SOME ASPECTS OF NON-DARCY BEHAVIOR OF GAS FLOW IN WOOD[1]

A. T. Wiley[2] and *E. T. Choong*

School of Forestry and Wildlife Management, Louisiana State University, Baton Rouge, LA 70803

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ABSTRACT

A study was undertaken to determine if the Klinkenberg equation for gas flow through porous media could be applied to a wide variety of wood species. When the results proved negative, a second study was initiated to determine if the cause of the nonconformities could be traced to turbulence. The combined experiments indicated that neither molecular slippage, as described by the Klinkenberg equation, nor turbulence can adequately explain the reduction in apparent permeability with increasing mean pressure.

Additional keywords: Hardwoods, softwoods, gas permeability, turbulent flow, molecular flow.

INTRODUCTION

A large number of studies on the permeability of wood to fluid have been made using Darcy's law. However, Darcy did his work for flow through sand, a porous medium that would be expected to have a different pore structure than does wood; therefore deviations from Darcy's law are expected in wood. In the flow of gases, equations have been proposed by Adzumi and Klinkenberg, and used in wood (Pfalzner 1950; Resch and Ecklund 1964; Comstock 1968; Sebastian et al. 1965; Siau 1971) to account for molecular slippage because of decreasing apparent permeability as mean pressure increases.

This study was originally undertaken to determine if the Klinkenberg equation could be applied to a wide variety of wood species. When the results showed considerable deviations, another study was undertaken to determine if those deviations could be attributed to nonlaminar or turbulent flow. Since more is known about the flow of fluids through rock, it was deemed relevant to compare its gaseous flow behavior with that in wood.

THEORY

Darcy's Law for gases expressed the apparent permeability K_a as follows:

$$K_a = \frac{Q_a \mu L P_a}{A\overline{P}(\Delta P)}$$

Darcy, [cm^3(fluid)-cp/sec-atm-cm(wood)] (1)

where Q_a (cm^3/sec) is the flow rate at atmospheric pressure, P_a = 1 atm, ΔP (atm) is the pressure drop ($P_1 - P_2$) between upstream pressure P_1 and downstream pressure P_2, $\overline{P}$ is the mean pressure $(P_1 + P_2)/2$, μ (cp) is the viscosity of the permeating gas, A (cm^2) is the cross-sectional area of the sample, and L (cm) is the sample length.

Klinkenberg (1941) derived the following expression for materials involving molecular flow:

$$K_a = K_s(1 + b/\overline{P}) \qquad (2)$$

where K_s is the true permeability, b is a material constant equal to $(4c\lambda\overline{P})/r$, λ is the mean-free-path of gas molecules, c is a constant assumed equal to 1.0, and r is the radius of the capillary.

Equation 2 indicates the following flow characteristics, which Klinkenberg verified with rock and glass filters:

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[2] Present address is Texas Forest Products Laboratory, Lufkin, TX 75901.

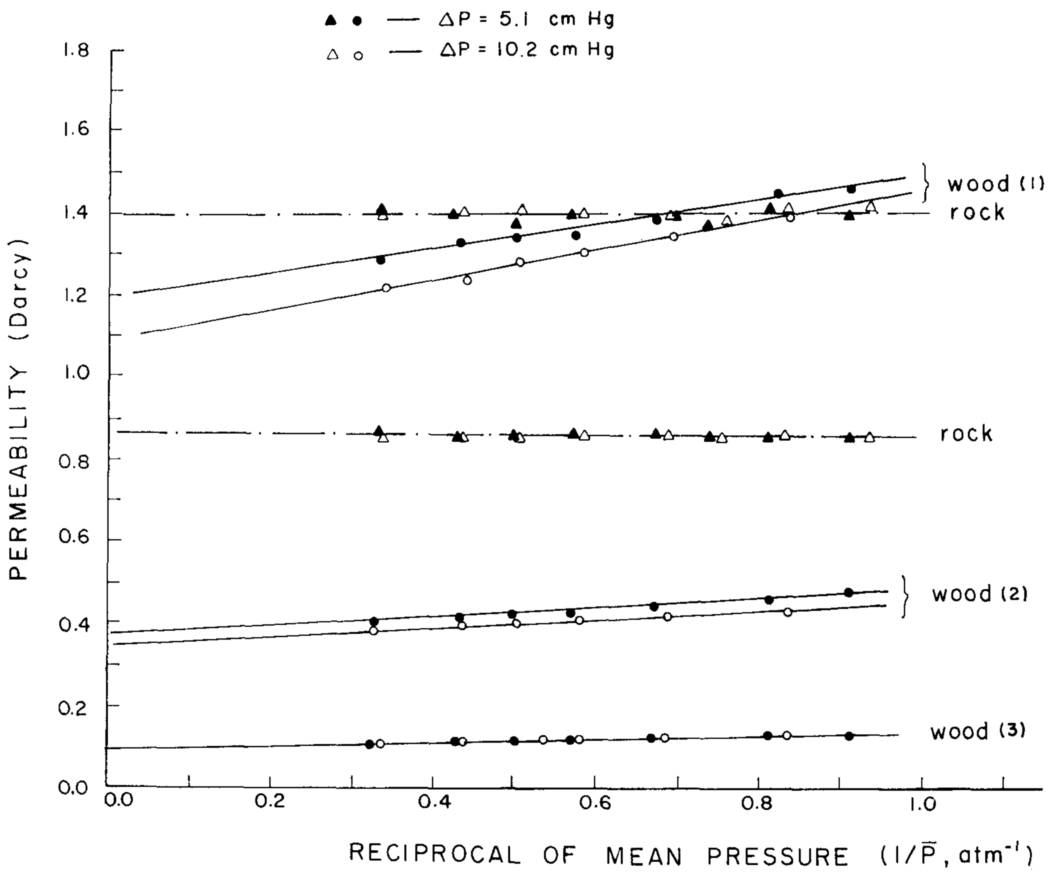

FIG. 1. Typical plot of nitrogen gas permeability as a function of reciprocal mean pressure for several wood and rock samples. [(1) white elm, (2) white pine, (3) fir.]

1. The apparent gas permeability is a linear function of the reciprocal of mean pressure. At high pressure, the permeability becomes a character of the material and not of the permeating fluid.
2. The apparent gas permeability is not dependent on the pressure difference.
3. The constant b is expected to be small for high permeability samples, since it is inversely proportional to the radius of the capillary.
4. The b-factor is directly proportional to the mean-free-path of the permeating gas molecules.

Both Eqs. 1 and 2 apply only when flow is laminar, i.e. in smooth streamlines. Turbulence is nonlaminar flow consisting of randomly occurring eddies that dissipate fluid motion as heat and increase the resistence to flow (Carman 1956).

The Reynolds number (R_e) is a dimensionless value given by:

$$R_e = \frac{2drv}{\mu} \qquad (3)$$

where d is the density and v the scalar velocity of the fluid. The Reynolds number is an expression of dynamic similitude, which is to say that flow systems having the same number will appear roughly the same. For a straight circular tube, the transition to full turbulence is abrupt and occurs at a Reynolds number of approximately 2200 (Scheidegger 1957). In curved tubes, however, the transition to turbulence is gradual, beginning as a regular circulatory motion, perpendicular to the axis of the tube. During the transition period, a less than proportional increase in flow with pressure drop occurs because of increasing eddy motion. Porous media, having tortuous flow paths, would be expected to behave more as curved tubes than as straight ones (Carman 1956). In wood, there are many flow channels that are not straight, particularly in the softwoods; therefore the Reynolds[3] number at which nonlaminar flow ensues can be expected to be low, and if the permeating fluid is compressible, a less than proportional increase in flow with mean pressure should be observed because of the changing density factor in Eq. 3.

EXPERIMENTAL PROCEDURE

Experiment I

In order to test the validity of the Klinkenberg equation, several species of wood, with two replications per species, were selected as follows: white elm[4], soft maple, red oak, sycamore, hickory, sweetgum, white pine, fir, Douglas-fir, and baldcypress. In addition, four sandstone rock samples were also used. Each of these samples was subjected to pressure drops of 5.1 and 10.2 cm Hg, respectively, at an upstream pressure ranging from 10 to 155 cm Hg above atmospheric. Nitrogen gas was used on all samples, and helium gas on eight wood samples and two rock samples. From these measurements, linear regression analyses were made to determine the slope of the relationship between K_a and $1/\overline{P}$ for the two pressure drops and fluid media.

Experiment II

The results of the above experiment showed almost no conformity to characteristics 2, 3, and 4 of the Klinkenberg equation; therefore, another experiment was undertaken to determine if the deviations could be traced to nonlaminar flow. The following wood species were used, also with two replications per species: honeylocust, soft maple, hard maple, Douglas-fir, hemlock, and southern pine, as well as the four aforementioned rock samples. Since molecular slippage is a function of the mean pressure, the mean pressure was maintained constant at 40 cm Hg above atmospheric, and only the pressure drop was varied from 0.147 to between 3.6 and 70 cm Hg. The upper limit of the pressure drops was dependent upon the specimen permeability, since large pressure drops could not be maintained across highly permeable specimens at the specified mean pressures. A water manometer was used when the pressure drop was below 4 cm Hg. The apparent permeability was plotted against pressure drop for each specimen. Since the flow appeared to be laminar at 0.37 cm Hg pressure drop for all specimens, Klinkenberg extrapolations (to infinite pressure) were carried out at this pressure drop as in Experiment I using both helium and nitrogen. Extrapolations were also conducted at 5 cm Hg pressure drop for comparison.

Apparatus

The apparatus used for permeability measurement was similar to the one described by Choong et al. (1974). The upstream pressure was adjusted by means of a pressure regulator, and the downstream pressure by a metering valve. Gas was prevented from flowing around the specimen by lining the holder with a pressurized rubber sleeve, which held the sample tightly so that no flow could occur around the edges. The rate of gas flow was measured by determining the time for a film of soap inside a buret to travel between two marks, delineating a known volume. Different burets were used, so that the measured volume ranged from 1 to 1000 cc. The size of the test sample was ⅞-inch in diameter and about ¾-inch

[3] The Reynolds number presented here is to show, in a general way, how different flow parameters could affect the degree of turbulence of a flow system.

[4] Botanical scientific names are omitted here since the woods were selected for convenience to provide a wide range of permeability values and not for species representation.

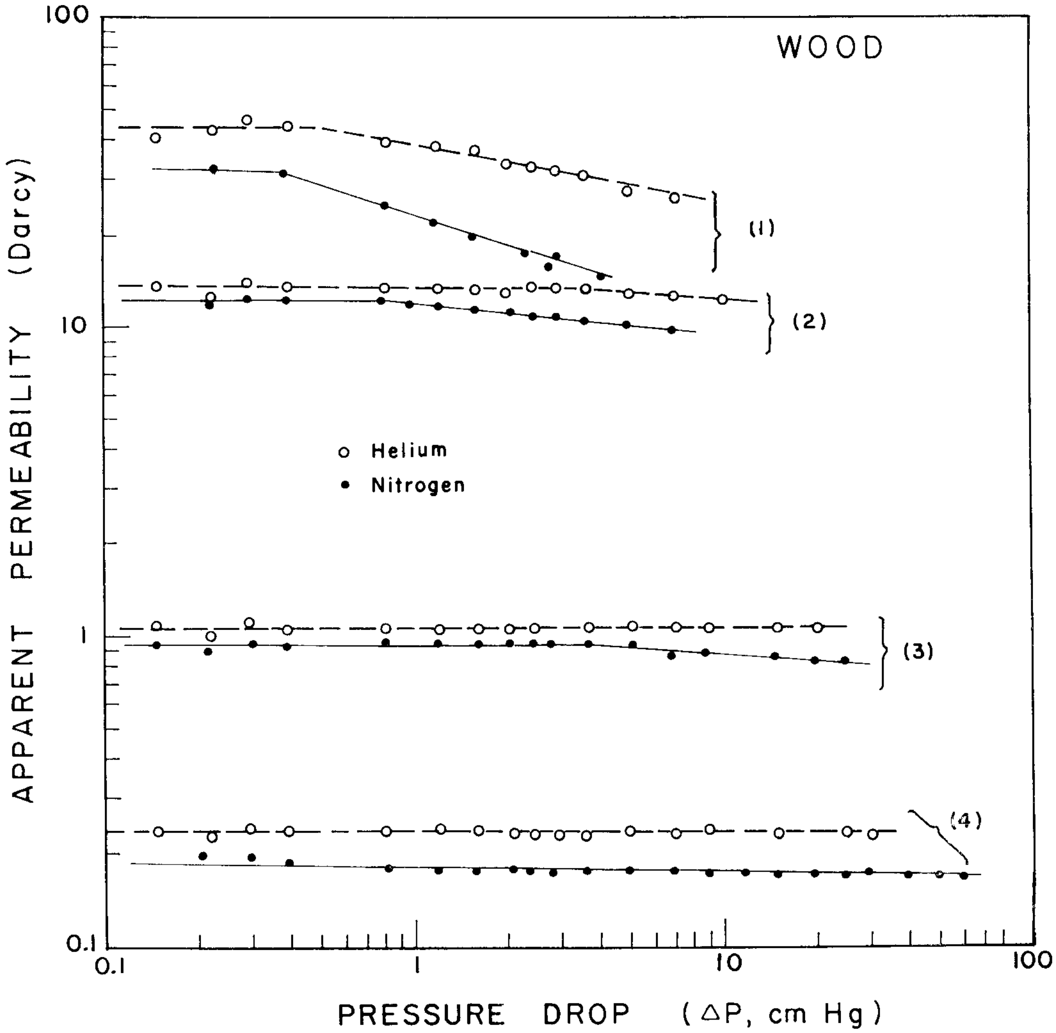

FIG. 2. Typical plot of the logarithm of apparent gas permeability as a function of the logarithm of pressure drop for several wood samples using helium and nitrogen. [(1) honeylocust, (2) soft maple, (3) hard maple, (4) hemlock.]

in length. Flow was in the longitudinal direction. All the samples were stored inside a small desiccator containing P_2O_5; thus their moisture contents were near zero percent. The fluids used were nitrogen ($\lambda = 9.29 \times 10^{-6}$ cm and $\mu = 1.78 \times 10^{-4}$ poise) and helium ($\lambda = 27.45 \times 10^{-6}$ cm, and $\mu = 1.94 \times 10^{-4}$ poise).

RESULTS

Typical plots of K_a vs. $1/\overline{P}$ obtained from Experiment I with nitrogen are shown in Fig. 1 for both rock and wood samples. In rock, the slope is horizontal, showing no apparent effect of pressure drop over the range studied. On the other hand, in wood the plots generally are lower and with somewhat steeper slopes at 10 cm than at 5 cm Hg pressure drop. The effect of pressure drop became more pronounced in the higher permeability range. For woods with permeability less than 0.2 Darcy, the effect of pressure drop was negligible.

The results of Experiment II are repre-

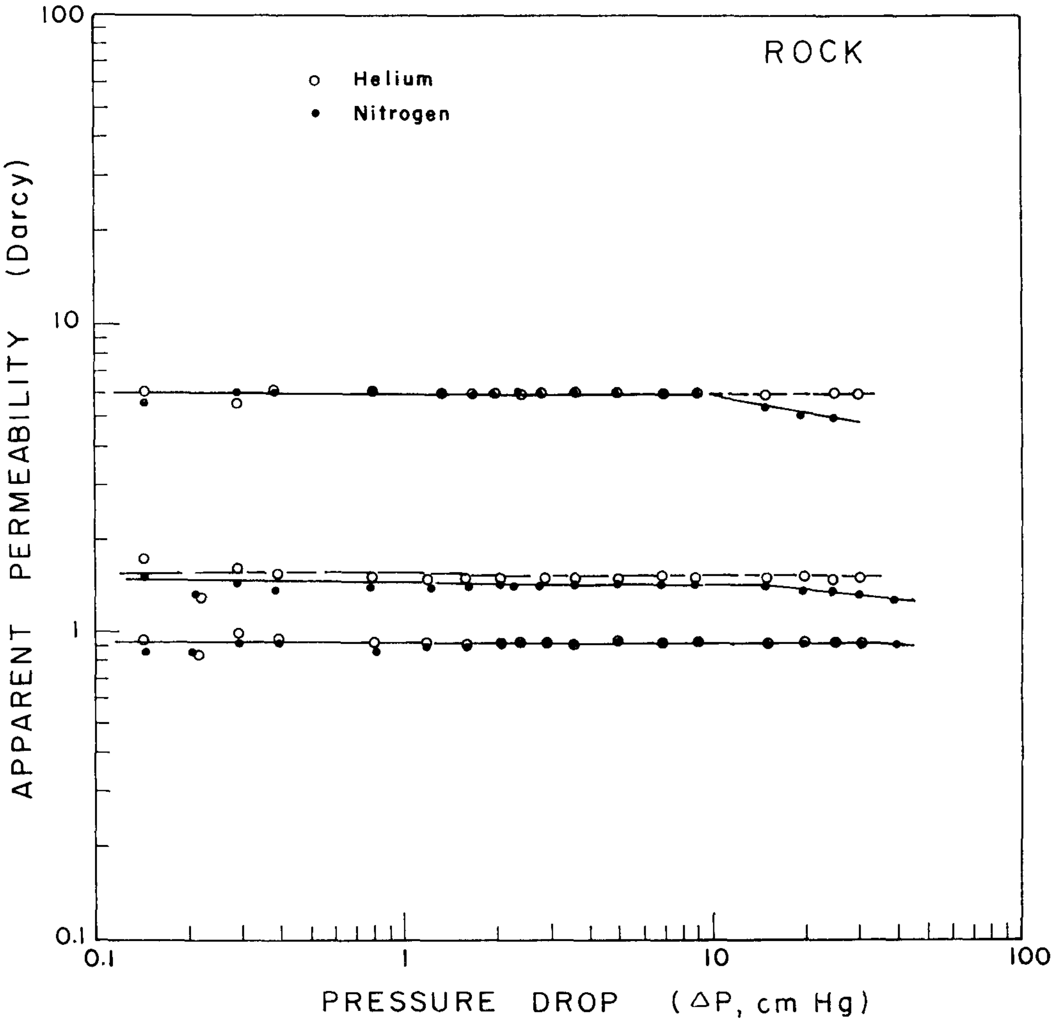

FIG. 3. Typical plot of the logarithm of apparent gas permeability as a function of the logarithm of pressure drop for three rock samples using helium and nitrogen.

sented by the relationship between permeability and pressure drop. Figure 2 shows typical plots of log K_a vs. log ΔP for representative wood samples with nitrogen and helium, whereas Fig. 3 shows similar plots for the rock samples. In most cases, the curves appeared horizontal up to some critical value of ΔP, then began to drop sharply with an increase in P due, presumably, to turbulence. This critical point is a function of permeability, type of porous media, and fluid medium. For the same comparative permeability value, the critical point occurred at a much higher ΔP value in rock than in wood; but apparently no turbulence (i.e. no critical point within the range pf pressures used) can be assumed to occur at permeabilities below 1 Darcy for rock and 0.2 Darcy for wood since the plot remained horizontal throughout the entire measured range of ΔP. When the permeating fluid was helium, the rock samples exhibited no critical point, while the wood samples show critical points only at permeabilities higher than about 1 Darcy.

TABLE 1. *Values for various ratios of b-factors in wood*

Ratio	Expected Value	Observed Value	Reference
b_{Oxygen}/b_{Air}	1.073	1.083	Pfalzner (1950)
b_{Oxygen}/b_{Air}	1.075	1.171	Resch and Ecklund (1964)
$b_{Helium}/b_{Nitrogen}$	2.950	1.51-2.35	Comstock (1968)
$b_{Helium}/b_{Nitrogen}$	2.950	0.00-2.32	Present study

DISCUSSION OF RESULTS

The more pronounced effect of pressure drop on apparent permeability of wood, as compared with rock may be traced to the structure of the porous media. It has been shown by Meyer (1971) that completely blocked flow paths within short distances of each other can be expected to take place in wood. Where paths are not completely blocked, only one bordered pit between two tracheids could be expected to remain open. This tortuosity and restriction to only one or a very few small flow channels per parallel path greatly restrict flow through wood and probably account for the deviations from permeability as predicted by the equations. Also, in view of the lack of similarity in flow paths between softwoods and hardwoods, other factors contributing to this effect should be considered. For instance, in capillary flow, a certain amount of inertial resistance due to velocity changes on entrance and exit of fluid from the capillary is created (Carman 1956). The amount of inertial resistance increases not only with a reduction in capillary length, but also with an increase in fluid density and velocity, and therefore could result in decreasing apparent permeability with increasing pressure drop and mean pressure. In softwoods, where most of the resistance to flow takes place in very short capillaries, namely the pits, inertial resistance may be pronounced. In hardwoods, particularly ring-porous, a large variation in pore sizes results in a high percent of flow volume taking place through a small fraction of the specimen cross section. In rock, the pore size is more homogeneous.

Figures 2 and 3 show that the permeability to helium was less subject to the effect of pressure drop than the permeability to nitrogen. The reason is due to the density factor of Eq. 3, for nitrogen is seven times more dense than helium. The b-factor for nitrogen (b_{N_2}), which was obtained from Eq. 2, was generally below 0.4 for permeabilities ranging from 0.1 to 10 Darcies. Only two samples show a high b-factor (i.e. over 1.0). These samples have low permeability values of about 0.1 Darcy. Contrary to the prediction of the Klinkenberg equation, there was no relationship between K_s and b_{N_2}, but this may have been due to large sample variation since permeability is a function of both pore size and pore density.

The mean-free-path of a helium molecule is 2.95 times greater than that of nitrogen (West 1971); thus, if the Klinkenberg equation is applicable to wood and its slope bK_s is wholly due to molecular slippage, then the ratio b_{He}/b_{N_2} should be equal to 2.95. On the other hand, if a significant portion of the slope is due to molecular slippage and the rest to nonlaminar flow, the ratio should tend toward 2.95 with decreasing permeability as well as pressure drop. Analyses of the results show no correlation between this ratio and permeability. The ratio was less than 2.95 in all cases. Also, there was no tendency for the ratio to decrease with an increase in pressure drop. The extrapolations carried out at 0.37 cm Hg pressure drop indicated less tendency to conform to the Klinkenberg equation than at any other pressure drop used.

The values of the b_{He}/b_{N_2} ratio may be compared with data obtained in other studies. Pfalzner (1950) compared the flow of oxygen and air through an unidentified species of wood. Resch and Ecklund (1964) compared the flow of oxygen and nitrogen in redwood. Both used the Adzumi approach, which does not generate b-factors, but the values may be readily calculated from the data. Comstock (1968) used the Klinkenberg approach to correct for molecular slippage on helium and nitrogen in eastern hemlock. The results are summarized in Table 1.

The observed ratios of Pfalzner and of Resch and Ecklund are in close agreement with the expected values, probably because of the similarity in density, viscosity, and mean-free-path of the gases they studied. Comstock's data showed a smaller range of variation in the ratio, as compared with this study, which may be due to smaller sample variation since he studied only one species.

There was no tendency for the b_{He}/b_{N_2} ratios to tend toward 2.95 as the pressure drop was decreased; therefore it cannot be concluded that the slope of the K_a vs. $1/\bar{P}$ relation was due to a combination of molecular slippage and turbulence; nor can it be concluded that turbulence was the only factor causing the slope, as this would have given rise to the ratios being less than one since nitrogen is more subject to turbulence than helium. It appears that in wood a third factor that cannot be explained in terms of molecular slippage or by turbulence, as normally conceived, may also affect the flow of gases in wood.

CONCLUSION

From this study, it can be concluded that the flow of gas in wood is different from that in rock. The gas permeability of wood is greatly affected by pressure drop at a high range of pressures and permeabilities. Molecular slippage alone cannot account for this deviation from Darcy's law; effect of turbulence and other factors must also be taken into consideration.

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