The problem of describing the migration of dissolved boron in wood is treated with special reference to the commonly used remedial treatment of wooden poles. The governing equations are derived and discussed together with some of the material parameters required. The equations are solved by the finite element method and, finally, results showing the effect of different treatment strategies are presented.

**Keywords:** Wood, boron, remedial treatment, diffusion, finite elements, poles.

**INTRODUCTION**

Remedial treatment of wooden poles embedded in soil is frequently required. One such treatment consists of placing concentrated boron deposits in the pole near the ground level. In practice, two or more holes are drilled into the pole from a point above the ground level extending downwards and to the center of the pole, (Fig. 1). These holes are filled with boric acid in solid form, which reacts with the water contained in the wood. The dissolved boric acid is then transported throughout the pole by diffusion with simultaneous leaching into the soil.

The transfer of boron is usually assumed to be a diffusive process (Ra 1999; Vianez 1993) with the diffusion coefficient depending on the wood moisture content and the temperature. The wood
moisture content and temperature are functions of the conditions in the surrounding environment, i.e., the moisture content and temperature in the soil and in the air. Also, the leaching of boron may be expected to depend on moisture content and temperature and, possibly, on the particular soil, e.g., sand or clay, in which the pole is embedded. Together with often rather complex geometries, (Fig. 1), this makes the description of the resulting transfer quite complicated. However, the constitutive relations governing each of the processes involved are well described, i.e., Fick’s law for diffusive boron and bound water transfer, Darcy’s law for the transfer of free water in the pole, Fourier’s law for heat conduction, etc.

By considering heat and mass conservation and using the above-mentioned constitutive laws, a set of governing equations can be derived. These are all of the diffusive type as, e.g., the well-known unsteady-state heat conduction equation. However, only for very simple geometries can these equations be solved analytically, or otherwise, by hand calculation methods; and we therefore have to resort to numerical methods. Traditionally, the finite difference method has been popular for the type of transfer problems considered here. However, for complex three-dimensional geometries, the finite element method is much better suited.

In the following the problem of boron transfer in poles such as the one shown in Figure 1 is treated. First the governing equations are discussed together with some of the material parameters required, after which the actual finite element formulation is derived. Finally, numerical examples are given. In these examples we focus particularly on the geometry of the problem, i.e., the efficiency of the treatment as function of the number of boron rods and their placement in the pole.

MOISTURE DISTRIBUTION

Wooden poles partially embedded in soil interact with the moisture contained in the air and in the soil, where the moisture content in the soil is again a function of the climate, type of soil, vegetation, and a number of other factors. Thus, the accurate determination of the soil moisture content, which is by far the most important for the moisture content in the pole, is rather complicated. However, the field observations of Peylo and Bechgaard (1999) suggest that in practice there seems to be little difference between the wood moisture contents found in poles at different locations, i.e., embedded in different soils. Thus, at the surface of the poles inspected, moisture contents between 40 and 90% were found, irrespective of whether the surrounding soil was sand or clay. This is quite a surprising observation which can, however, be easily explained by considering the capillary pressure-saturation curves for sand, clay, and wood. At a soil-wood interface the relevant quantity to consider is the capillary pressure rather than the saturation or moisture content. In Fig. 2, typical capillary pressure-saturation curves for the three materials are shown. For a given de-

![Fig. 1. Boron treatment of wooden poles.](image)

![Fig. 2. Capillary pressure-saturation curves for wood, sand, and clay.](image)
gree of saturation in the soil, the corresponding
degree of saturation in the wood should be deter-
mimed by considering continuity of the capillary
pressure across the soil-wood interface. If we
assume a degree of saturation in the wood of ap-
proximately $S = 0.3$ ($MC = 60\%$), the corre-
sponding degrees of saturation in the clay and in
the sand are approximately $S = 0.4$ and $S = 0.15$
respectively, which are in fact quite reasonable
values for these two types of soils. In the follow-
ing we assume a constant moisture content of $MC$
5 60% at the surface below the ground. Above the
ground, the relative humidity can vary signifi-
cantly over a day, especially during the summer
period. However, the yearly variation, in Northern
European countries are much smaller, and the rel-
ative humidity can with good approximation be
assumed to be constant and equal to $RH = 80\%$.
Furthermore, since moisture transfer below the
fiber saturation point is a very slow process, the
assumption of a constant relative humidity is quite
reasonable. In other words, the dimensions of a
typical pole taken into consideration, the daily
variations in relative humidity do not cause any
significant deviations from the year-average
moisture content in the pole.

Governing equations

Water in wood appears in three different forms:
water vapor, bound water, and free or capillary
water. For each of these phases of water a conser-
vation equation may be derived. These conserva-
tion equations can then be added (Perre and
Turner 1999), to yield one equation for the total
transfer of water. This equation can be written as

\[ \frac{\partial X}{\partial t} = \nabla \cdot \left( K \nabla (P + \gamma_w z) + D_b \nabla X_b + D_v \nabla P_v \right) \]

where $X$ is the moisture content, $P_c$ capillary
pressure, $X_b$ bound water content, and $P_v$ the
vapor pressure. Material parameters required are
the effective conductivity for free water $K$, the
bound water diffusivity $D_b$, the vapor diffusivity
$D_v$, the density of gross wood $\rho_w$, and the effec-
tive weight of water $\gamma_w = 10 \text{kN/m}^3$. Gravity,
which is often neglected but in this application is

BORON DIFFUSION

In the following, the transfer of boron in the
pole is treated. The governing equations are de-
rived, and the variations of the boron diffusion
coefficients with moisture content, temperature,
flow direction and concentration are discussed.

Governing equations

The conservation equation for soluble boron
takes the following form

\[ \frac{\partial \phi SC_i}{\partial t} + \nabla \cdot (C_i \mathbf{v}) = \nabla \cdot (\phi SD_b \nabla C_i) \]

where subscript “$i$” indicates that the concentra-
tion $C_i$ is the intrinsic concentration, i.e., the mass
of boron per unit volume of water within the
wood. By multiplying by the porosity $\phi$ and the
saturation $S$, we get the extrinsic concentration,
i.e., the mass of boron per unit volume of wood.
Apart from the left-hand side accumulation term,
the above equation contains two contributions to
the overall transfer of boron, namely a convective
term, $\nabla \cdot (C_i \mathbf{v})$ where is the mass average velo-
city of the water phase, accounting for boron car-
ried in the water flow, and a diffusive term
$\nabla \cdot (\phi SD_b \nabla C_i)$ accounting for transfer resulting
from concentration gradients.

Under normal circumstances with a steady-
state water phase, we can assume that diffusive
boron transfer is dominant over the convective
transfer, and thus we end up with the following
equation

\[ \frac{\partial C}{\partial t} = \nabla \cdot \left( \phi SD_b \left( \frac{\partial C_i}{\partial C} \right) \nabla C \right) \]

where $C$ is the extrinsic concentration. The in-
trinsic concentration $C_i$ is limited by the satura-
tion concentration of boron in water, at 20°C
approximately 50 kg/m$^3$. Thus, we may relate
the intrinsic and extrinsic concentrations by

$$C_i = \min \left( \frac{1}{\rho}, C^\text{sat}_i \right)$$

(4)

where $C^\text{sat}_i$ is the saturation concentration of boron in water. The governing Eq. (3) can then be written as

$$\frac{\partial C}{\partial t} = \nabla \cdot (\mathbf{D} B \nabla C)$$

(5)

where

$$\mathbf{D} B = \begin{cases} D_B & \text{if } C \leq \rho S C^\text{sat}_i \\ 0 & \text{otherwise} \end{cases}$$

(6)

Thus, if at any point the maximum concentration is reached, no transport occurs before the intrinsic concentration is again within physically possible limits. With respect to the convective transfer, it should be borne in mind that this in certain cases could be significant, e.g., in connection with heavy rainfall or large abrupt temperature variations.

**Boron diffusion coefficients**

Although several authors, see e.g. (Wickens), have studied the migration of boron as influenced by moisture content, to our knowledge very few works have been concerned with determining boron diffusion coefficients in the framework of the theory outlined in the above. Recently, however, Ra (1999) attempted this in connection with dip-treatment. Although many discrepancies such as sample length and time dependent diffusion coefficients were reported, the results do give some important indications regarding the actual values and their variation with temperature and moisture content as well the variation with respect to the three principal directions of wood.

**Moisture dependence**

The influence of wood moisture content on the migration of boron is qualitatively well described. Morell et al. (1990) have shown that diffusion of boron begins at moisture contents around 20%. However, when the fiber saturation point, i.e., $MC = 30\%$, is approached, the diffusion increases significantly to reach a maximum at around 100% MC (Smith and Williams 1969). Thus, it can be expected that the free water contained within the wood provides the principal pathway of boron diffusion. At low moisture contents, this free water does not constitute a continuous phase, and thus the transfer of boron is hindered. However, as the moisture content increases and the free water phase becomes more and more continuous, the transfer speed of boron is also increased. Thus, it could be expected that the effect of raising the moisture content would gradually become less and less significant as full saturation is approached.

**Directional dependence**

Usually, transfer coefficients in wood, be they electrical resistance, heat conductivity, or bound water diffusivity, are given in terms of a single coefficient, which may depend on moisture content, temperature, etc. The transfer coefficients in the three principal directions are then given as fractions of this reference coefficient. An exception from this rule, however, appears in the transfer of free water and is related to the above-mentioned continuity of the water phase. Thus, at high degrees of saturation the ‘conductivity’ in the longitudinal direction is much higher than in the other directions. However, at low moisture contents this situation reverses. (Fig. 3) Since boron is carried in the free water, it could be expected that a similar rule would apply with respect to boron diffusion coefficients.

**Temperature dependence**

As is the case with most diffusive processes the diffusion coefficient can be expected to increase with increasing temperature. Thus, Ra (1999) found an increase in boron diffusion coefficients with increasing temperature. Measurements were performed at 30, 50, and $70^\circ\text{C}$ and the temperature variation fitted by a straight line. This, however means that for low temperatures ($\sim 5$–$10^\circ\text{C}$),
some of the coefficients become negative, which is clearly not satisfactory. A more commonly used relationship between diffusion coefficients and temperature is given by the Arrhenius relation which states that the diffusion coefficient $D$ at a certain temperature is given by

$$D = D_0 e^{-E_a / RT}$$

(7)

where $D_0$ is a reference diffusion coefficient, $R$ the universal gas constant, $T$ the absolute temperature, and $E_a$ the activation energy. On examining Ra’s results, we found that the temperature variation could in fact be described very accurately by the Arrhenius relation using an activation energy of approximately $E_a = 20$ kJ.

Concentration dependence

In the experiments performed by Ra (1999) and Vianez (1993) time-dependent diffusion coefficients were reported. Although time-dependent diffusion coefficients do occur, it is usually as a result of some underlying process, e.g., swelling and subsequent relaxation in polymers. Since it is hard to find such explanations in connection with boron diffusion in wood, an obvious possibility is that the diffusion coefficient varies with the concentration of boron. To our knowledge, this possibility has not previously been considered. There is, of course, also the possibility that the apparent time-dependence is a result of a faulty experimental procedure, but under all circumstances this is an issue which needs further clarification.

Choice of diffusion coefficients

In the light of the foregoing discussion, it is clear that estimating diffusion coefficients as input for the diffusion equation is a rather delicate issue. Based on previous attempts (Krabbenhoft et al. 2002) at simulating the experimental results of Wickens (1997) we have chosen a longitudinal diffusion coefficient of

$$D^0_L = 3 \times 10^{-10} \text{ m}^2 / \text{s}, \quad D_L = 0.55 D^0_L e^{2.15X}$$

(8)

where $X$ is the moisture content (kg/kg). Below the fiber saturation point, in this work defined as $X_{FSP} = 0.28$, the diffusion coefficients are set equal to zero. The above relationship gives an approximate doubling when raising the moisture content from 30% to 60%, see Fig. 4. The radial

![Fig. 3. Free water in wood at high (longitudinal transfer dominant) (a) and low (transverse transfer dominant) (b) degrees of saturation.](image)

![Fig. 4. Assumed diffusion coefficients.](image)
and tangential diffusion coefficients are related to the longitudinal coefficient by

$$D_L : D_R : D_T = 20 : 2 : 1$$  \hspace{1cm} (9)

These values are, admittedly, somewhat arbitrarily chosen. However, apart from calibration with the above-mentioned experiments, the values are also within the limits of what has been reported by Ra (1999).

**Leaching**

When the boron reaches the surface of the pole, it will leach into the soil. The migration of boron in soil can, in analogy with the transfer in wood, be assumed to be a diffusive process. Thus, in principle we need not only diffusion coefficients for wood but also for soil. In addition, the domain of interest is increased significantly. To circumvent these difficulties, we propose a Robin boundary condition for the leaching of boron into the soil. If the concentration at the surface is denoted \(C_s\) and the concentration at an infinite distance from the pole \(C_\infty\), the mass flux of boron into the soil is approximated as

$$j_s = \beta(C_s - C_\infty)$$  \hspace{1cm} (10)

where \(C_\infty\) should be taken as being equal to zero. Thus, the rate of leaching is assumed to be directly proportional to the concentration at the surface. This type of boundary condition is commonly applied in connection with heat transfer from the air to a solid or vice versa. Here one would write

$$q = h(T - T_s)$$  \hspace{1cm} (11)

where \(q\) is the heat flux and \(h\) is known as the convective heat transfer coefficient. This analogy provides a way of determining the convective mass transfer coefficient, or at least its magnitude. If the diffusion of boron in wood is assumed to be equivalent to heat conduction, the diffusion coefficient \(D\) and the convective mass transfer coefficient \(\beta\) should be related to the convective heat transfer coefficient \(h\) and the heat conductivity \(\lambda\) by

$$\frac{\beta}{D} = \frac{h}{\lambda}$$  \hspace{1cm} (12)

For wood \(h/\lambda \approx 30-60 \text{ m}^{-1}\), see e.g., (Perre and Turner 1999), and at 60% MC \(D_R = 6 \times 10^{-11} \text{ m}^2/\text{s}, 2D_T = D_R\). Thus, a value of \(\beta = 12 \times 10^{-12} \text{ m/s}\), giving \(\beta D_R = 20 \text{ m}^{-1}\) and \(\beta D_T = 40 \text{ m}^{-1}\) is not unreasonable and will be used in the following.

**FINITE ELEMENT FORMULATION**

The basis of the numerical solution is the Galerkin finite element method. This procedure is now illustrated by the discretization of the diffusion Eq. (5)

$$\frac{\partial C}{\partial t} - \nabla \cdot (D \nabla C) = 0$$  \hspace{1cm} (13)

Instead of trying to fulfill this equation exactly, the finite element method considers an average integral fulfillment of the equation. This is accomplished by multiplying the original equation by a suitable weight function \(w = w(x, y, z)\) and integrating over the domain \(\Omega\)

$$\int_{\Omega} w \left( \frac{\partial C}{\partial t} - \nabla \cdot (D \nabla C) \right) d\Omega = 0$$  \hspace{1cm} (14)

By application of Green’s theorem, this equation may be recast as

$$\int_{\Omega} w \frac{\partial C}{\partial t} + \nabla w \cdot D \nabla C d\Omega = \int_{\Gamma} w_j n \cdot d\Gamma$$  \hspace{1cm} (15)

where \(\Gamma\) denotes the boundary of the domain and \(j_s\) is the outward directed flux. The concentration and weight functions are now approximated by finite element functions as

$$C(x, y, z, t) = N(x, y, z)C(t),$$
$$\nabla C(x, y, z, t) = \nabla N(x, y, z)C(t)$$
$$w(x, y, z, t) = N(x, y, z)w(t),$$
$$\nabla w(x, y, z, t) = \nabla N(x, y, z)w(t)$$  \hspace{1cm} (16)

For a one-dimensional element, with \(x \in [0;L]\), where \(L\) is the length, \(N\) and \(C\) may be chosen as
where \( C_1 = C(x=0) \) and \( C_2 = C(x=L) \) are the concentrations at the beginning and end of the element. Thus, \( C \) is interpolated linearly between the nodal values. By inserting the functions defined in Eqs. (16) into (15), a set of equations are obtained and can be written in matrix form as

\[
N(x) = \left[ \begin{array}{c} \frac{1-x}{L} \\ \frac{x}{L} \end{array} \right] C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}
\] (17)

where \( n+1 \) denotes the unknown state and \( n \) the current state. Since \( M, K, f \) may depend on the unknown state an iterative solution procedure should generally be considered. Further details on the numerical method with reference to the problem of boron diffusion can be found in Krabbenhoft et al. 2002.

**Numerical Results**

In the following, numerical results demonstrating some of the possibilities of the above outlined theory are presented. We focus particularly on the advantages and disadvantages of using several rods (two, three, and four) and on how these should be placed in the pole. In all the examples, the holes are filled with three 10 mm rods such that each hole contains approximately 50 g of boron. The rods are placed as shown in Fig. 5. With respect to the finite element discretization, the symmetry inherent in each of the problems is utilized such that only part of the pole is discretized as indicated by the shaded parts in Fig. 5. Also shown in the figure are the critical points, i.e. the points in the horizontal plane with the farthest distance from the rods.

**Moisture Distribution**

As already discussed, a steady-state moisture field is considered. The moisture content in the soil is assumed to correspond to 60% MC in the
pole, whereas above ground the relative humidity is 80% corresponding to a moisture content of $MC = 15\%$. The solution of (1) provides the moisture distribution shown in Fig. 6.

### Influence of number of rods

A pole with a diameter of 22 cm is analyzed. Two, three, or four rods are inserted with the distances $a$ and $b$ being $a = 7$ cm and $b = 4$ cm. These distances correspond to those used in practice for poles with a diameter of 22 cm.

In Fig. 7(a), the cumulative release of boron from the rods as well as the cumulative leaching into the soil is shown for the cases of two, three, and four rods. Quite surprisingly, the results for two and three rods are very similar. We should mention that this is the case over a large range of the parameters, i.e., diffusion coefficients and convective mass transfer coefficients, used in the simulations; and the fact that the results are so similar thus appears to be a geometric effect. In the case of four rods, the release and leaching (per rod) are somewhat smaller. Thus, it appears that the more rods are inserted, the longer time passes until the reservoirs are depleted. In other words, the release per rod is influenced by the number of rods present, although this influence is very small when comparing the cases of two and three rods.

In Fig. 7 the concentrations in the critical points at ground level are shown as function of time. As could be expected, the maximum concentration increases as the number of rods is increased, and this in a way which is almost proportional to the number of rods. Also, the more rods that are inserted, the faster a given concentration is reached; and since the maximum concentration increases the same is the case with respect to the time which passes after depletion and until the concentration has dropped to a certain level. This is illustrated in the figure with respect to a concentration of 1 kg/m$^3$. Although the difference between the times to depletion in the case of three and four rods is only around 1.5 years, the difference between the periods of time where the concentration is above 1 kg/m$^3$ is more than twice this.

Thus, regarding the effect of increasing the number of rods in a pole it can be concluded that there is a pronounced effect and that this effect is more than proportional to the number of rods. In other words, if the number of rods is doubled, from two to four, the preservative effect is more than doubled.

In Figs. 8–10, the distributions of boron in horizontal sections are shown in the three cases considered. The sections are located 0, 15, and 30 cm below ground level. As has already been discussed, increasing the number of rods has a positive effect at the ground level, and from the figures it can be seen that this effect is not smaller below ground level.

### Influence of rod placing

When the number of rods to be inserted has been determined, it must be decided how the...
Fig. 7. Cumulative release and leaching (a) and concentration in critical points (ground level) (b) as function of number of rods. Pole diameter of 22 cm.

Fig. 8. Boron distributions (kg/m$^3$). Two rods, pole diameter of 22 cm. Red line indicates 0.5 kg/m$^3$ limit and green line 1.0 kg/m$^3$ limit.

Fig. 9. Boron distributions (kg/m$^3$). Three rods, pole diameter of 22 cm. Red line indicates 0.5 kg/m$^3$ limit and green line 1.0 kg/m$^3$ limit.
rods should be placed in the pole. As shown in Fig. 5, this entails determining the distance \( a \) from the bottom of the rod to the pole surface. As a function of this distance, cumulative release and leaching rates have been computed, (Fig. 11 (a)). Also, the concentration in the critical point has been determined as a function of time for each of the distances to the surface, Fig. 11(b).

The results are intuitively reasonable: the smaller the distance from the rod to the surface, the shorter time to depletion; and since the distance from the critical point to the rod increases with decreasing distance from the rod to the surface, we see an increasing maximum concentration in the critical point as \( a \) increases. These two parameters, the time to depletion and the maximum concentration in the critical point, seem to be more or less proportional to the distance from the rod to the surface. However, the time interval over which a certain concentration is maintained increases more than proportionally with the rod-surface distance; and placing the rod as far away
from the surface is thus more beneficial than may appear from Fig. 11(b).

ALTERNATIVE PLACEMENT OF RODS

The next question which naturally arises is to whether the rod placement used in the foregoing is optimal. To answer this question partly, we have experimented with the alternative placement shown in Fig. 12(a). The rods still have a length of 35 cm and are placed such that 20 cm is below ground. Plotted in Fig. 12(b) are the concentrations in the critical points at ground level and 15 cm below. These are compared to the conventional case ($a = 7$ cm) and as can be seen the alternative placement produces higher maximum concentrations. However, leaching is greater since the rods are closer to the surface. This could be reduced by pointing the rods slightly more to the center of the pole, which then again would result in slightly lower concentrations in the critical points.

CONCLUSIONS

The conclusions of this paper are two-fold. First, concerning the obtained results the following can be concluded

- The boron rods should be placed as far away from the pole surface as possible. This increases the time to depletion as well as the maximum concentration in the critical point and the time over which a certain concentration is maintained in these points.
- Increasing the number of rods increases the maximum concentration in the critical points and the time over which a certain concentration is maintained in these points. A smaller effect is observed with respect to the time to depletion. The trend, however, is towards longer times to depletion with increasing number of rods.
- Some improvement can be achieved by alternative rod placements as shown in Section 5.4. Whether these are significant enough to justify the additional practical difficulties involved is of course another question.

Secondly, concerning the nature of the physical problem considered. There is no doubt that the problem treated is extremely complicated regarding the different mechanisms involved and their mutual coupling. A further complication, however, is the fact that the most basic material parameters, e.g., boron diffusion coefficients, are non-existent. To our knowledge no rigorous experimental work has ever been undertaken to determine these as function of moisture content, temperature and possibly boron concentration, which in light of the numerous works concerned with observing and measuring the migration of boron in wood, is quite surprising. However, with the governing equations identified here we
feel that we have established a scientific basis for further study of the problem.

REFERENCES


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