FINITE ELEMENT MODEL FOR THE HEATING OF FROZEN WOOD

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ABSTRACT

A finite element method for calculating the transient heating of frozen log and lumber subject to different initial and boundary conditions is described. The governing equation used in the study accounts for thawing by incorporating the latent heat of fusion into the heat capacity term. The heat capacity and thermal conductivity values for wood in the radial direction for temperatures below and above 0°C were obtained from Steinhagen and Lee (1988). For two- and three-dimensional modeling, it was assumed that the thermal conductivity in the tangential direction is equal to, while that in the longitudinal direction is 2.5 times, the thermal conductivity in the radial direction. The finite element model was validated successfully using the experimental results of Steinhagen (1977) for frozen logs. The model was applied to one-, two-, and three-dimensional heating of lumber and to cases where surface resistance to heat transfer was significant. The thawing of frozen free water in the cell lumina involves the consumption of heat and therefore must be considered in the heating of frozen wood; otherwise the heating time so calculated will be underestimated. A table showing the times needed to heat the center of lumber with different cross-sectional dimensions, green specific gravities, and moisture contents to a temperature of 56.1°C is presented.

Keywords: Unsteady-state heat transfer, thermal conductivity, heat capacity, thermal diffusivity, latent heat of fusion, thawing.

INTRODUCTION

The heating of wood is required in many wood manufacturing operations such as veneer manufacture, preservative treatment, lumber drying, and wood sterilization. Since heating is an energy-intensive and time-consuming operation, estimation of the heating time to a desired temperature is important to save energy and to expedite the manufacturing process. Several research papers have been published for the heating of round and rectangular cross-sections of wood from ambient condition (Simpson 2001); but except for those dealing with logs (Feihl 1972; Steinhagen 1991), few papers have addressed the heating of frozen wood. This paper describes a finite element method to calculate the one-, two-, and three-dimensional transient heating of frozen wood subject to different initial and boundary conditions.

METHODOLOGY

Unsteady-state thermal conduction problems are, in general, described mathematically by the heat equation, which is given by:

$$\rho C \frac{\partial T}{\partial t} + \nabla \cdot (-k \nabla T) = \bar{q} \qquad (1)$$

where ρ is density, C is heat capacity, T is temperature, t is time, k is thermal conductivity, and $\bar{q}$ is the rate of energy generation per unit volume. For problems involving phase change such

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as the heating of frozen wood with moisture content above the fiber saturation point, the latent heat associated with the phase change must be considered by modifying the heat equation as follows (COMSOL 2004b):

$$\rho(C + D\lambda)\frac{\partial T}{\partial t} + \nabla \cdot (-k\nabla T) = \bar{q} \qquad (2)$$

which accounts for thawing by incorporating the latent heat of fusion (λ) into the heat capacity term. In the above equation, the quantity D represents a normalized pulse around the melting point. In the implementation of the finite element solution, a Gauss pulse given by the following expression was used:

$$D = \frac{e^{\frac{-(T-T_m)^2}{dT^2}}}{\sqrt{\pi dT^2}} \qquad (3)$$

where T_m is the melting point, and dT is the width of the pulse.

In solving Eq. (2), it was assumed in this paper that the wood initially had a uniform temperature distribution. Boundary condition of the first (constant surface temperature), second (constant surface heat flux), or third (convective surface condition) kind may be imposed. The wood density (ρ) was assumed to be constant during the heating process and was calculated using the following equation:

$$\rho = G_g \rho_w \left(1 + \frac{MC}{100}\right) \qquad (4)$$

where G_g is the wood specific gravity based on oven-dry mass and green volume, ρ_w is the density of water, and MC is the wood moisture content. The thermal conductivity (k) in the radial direction and the heat capacity (C) at temperatures below and above 0°C were calculated from equations given by Steinhagen and Lee (1988). Those equations are applicable for temperature between –40°C and 100°C, green specific gravity between 0.3 to 0.7, and moisture content between 30% and 130%. For two- and three-dimensional modeling, it was assumed that the thermal conductivity in the tangential direction is equal to, while that in the longitudinal direction is 2.5 times, the thermal conductivity in the radial direction. The latent heat of fusion for water (λ), on a per unit mass of wood basis, was calculated using the following equation:

$$\lambda = \frac{\lambda_w (MC - 30)}{100 + MC} \qquad (5)$$

where $\lambda_w = 334{,}000$ J/kg is the latent heat of fusion for water on a per unit mass of water basis. The rate of energy generation per unit volume ($\bar{q}$) was assumed to be zero during the heating of wood.

The values of the parameters in Eq. (2), together with the boundary and initial conditions, were entered in FEMLAB 3.0, a finite element modeling software package (COMSOL 2004a). Since the thermal conductivity and heat capacity are discontinuous at the phase transition, the steps in k and C were represented using a FEMLAB built-in function called flc1hs, a smoothed Heaviside function with continuous first derivative. Therefore k and C are represented in the model by the following equations:

$$k = k_{low} + (k_{high} - k_{low})(\mathrm{flc1hs}(T - T_m, \mathrm{scale})) \qquad (6)$$

$$C = C_{low} + (C_{high} - C_{low})(\mathrm{flc1hs}(T - T_m, \mathrm{scale})) \qquad (7)$$

where k_{low} and C_{low} are the thermal conductivity and heat capacity below the melting point of water, while k_{high} and C_{high} are those above the melting point of water. The scale term in the flc1hs function defines the interval over which the function is to be smoothed.

RESULTS AND DISCUSSION

The finite element model was validated using the experimental results of Steinhagen (1977) for frozen logs. Figure 1 shows the temperature plotted against heating time for three different points located 22.9 cm, 10.2 cm, and 2.5 cm from the surface of an eastern white pine log designated as log #10 in Steinhagen's study. The log had a moisture content of 97%, green spe-

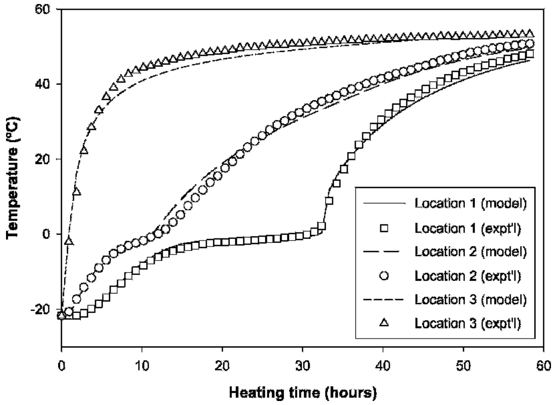

FIG. 1. Graph of temperature against heating time for three different points located 22.9 cm (location 1), 10.2 cm (location 2), and 2.5 cm (location 3) from the surface of an eastern white pine log. The solid and dashed lines represent values calculated using the finite element model, while the markers represent experimental values.

cific gravity of 0.32, diameter of 45.7 cm, and was heated from an initial temperature of –23°C under a constant surface temperature of 54°C. The calculations closely agree with the experimental data, the calculated values not deviating by more than 5°C from the actual numbers. In Steinhagen's experiments, the heated water was agitated vigorously such that the surface of the log came into immediate thermal equilibrium with the heating medium. In cases where surface heat transfer resistance is not negligible, a convective boundary condition must be used. Steinhagen noticed for instance that when the water agitator was shut off momentarily in the course of his experiments, the temperature difference between the heated water and the log surface was 5 to 10 times larger than with agitation. A convective boundary condition can be easily implemented in the finite element model by entering the heating medium's temperature and the convective heat transfer coefficient. If a convective heat transfer coefficient of 15 W/m^2K and a fluid temperature of 54°C are used, the temperature profiles in the eastern white pine log considered above will be those shown by the markers in Fig. 2. These temperature profiles differ from those when constant surface temperature is assumed (represented by the solid and dashed lines in Fig. 2) in that the time required for a certain point in the log to reach a given temperature is longer. It is therefore critical that surface heat transfer resistance be considered in processes involving the heating of logs so as not to underestimate the heating time.

The finite element program may also be used to model the heating of frozen lumber. Using the same thermal properties as those used to model

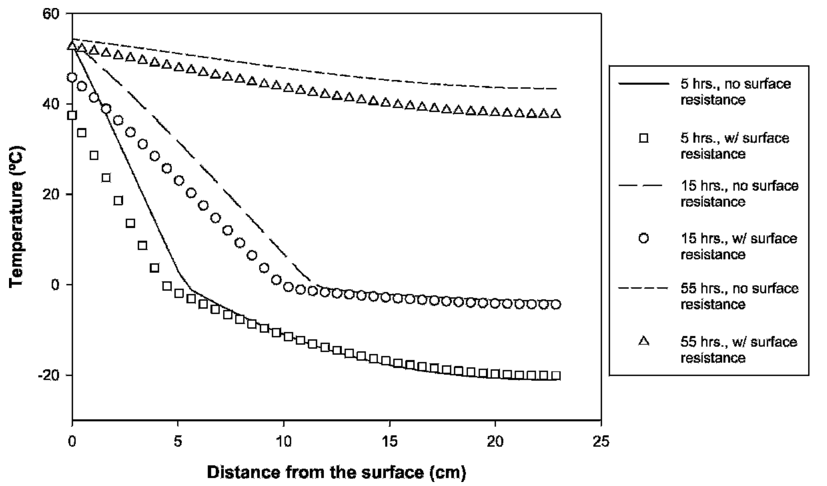

FIG. 2. Radial temperature profiles at different times during the heating of an eastern white pine log. The solid and dashed lines represent the temperature profiles when surface resistance to heat transfer is negligible, while the markers represent the temperature profiles when surface resistance to heat transfer is significant.

the heating of frozen logs, the one-, two-, and three-dimensional heating of lumber can be calculated. Figure 3 shows the temperature distribution across the thickness of a 5.08-cm-thick, flatsawn lumber with a green specific gravity of 0.35 and moisture content of 70%. The lumber was subjected to one-dimensional heating from a uniform initial temperature of –10°C by maintaining a constant temperature of 98.9°C on the two faces of the board. Also included in the figure is the temperature distribution for the case where the latent heat was not included in the governing equation. The figure shows that the heating of lumber is delayed in the case where the latent heat of fusion is taken into consideration. Its temperature distribution has a flat profile near the center of the lumber compared to the parabolic distribution when the latent heat is ignored. If the temperature is plotted against the heating time for a given location in the case where the latent heat is included in the governing equation, the curve has an inflection point at temperatures near the melting point, similar to that displayed in Fig. 1 for round cross-section. Such inflection point is not evident when the latent heat is ignored. These show that a large amount of energy is required to melt the ice in the wood, and therefore sensible heating of the material can not proceed while it is being subjected to latent heating.

The same phenomena observed in one-dimensional heating of frozen lumber are also evident in two-dimensional heating. The times required in heating the center of lumber with different cross-sectional dimensions, green specific gravities, and moisture contents to a temperature of 56.1°C are shown in Table 1. In all cases, the lumber was at an initial uniform temperature of –10°C and subjected to a constant surface temperature of 98.9°C. The table extends Simpson's results (Simpson 2001) to frozen lumber. The heating time increases with lumber size, specific gravity, and moisture content and is higher when the latent heat of fusion is taken into consideration. The heating dynamics also change when surface resistance to heat transfer is significant. For instance, a surface convective heat transfer coefficient of 15 W/m^2K results in the heating times shown in column 5 of Table 1. These values are considerably higher than the heating times for the case when the surface immediately comes into equilibrium with the temperature of the surrounding medium (column 4 of Table 1).

Most published work on the heating of log and lumber assumes that the material is "long,"

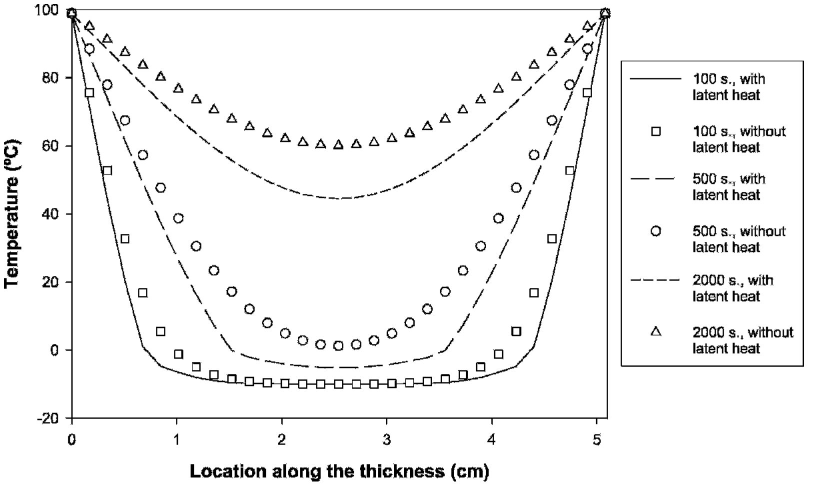

FIG. 3. Temperature profiles in the thickness direction at different times during the heating of a 5.08-cm-thick, flatsawn lumber. The solid and dashed lines represent the temperature distributions when the latent heat was included in the governing equation, while the markers represent the temperature distributions when the latent heat was not included in the governing equation.

TABLE 1. *Heating times for frozen lumber of different moisture contents (MC), green specific gravities (G_g), and dimensions. The lumber was initially at a uniform temperature of −10°C and then was subjected to a fluid temperature of 98.9°C until the center reached a temperature of 56.1°C.*

			Heating time		
MC (%)	G_g	Cross-sectional dimensions (cm × cm)	Constant surface temperature, with latent heat (minutes)	Convective boundary condition with latent heat (minutes)	Constant surface temperature, w/o latent heat (minutes)
70	0.35	2.5 × 10.2	10	37	8
70	0.35	2.5 × 15.2	10	38	8
70	0.35	5.1 × 10.2	37	82	29
70	0.35	5.1 × 20.3	40	97	31
70	0.35	10.2 × 10.2	95	158	75
70	0.35	10.2 × 20.3	148	243	116
70	0.35	10.2 × 30.5	158	271	123
70	0.45	2.5 × 10.2	10	45	8
70	0.45	2.5 × 15.2	10	47	8
70	0.45	5.1 × 10.2	38	95	30
70	0.45	5.1 × 20.3	41	114	32
70	0.45	10.2 × 10.2	97	178	78
70	0.45	10.2 × 20.3	152	272	120
70	0.45	10.2 × 30.5	163	305	126
70	0.55	2.5 × 10.2	10	52	8
70	0.55	2.5 × 15.2	10	55	8
70	0.55	5.1 × 10.2	39	107	31
70	0.55	5.1 × 20.3	42	130	32
70	0.55	10.2 × 10.2	99	197	79
70	0.55	10.2 × 20.3	155	300	122
70	0.55	10.2 × 30.5	166	338	129
100	0.35	2.5 × 10.2	11	49	8
100	0.35	2.5 × 15.2	11	51	8
100	0.35	51. × 10.2	42	103	30
100	0.35	5.1 × 20.3	45	124	32
100	0.35	10.2 × 10.2	106	193	78
100	0.35	10.2 × 20.3	166	296	121
100	0.35	10.2 × 30.5	178	332	127
100	0.45	2.5 × 10.2	12	59	8
100	0.45	2.5 × 15.2	12	62	8
100	0.45	51. × 10.2	43	120	31
100	0.45	5.1 × 20.3	46	146	33
100	0.45	10.2 × 10.2	109	220	80
100	0.45	10.2 × 20.3	171	335	124
100	0.45	10.2 × 30.5	183	379	131
100	0.55	2.5 × 10.2	12	69	8
100	0.55	2.5 × 15.2	12	73	8
100	0.55	5.1 × 10.2	44	136	32
100	0.55	5.1 × 20.3	47	168	34
100	0.55	10.2 × 10.2	111	245	82
100	0.55	10.2 × 20.3	175	371	127
100	0.55	10.2 × 30.5	187	423	134

[1] Convective heat transfer coefficient equal to 15 W/m^2 K.

that is, the log is considered as an infinite cylinder and the lumber is considered either a plane wall or an infinite rectangular bar. Those studies therefore considered only one- or two-dimensional heating. Since the thermal conductivity of wood in the longitudinal direction is about 2.5 times those in the radial and tangential directions, the heating of wood must be analyzed as a three-dimensional problem if short pieces are involved. Figure 4 shows the temperature

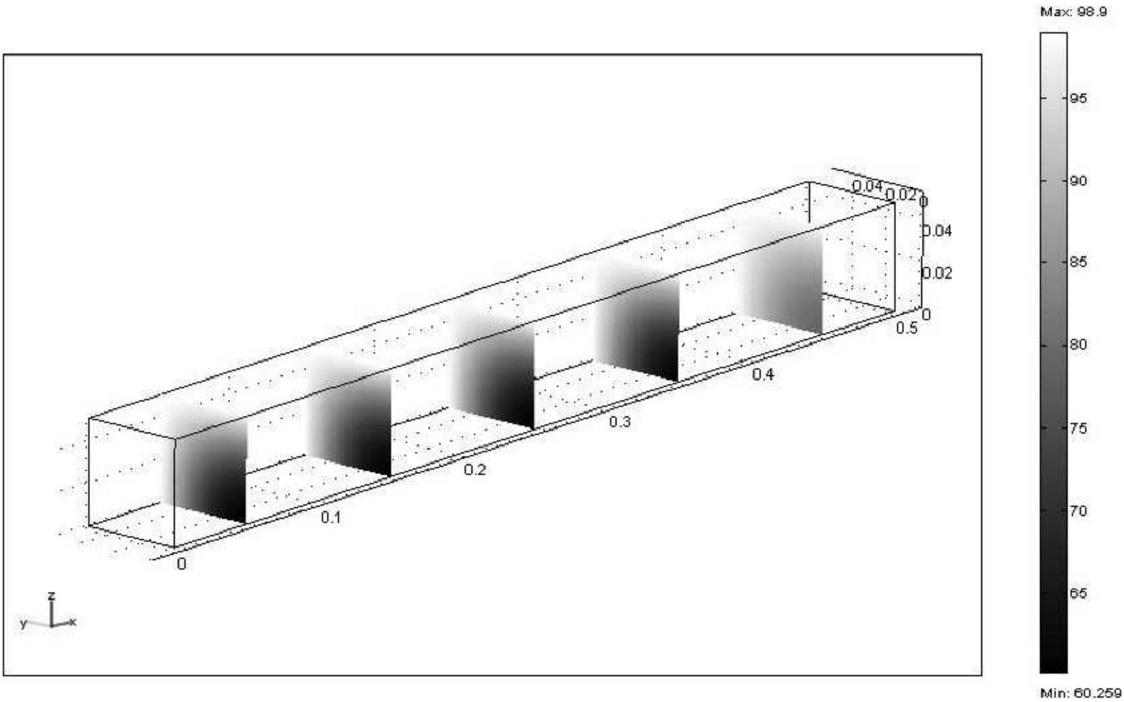

FIG. 4. Temperature profiles after heating a 0.102-m × 0.102-m × 1.00-m frozen lumber for 100 minutes from an initial temperature of −10°C under a constant surface temperature of 98.9°C. Only ¼ of the lumber is shown, with the top, left, and far-end faces being maintained at the constant temperature.

profiles after heating a 0.102-m × 0.102-m × 1.00-m frozen lumber with a green specific gravity of 0.35 and moisture content of 70% for 100 min from –10°C under a constant surface temperature of 98.9°C. Due to symmetry, the figure shows only one-fourth of the lumber, with the top, left, and far-end faces being maintained at the constant temperature. Longitudinal heat transfer becomes significant when the ratio of the longitudinal dimension to transverse dimension is less than 4. The finite element model calculates that it will take 95 min to heat up the center of the lumber to 56.1°C if the lumber length is 1.00 m but it will take only 82 minutes to heat up a 0.20-m long lumber.

CONCLUSIONS

The finite element modeling approach can be used to analyze the heating of frozen wood by solving the heat equation that incorporates the latent heat of fusion in the heat capacity term. It can describe one-, two-, and three dimensional heating of logs and lumber subject to different boundary and initial conditions.

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