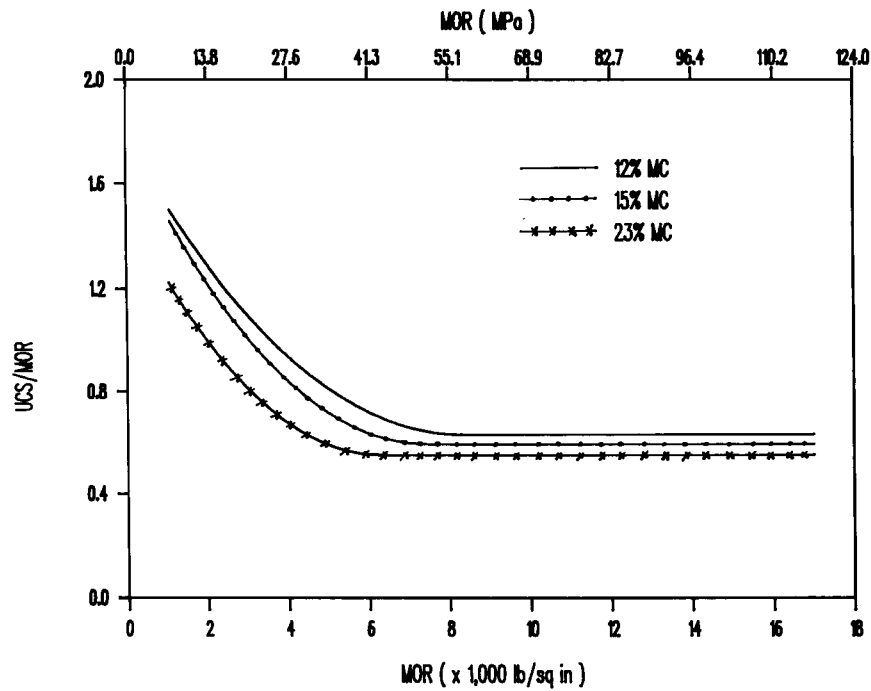


FIG. 5. Relationship of ultimate compression stress (UCS) to modulus of rupture (MOR) for 2 by 8 lumber at various moisture content (MC) levels.

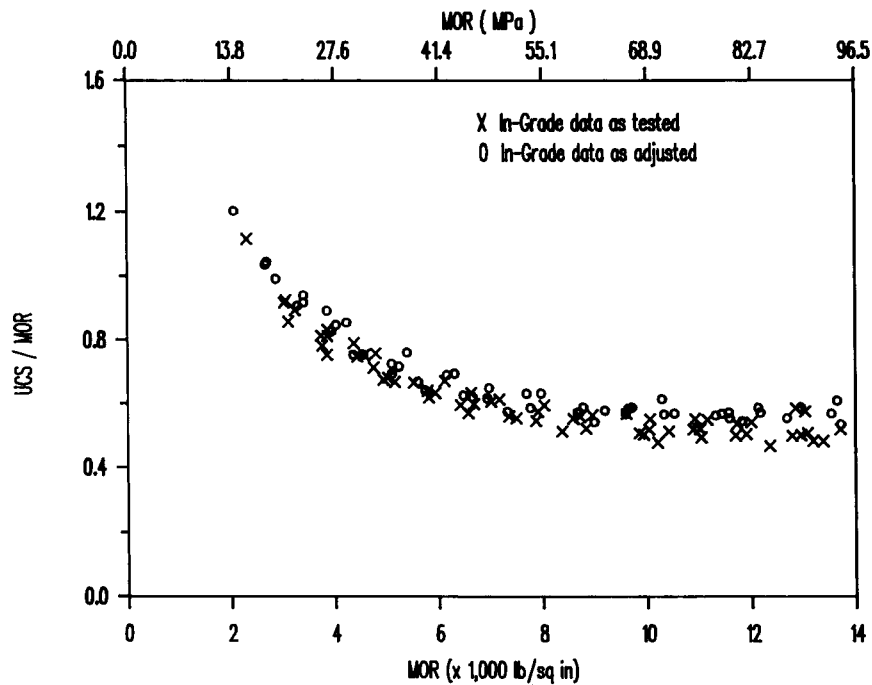


FIG. 6. Relationship of ultimate compression stress (UCS) to modulus of rupture (MOR) at 15% moisture content for 2 by 4 lumber adjusted to 12-ft (3.7-m) length.

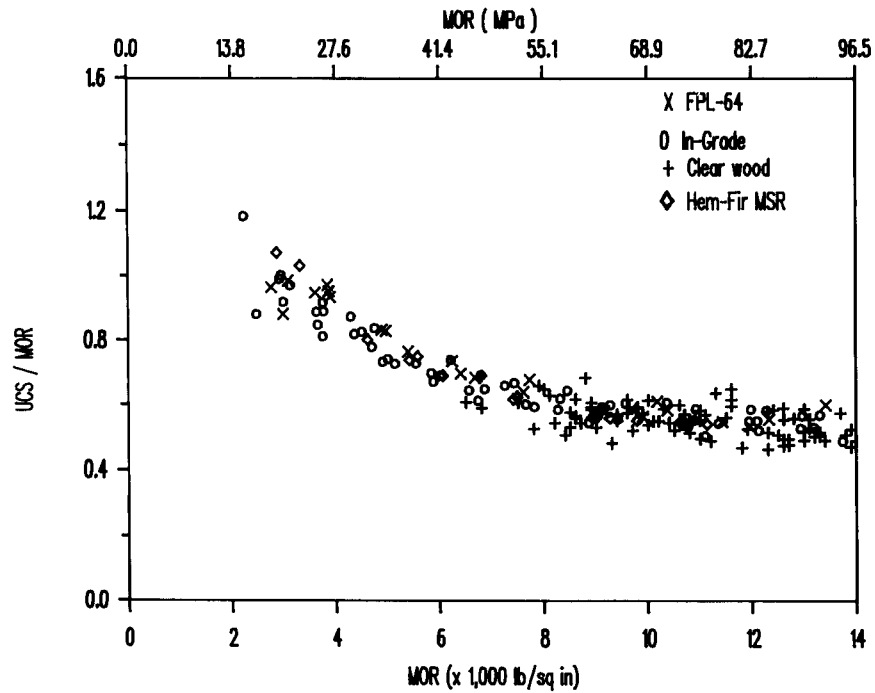


FIG. 7. Relationship of ultimate compression stress (UCS) to modulus of rupture (MOR) for 2 by 4 lumber at 12% moisture content.

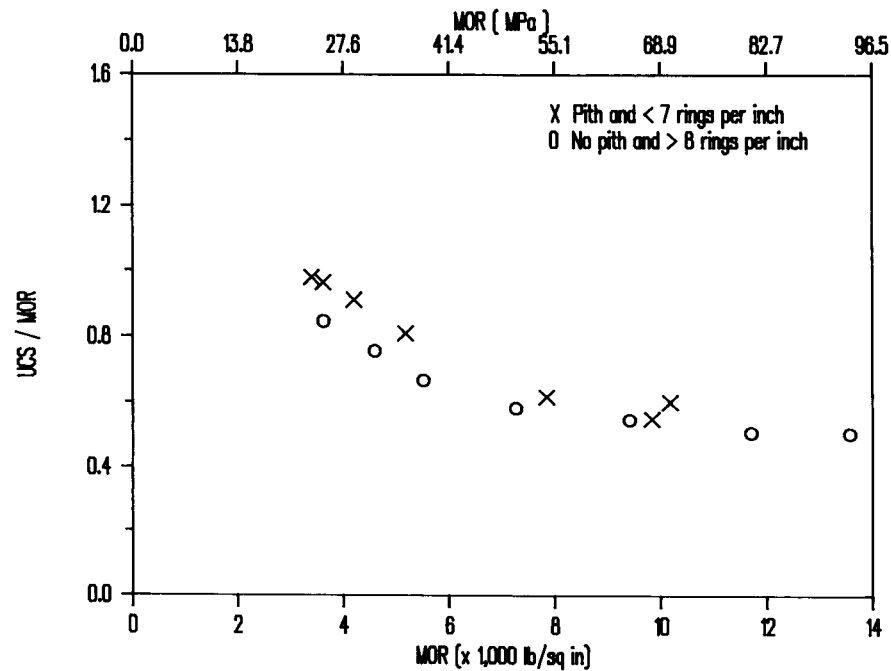


FIG. 8. Relationship of ultimate compression stress (UCS) to modulus of rupture (MOR) for southern pine 2 by 4s at 15% moisture content and sorted by growth rate and presence of pith.

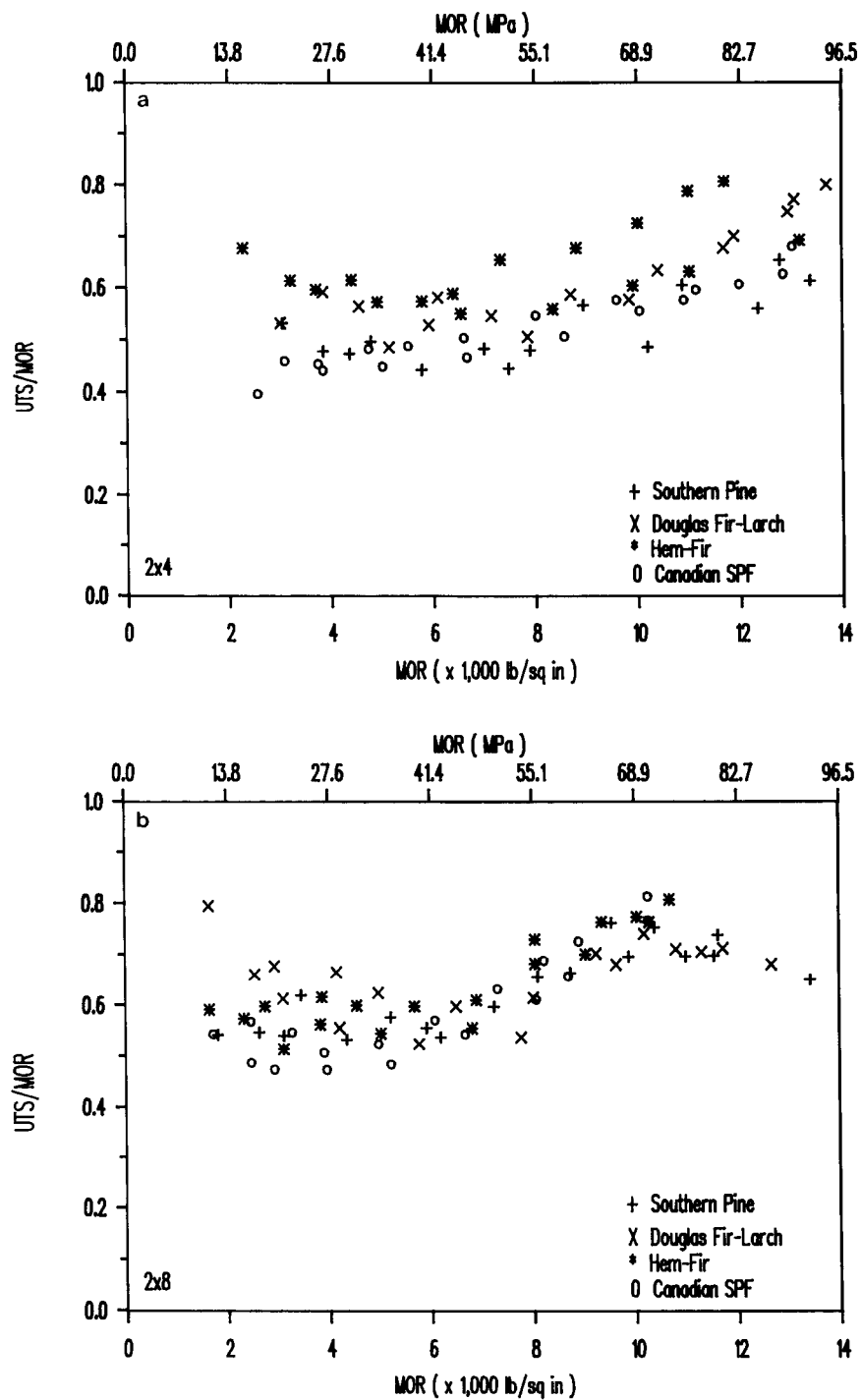


FIG. 9. Relationship of modulus of rupture (MOR) to ultimate tensile stress (UTS) at 15% moisture content for lumber of varying width.

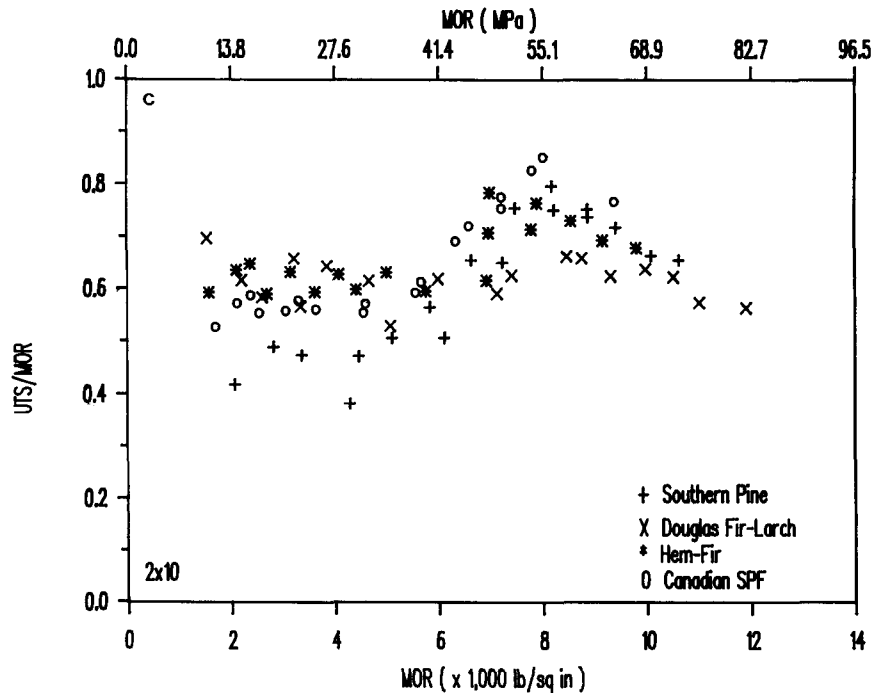


FIG. 9. Continued.

an MOR value of about 7×10^3 lb/in.² (48.3 MPa) (WWPA 1965). For MOR values below 7×10^3 lb/in.² (48.3 MPa), the average ratio of UTS/MOR is 0.56 averaged over all grades, species, and sizes (Table 6). This figure agrees well with the value of 0.60 determined by Curry and Fewell (1977) and assumed in Eurocode 5 (Glos and Fewell 1989).

The average UTS/MOR ratio varies with moisture content. For 2 by 8 lumber adjusted to 12% moisture content, the average ratio for MOR values below 7×10^3 lb/in.² (48.3 MPa) is 0.55, whereas the ratio at 15% is 0.56 and the ratio for green lumber (assumed 23% moisture content) is 0.74. At 12% moisture content, the In-Grade UTS/MOR ratio agrees with the ratio of 0.49 that can be derived from FPL-64 and FPL-84 values for southern pine (Doyle and Markwardt 1966, 1967).

If the MOR and UTS data are adjusted to a constant span of 12 ft (3.7 m) using Eq. (2), the UTS/MOR ratio changes very little because the test span is close to 12 ft (3.7 m) (Table 3). As with the UCS/MOR ratio, the data on southern pine 2 by 4s were sorted by rings per inch and presence or absence of pith. For lumber containing the pith and having six or more rings per inch, the average UTS/MOR ratio of all pieces having an MOR value less than 7×10^3 lb/in.² (48.3 MPa) was 0.44. This compares to an average of 0.50 for lumber without the pith and having nine or more rings per inch. Slight reductions in the UTS/MOR ratio were also noted with high-temperature-dried boxed-pith (*Pinus radiata*) lumber (Tsehay et al. 1989). In this study, the average ratio was found to be 0.58 compared to an expected value of 0.60.

TABLE 6. Variation in ratio of ultimate tensile stress to modulus of rupture for low MOR values.^a

Lumber size	Species group	Lumber grade	Average UTS/MOR ratio
All	All	All	0.56
2 by 4	All	All	0.52
2 by 6	All	All	0.53
2 by 8	All	All	0.57
2 by 10	All	All	0.59
All	All	S.S. ^b	0.58
All	All	No. 2	0.55
2 by 4	Southern pine	All	0.49
	Douglas fir-larch	All	0.55
	Hem-fir	All	0.60
	Canadian SPF	All	0.49
2 by 8	Southern pine	All	0.56
	Douglas fir-larch	All	0.64
	Hem-fir	All	0.58
	Canadian SPF	All	0.52
2 by 10	Southern pine	All	0.50
	Douglas fir-larch	All	0.61
	Hem-fir	All	0.64
	Canadian SPF	All	0.59

^a MOR values below 7×10^3 lb/in.² (48.3 MPa). Moisture content of lumber was 15 %.

^b Select Structural.

For MOR values below 7×10^3 lb/in.² (48.3 MPa), the UTS/MOR ratio tends to increase with lumber size and grade (Table 6). There also may be slight differences between species. However, for standards, the preferable course of action might be to assume a conservative value and to apply this value to all species. A value of 0.45 to 0.50 would seem appropriate.

Property ratios based on ultimate tensile stress

Currently, considerable interest has been shown in determining the UTS of MSR lumber in quality control programs. If allowable properties were set on the basis of UTS, it might be desirable to estimate the MOR/UTS ratio as a function of UTS. The MOR/UTS ratio for 2 by 4s is shown in Fig. 10. Likewise, the UCS/UTS may be estimated as a function of UTS (Fig. 11). The mean trend for the UCS/MOR ratio for 2 by 8 lumber is

$$\begin{aligned}
 \text{UCS/UTS} &= 2.724 - 0.678(\text{UTS}) \\
 &\quad + 0.0608(\text{UTS})^2 \quad [\text{UTS} < 5.6 \times 10^3 \text{ lb/in.}^2 (38.6 \text{ MPa})] \\
 &= 0.837 \quad [\text{UTS} \geq 5.6 \times 10^3 \text{ lb/in.}^2 (38.6 \text{ MPa})] \quad (4)
 \end{aligned}$$

where UTS is entered in thousands of pounds per square inch (38.6 MPa). Both these ratios would be expected to vary somewhat with such factors as species, lumber size, and lumber grade.

Fundamental considerations

The relationships between strength properties presented in his paper are a result of the influences of inherent growth characteristics on lumber properties. The UCS-MOR relationship can primarily be explained from the relationship between

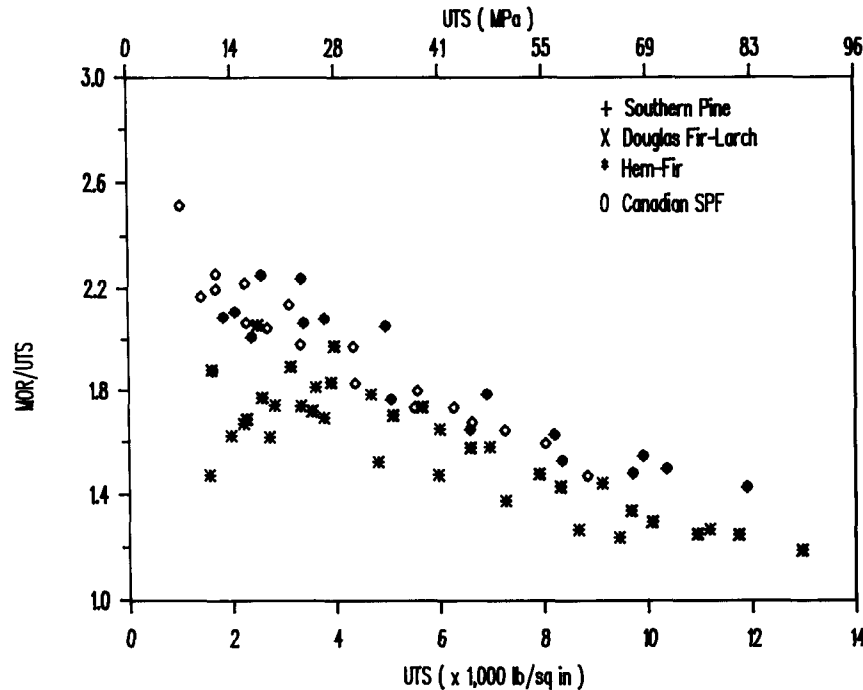


FIG. 10. Relationship between modulus of rupture (MOR) and ultimate tensile stress (UTS) as a function of UTS for 2 by 4s at 15% moisture content.

strength and MOE. Equations relating MOE and MOR and MOE and UCS for southern pine are given in FPL-64 (Doyle and Markwardt 1966). For all sizes and grades combined, these equations are

$$\text{MOR} = -338 + 4,800(\text{MOE}_f), R^2 = 0.429 \quad (5)$$

where MOE_f is the flatwise modulus of elasticity, and

$$\text{UCS} = 1,881 + 1,767(\text{MOE}_c), R^2 = 0.448 \quad (6)$$

where E_c is the modulus of elasticity in compression parallel to the grain. In Eqs. (5) and (6), MOE_f and MOE_c are entered in millions of pounds per square inch (38.6 MPa).

The UCS-MOR relationship predicted using Eqs. (5) and (6) (Fig. 12) is very similar in form to that given in Fig. 4. A UCS-MOR relationship of a similar form could have been predicted using the equations given in Hoyle (1968). However, this early work of Hoyle gave slightly higher UCS/MOR ratios than those presented in our paper. A relationship of the correct form can also be predicted from density.

A curve of the relationship of UTS to MOR as a function of MOR similar in form to that presented in Fig. 9 can be obtained using the strength-MOE relationships presented in Table 5 (Fig. 12). However, the predicted UTS/MOR ratio at lower MOR levels is smaller than that presented in Fig. 8. We conclude that more sophisticated predictive models involving combinations of characteristics would be needed to accurately predict the UTS-MOR relationship (Orosz 1968;

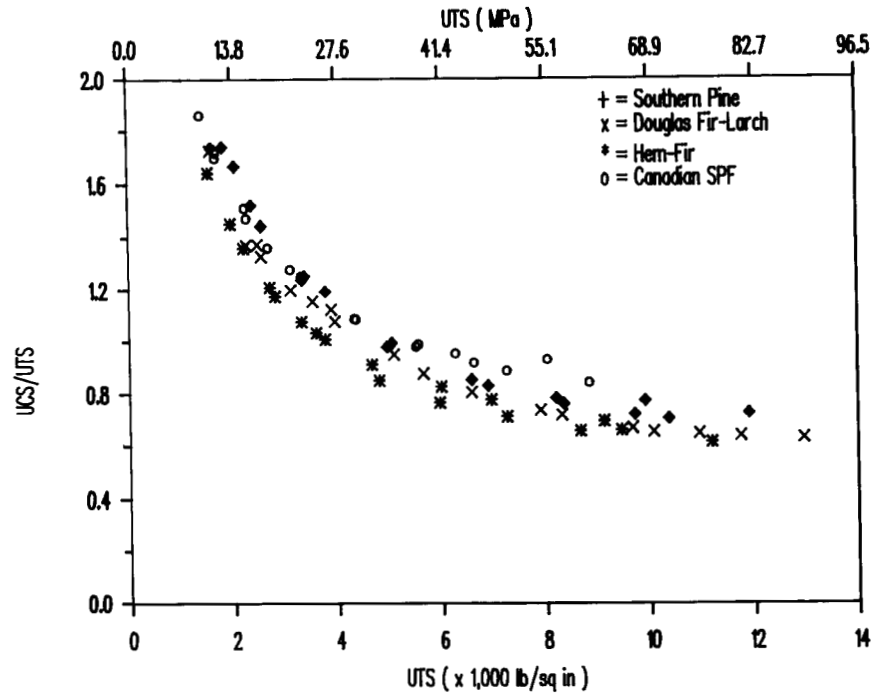


FIG. 11. Relationship between ultimate compression stress (UCS) and ultimate tensile stress (UTS) as a function of UTS for 2 by 4s at 15% moisture content.

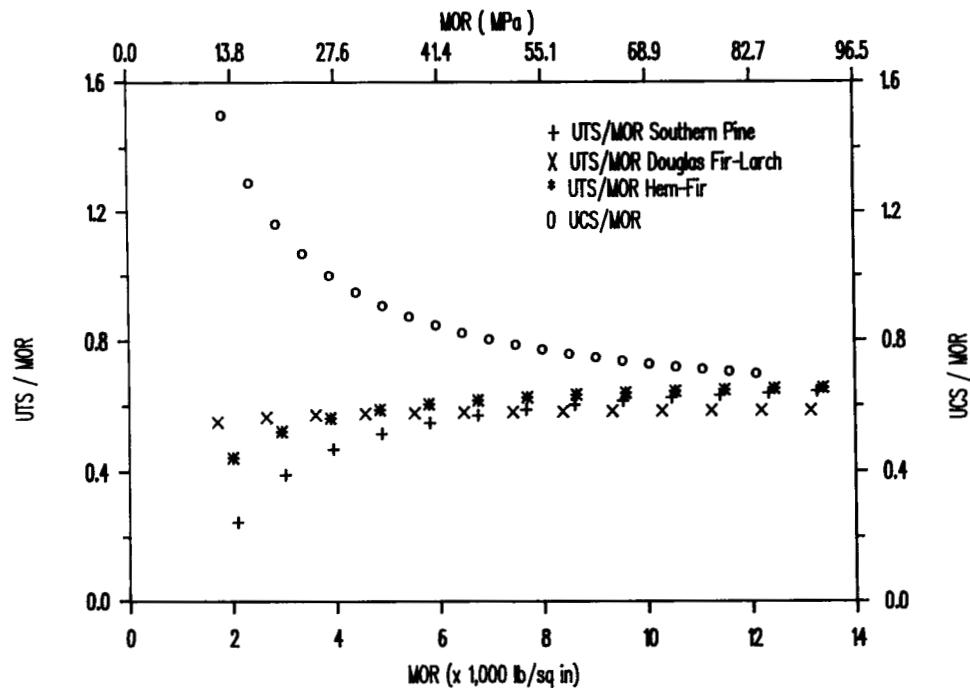


FIG. 12. Relationship of ultimate tensile stress (UTS) and ultimate compression stress (UCS) to modulus of rupture (MOR) predicted from modulus of elasticity. UCS/MOR relationship predicted from Eqs. (5) and (6). UTS/MOR relationships predicted from In-Grade data.

TABLE 7. Coefficient of variation of In-Grade data for Douglas fir-larch, hem-fir, and southern pine 2 by 4s at 12% moisture content.^{a,b}

Clear wood	Test mode	A(SR) + B		R ²
		A	B	
Not included	UTS	-85.02	+85.78	0.767
	MOR	-40.09	+55.84	0.746
	UCS	-26.97	+35.58	0.497
Included ^b	UTS	-59.15	+73.81	0.756
	MOR	-40.40	-55.40	0.829
	UCS	-17.41	+31.59	0.592

^a Assumed COV values for clear wood at a strength ratio of 1.0 are UTS = 25%, MOR = 16%, UCS = 18% (ASTM D2555 1989).

^b Strength ratio is expressed in decimal form (0 to 1.0); COV is predicted as a percentage.

^c No. 1 and No. 3 grades dropped.

Gerhards and Ethington 1974). Note that current industry practice utilizes both MOE and edge-knot size to set grade boundaries for MSR lumber.

ESTIMATION OF COEFFICIENT OF VARIATION

Development of reliability-based design procedures requires the estimation of the coefficient of variation (COV) as well as of the properties at some specified percentile level (Murphy 1988). The measured COV should be used whenever data are available. However, to reduce testing costs, lumber may be tested in one mode only. In this instance, it may be beneficial to evaluate COV trends with lumber quality using In-Grade data.

To evaluate variation in COV with lumber grade, COV estimates were obtained using an assumed normal distribution fit to the entire strength distribution. The values for Douglas fir-larch, hem-fir, and southern pine were then plotted against the assumed minimum bending strength ratio for the grade. The assumed minimum strength ratios for the In-Grade data used in this paper are as follows: Select Structural, 0.65; No. 1, 0.55; No. 2, 0.45; Construction, 0.34; Standard, 0.19; and Utility, 0.09.

For complete data sets, the relationship between the COV of MOR and assumed minimum strength ratio is shown in Fig. 13 for a moisture content of 12%. Similar relationships are also shown for COV-UTS and COV-UCS. Because these ratios show little variation with lumber width, the 2 by 4, 2 by 8, and 2 by 10 data are plotted on the same figure for Douglas fir-larch, southern pine, and hem-fir. The relationship of COV to strength is plotted with and without assumed clear wood COV values (ASTM 1989). For MOR, data are available for a wide range of grades, and there is no difference between the COV-strength curves with or without the clear wood data point. For UCS and UTS, however, data are only available for strength ratios between 0.45 and 0.55. In these instances, addition of COV values for clear wood at a strength ratio of 1.0 has a marked effect on the COV-strength relationship (Table 7). The COV would be expected to decrease slightly as moisture content is increased (McLain et al. 1984; Aplin et al. 1986; Green et al. 1990). The relationships presented in Table 7 might also be altered significantly if another distributional form were assumed or if the distribution were fit to only a portion of the data. Although we have obtained some reasonable results using an assumed two-parameter Weibull distribution and various degrees of data censoring, illogical results can also be obtained if data are available for only a limited

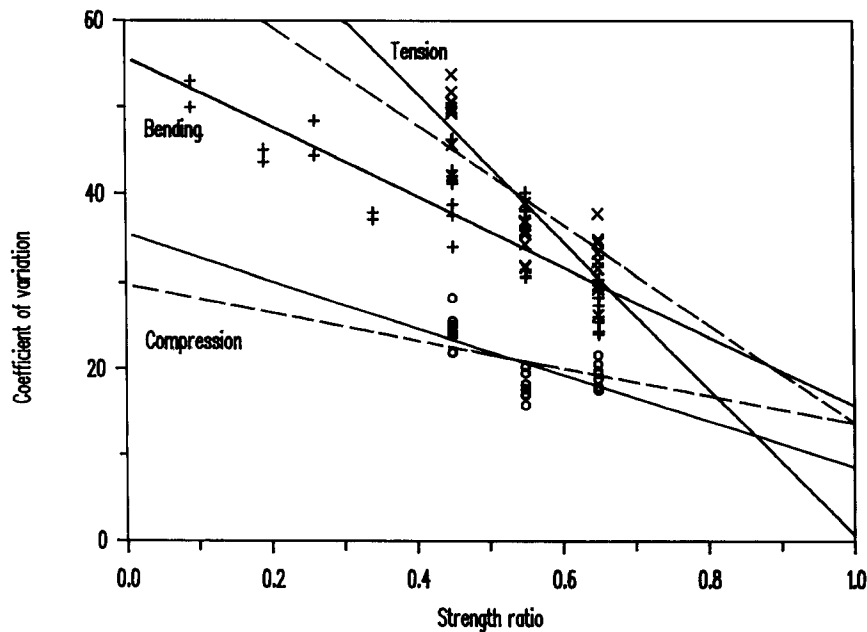


FIG. 13. Coefficient of variation of lumber strength with (-----) and without (—) clear wood data at 12% moisture content.

range of strength ratios. We suggest that generalizations such as those presented in Fig. 13 be limited to the more conservative estimates obtained using full data sets (Table 7).

CONCLUSIONS

1. The relationship between modulus of elasticity (MOE) and modulus of rupture (MOR) is similar for Douglas fir-larch, hem-fir, and southern pine dimension lumber.
2. The slope of the MOE-ultimate tensile stress (UTS) relationship varies little among these three species groups.
3. The MOE-UTS relationship varies little with moisture content. However, the slope of the MOE-MOR relationship decreases with increasing moisture content.
4. A good relationship exists between ultimate compression stress (UCS) and MOR and between UCS and UTS. These relationships vary slightly with moisture content, species, and lumber width.
5. The relationship between UTS and MOR is not nearly as good as that found between UCS and MOR. A conservative relationship may be obtained by assuming that one property is a constant percentage of the other property.
6. Presence or absence of pith has little effect on the relationships between UCS and MOR. Thus, the relationships summarized in this paper should also apply to lumber containing juvenile wood. A slight reduction in the UTS/MOR ratio would be anticipated with juvenile wood.
7. Conservative estimates of coefficient of variation may be obtained by expressing coefficient of variation as a function of assumed minimum strength ratio.

The results of this paper suggest the need to reconsider the lumber property relationships currently assumed for assigning allowable tensile and compression stresses to machine stress-rated lumber. A more fundamental understanding of mechanical properties is required to fully understand the relationships between UTS and MOR.

REFERENCES

- APLIN, E. N., D. W. GREEN, J. W. EVANS, AND J. D. BARRETT. 1986. The influence of moisture content on the flexural properties of Douglas-fir dimension lumber. Res. Pap. FPL 475. U.S. Department of Agriculture, Forest Service, Forest Products Laboratory, Madison, WI.
- ASTM. 1989. Designations D198-84 (p. 80), D245-88 (p. 99), D2555-88 (p. 368), D2915-88 (p. 404), and D4761-88 (p. 507). Annual Book of ASTM Standards, Wood, vol. 04.09. American Society for Testing and Materials, Philadelphia, PA.
- BARRETT, J. D., AND H. GRIFFIN. 1989. Size effect and property relationship for Canadian 2-inch dimension lumber. Proceedings of CIB W18A, Berlin, German Democratic Republic.
- CANADIAN WOOD COUNCIL. 1988. Mechanical properties of visually graded lumber: A summary. Ottawa, Ontario. 38 pp.
- CURRY, W. T., AND A. R. FEWELL. 1977. The relations between the ultimate tension and ultimate compression strength of timber and its modulus of elasticity. Building Research Establishment, Current Paper CP 22/77. Princes Risborough Laboratory, Aylesbury, Bucks, England.
- DOYLE, D. V., AND L. J. MARKWARDT. 1966. Properties of southern pine in relation to strength grading of dimension lumber. Res. Pap. FPL 64. U.S. Department of Agriculture, Forest Service, Forest Products Laboratory, Madison, WI.
- , AND ———. 1967. Tension parallel-to-grain properties of southern pine dimension lumber. Res. Pap. FPL 84. U.S. Department of Agriculture, Forest Service, Forest Products Laboratory, Madison, WI.
- EVANS, J. W., R. E. JOHNSON, AND D. W. GREEN. 1984. Estimating the correlation between variables under destructive testing: Or how to break the same board twice. *Technometrics* 26(3):278–297.
- FEWELL, A. R. 1989. Stress classes for structural lumber. Fourth draft of CEN standard xxx2. Document CEN/TC/WG 2 N66E. British Standards Institute.
- GALLIGAN, W. L., C. C. GERHARDS, AND R. L. ETHINGTON. 1979. Evolution of tensile design stresses for lumber. Gen. Tech. Rep. FPL 28. U.S. Department of Agriculture, Forest Service, Forest Products Laboratory, Madison, WI.
- GERHARDS, C. C., AND R. L. ETHINGTON. 1974. Evaluation of models for predicting tensile strength of 2- by 4-inch lumber. *Forest Prod. J.* 24(12):46–54.
- GLOS, P., AND A. R. FEWELL. 1989. The determination of characteristic values of mechanical properties and density of timber. Fourth draft of CEN standard xxx1. Document CEN/TC/WG 2 N66E. British Standards Institute.
- GREEN, D. W., AND J. W. EVANS. 1987. Mechanical properties of visually graded lumber. Volume 1: A summary. Publication PB-88-159-389. National Technical Information Service, U.S. Department of Agriculture, Springfield, VA.
- , AND ———. 1988. Evaluating lumber properties: Practical concerns and theoretical restraints. Proceedings of the 1988 International Conference on Timber Engineering. Pages 203–217 in R. Y. Itani, ed. Weston Hotel, Seattle, WA, September 19–22.
- , AND D. E. KRETSCHMANN. 1989. A discussion of lumber property relationships in Eurocode 5. Proceedings of CIB W18A, Berlin, German Democratic Republic.
- , AND ———. 1990. Stress class systems: An idea whose time has come. Res. Pap. FPL 500. U.S. Department of Agriculture, Forest Service, Forest Products Laboratory, Madison, WI.
- , B. E. SHELLEY, AND H. P. VOKEY, eds. 1989. Proceedings: In-grade testing of structural lumber. Forest Products Laboratory, April 1988, Proceedings 47363. Forest Products Research Society, Madison, WI.
- , R. F. PELLERIN, J. W. EVANS, AND D. E. KRETSCHMANN. 1990. Moisture content and the tensile strength of Douglas-fir dimension lumber. Res. Pap. FPL 497. U.S. Department of Agriculture, Forest Service, Forest Products Laboratory, Madison, WI.
- HOYLE, R. J. 1968. Background to machine stress grading. *Forest Prod. J.* 18(4):87–98.
- JOHNSON, L. A., J. W. EVANS, AND D. W. GREEN. 1989. Volume effect adjustments of the in-grade

- data. D. W. Green, B. E. Shelley and H. P. Vokey, eds. Pages 56–67 in *Proceedings: In-grade testing of structural lumber*. Forest Products Laboratory, April, 1988, Proceedings 47363. Forest Products Research Society, Madison, WI.
- McKEAN, H. B., AND R. J. HOYLE. 1964. Stress-grading method for dimension lumber. ASTM Special Technical Publication 353. American Society for Testing and Materials, Philadelphia, PA.
- McLAIN, T. E., A. L. DE BONIS, D. W. GREEN, F. J. WILSON, AND C. L. LINK. 1984. The influence of moisture content on the flexural properties of southern pine dimension lumber. Res. Pap. FPL 447. U.S. Department of Agriculture, Forest Service, Forest Products Laboratory, Madison, WI.
- MURPHY, J. F. CHAIR. 1988. Load and resistance factor design for engineered wood construction. ASCE prestandard report. March. American Society of Civil Engineers, New York, NY.
- OROSZ, I. 1968. Some nondestructive parameters for prediction of strength of structural lumber. Res. Pap. FPL 100. U.S. Department of Agriculture, Forest Service, Forest Products Laboratory, Madison, WI.
- TSEHAYE, A., J. C. F. WALKER, AND A. H. BUCHANAN. 1989. In-grade testing of juvenile wood. Pages 87–91 in *Proceedings of the Second Pacific Timber Engineering Conference*. University of Auckland, New Zealand. August 28–31, Centre for Continuing Education, U. of Auckland, vol. I.
- USDA. 1987. Wood handbook: Wood as an engineering material. Agric. Handb. 72. U.S. Department of Agriculture, Forest Service, Forest Products Laboratory, Madison, WI.
- WWPA. 1981. Standard lumber grading rules. Western Wood Products Association, Portland, OR.