DEVELOPMENT OF A BENDING STIFFNESS MODEL FOR WET PROCESS FIBERBOARD

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(Received November 2005)

ABSTRACT

In traditional mechanics of materials, the stiffness of a beam or plate in bending is described by its cross-sectional shape as well as its material properties, primarily the modulus of elasticity. Previous work at the USDA Forest Products Laboratory, Madison, Wisconsin, has shown that modulus of elasticity has a strong correlation to the density of the fiberboard. Examined here are experimental and theoretical relationships between process variables of bulk density and area density and the transverse bending stiffness of the fiberboard. A model is developed and evaluated for the relationship between area density, bulk density, and bending stiffness.

Keywords: National fire plan, fiberboard, bending stiffness, modulus of elasticity, dynamic testing, dynamic modulus, density, area density, sheet weight, process variable, value-added.

INTRODUCTION

For fiber-based products, performance of a product is linked to factors that drive the cost of the raw material. Costs are driven by fiber furnish and how much of that fiber it takes to make a certain product achieve the desired performance. Engineers can optimize product performance by adjusting process variables, thus adding value to fiber composite products. To optimize product performance, however, relationships between process variables and the final product performance must be understood and quantified.

This paper describes a model that can be used to determine the potential bending stiffness of fiberboard over a wide range of board densities and thicknesses. In this case, bending stiffness is the product performance characteristic, and area density and bulk density are considered as process variables. The model relies on establishing the correlation between the fiberboard bulk density and modulus of elasticity and extends that correlation to predict the relationship between area density and bending stiffness.

BACKGROUND

The Forest Service's National Fire Plan (USDA Forest Service 2005) seeks to develop

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Wood and Fiber Science, 39(1), 2007, pp. 196–203

more outlets for underutilized or low-value biomass material such as small-diameter thinning and logging residuals (Barbour 1999). This material represents a significant amount of hazardous fuel for many forests in the western United States. Currently, only a few outlets exist for this material. The amount of low-value biomass is vast, and because this material poses a risk to life and property, it needs to be reduced. There is also a need to develop more outlets for waste paper to reduce the amount of fibrous material going to landfills. North Dakota alone has 22,000 dry tons of forest residues, 326,510 dry tons of urban waste, and 4,000 dry tons of fill residue that have no specific outlet (ORNL 1999). An outlet for these materials could be created by incorporating them into wood–fiber-based composites with specific performance requirements. The difficulty in incorporating these underutilized materials results from variability in material properties associated with fiber source, processing method, and the composite fabrication process. This combination of options makes it extremely difficult for an engineer to incorporate materials on the basis of performance for wood-based composites. Optimal incorporation requires decision-making tools or models based on fundamental properties.

As part of an appropriate set of tools, this model may provide a methodology to determine potential performance of fiberboard and wood–fiber composites over a vast array of potential processing variables. This will allow engineers to tailor products to specific performance characteristics based on fiber characteristics. The USDA Forest Products Laboratory in Madison, Wisconsin, is developing an engineered approach to fabricating value-added three-dimensional (3D) engineered fiberboard from underutilized and low-value fibers. Through a structural 3D design, potential performance characteristics of each fiber type may be considered in engineering a product that has the desired performance characteristics. Development of 3D engineered fiberboard will require new techniques based on improved structural and material efficiency, both of which can have direct implications on manufacturing costs. When value is added to a product in this way, it can compete in a niche market and its return on investment can provide incentive for private investors to use these underutilized and low-value fiber resources, thus reducing hazardous fuels and pressure on landfills.

Panels used for this analysis were produced as part of a larger study aiming to fully quantify the mechanical properties of wet-formed fiberboard across a range of thicknesses, densities, resin contents, and fiber types. Because of the scope of that study, complete results are not yet available. The data presented here are limited to a recycled corrugated cardboard fiber furnish unless otherwise noted. The goal of the work is to provide a fundamental understanding of material-process–product relationships and then to quantify the relevant relationships. This work will enhance the abilities of engineers to design and model fiberboard structures either analytically or with computer-based finite element models.

EXPERIMENTAL METHODS

Fiber furnish

Recycled corrugated cardboard was produced for this study by atmospheric refinement of corrugated boxboard sheets obtained commercially. A low-consistency hydropulper was used to pulp the cardboard for 30 min at 46°C at a 2.35% consistency. The pulp was then mechanically dewatered and cold stored until used.

Processing method

Fiber and resin were added at the same time into the process water. The pH of the pulp slurry was reduced to approximately 4.5 using aluminum sulfate to precipitate the resin on the fiber. Resin contents of 0.0, 0.5, 1.5, 3.0, and 4.5% were used. Resin content was computed on the basis of resin solids content to dry fiber mass. Wet-formed fiber mats were formed in a 622- by 622-mm deckle box with vacuum drainage. Target forming consistency was 1.5%. Fiber mats were then hot-pressed with press platen temperature set at 175°C. Boards were placed between two stainless steel screens and two cauls before being placed into the press and pressed at constant pressure for 10 min. Target densities of 0.4, 0.6, 0.8, 1.0, and 1.15 (bulk density) were pressed at mat pressures of 103, 207, 517, 1379, and 2068 kPa, respectively.

Pressed boards were placed into conditioning rooms at 21°C and 50% relative humidity until they could be cut into specimens. Once the specimens were cut from the panels, they were placed back into the same conditioning environment and allowed to equilibrate before testing.

Testing

The dynamic modulus data presented here were obtained by measuring the frequency of the free vibration of a specimen in a cantilever support condition, the apparatus for which is described in a paper in progress (Hunt and Turk, in preparation). Specimens for the dynamic modulus testing were cut to nominal dimensions of 5.1 cm by 25.4 cm. A digital extensometer was used to measure the width and thickness to an accuracy of +/− 0.02 mm. The length was measured with digital calipers to an accuracy of +/− 0.1 mm. A digital balance was used to weigh the specimens to an accuracy of +/− 0.01 g. Specimens were clamped in place to obtain cantilever loading by restraining 5.1 cm of the total length with the necessary pressure to obtain 10% compression through the specimen thickness or a maximum pressure of approximately 2760 kPa, whichever was less. A displacement of approximately 8 mm was applied to the unsupported specimen edge, and then the specimen was released into free vibration. The resulting displacement over time during the vibration was observed and recorded using a displacement-measuring laser and high-speed data acquisition card.

The dynamic modulus was obtained from the recorded response using transformed equations of vibration.

The frequency of the first mode of free vibration of a cantilever beam is

$$\omega_{n1} = 2\pi f = \left(\frac{1.875^2}{l^2}\right)\sqrt{\frac{EI}{m_u}} \tag{1}$$

where

$\grave{u}_{n1}$ is frequency of the first mode of vibration (rad/s),
f detected frequency of the first natural mode of vibration (Hz),
l unclamped or "free length" of the cantilever beam (m),
E dynamic modulus of elasticity (N/m^2),
I centroidal area moment of inertia of the cross-section (m^4), and
m_u mass per unit length (kg/m).

Shear and rotary effects on vibration are neglected because they have little effect on the frequency of the first mode of vibration when the ratio of specimen thickness to length is kept small, as shown by Stokey (Harris and Piersol 2002). Equation (1) can be rewritten to provide the dynamic modulus of elasticity by substituting the appropriate specimen properties for the area moment of inertia and mass per unit length to provide the dynamic modulus of elasticity.

Dynamic modulus of elasticity in terms of specimen properties and free vibration frequency for a cantilever beam is

$$E = \frac{m}{L}\frac{12}{bt^3}(2\pi f)^2\left(\frac{l^2}{1.875^2}\right)^2 \tag{2}$$

where

m is mass of the specimen (kg),
L complete length of the specimen (m),
b base width of the specimen (m), and
t thickness of the specimen (m).

A custom-written data acquisition program calculated the frequency of the recorded displacement curve, applied Eq. (2) to determine the dynamic modulus, and included the result along with data files for easy analysis.

THEORY

Transverse bending stiffness (k) equations for prismatic beams are of the following forms.

The general stiffness for transverse bending of prismatic beams is

$$k = \frac{C_1 EI}{l^n} \tag{3}$$

where C_1, n are constants defined by loading and support conditions.

By substituting the definition of the area moment of inertia for a rectangular beam, Eq. (3) can be rewritten in terms of specimen dimensions.

The transverse stiffness of a rectangular beam is

$$k = \frac{C_1 Ebt^3}{l^n 12} \tag{4}$$

The stiffness equation describes the stiffness of a beam of a defined size in a specific loading condition. To examine the board stiffness independently of the beam dimension (b and l) or loading condition (C_1 and n), we will define the "transverse stiffness factor" as

$$K = Et^3 \tag{5}$$

Substitution of Eq. (5) into Eq. (4) provides the transverse stiffness in terms of the transverse stiffness factor (Eq. (6)). In this form, the transverse stiffness is dependent upon the specimen width and length, the loading and support conditions, and the transverse stiffness factor.

The transverse stiffness in terms of the transverse stiffness factor is

$$k = \frac{KC_1 b}{12 l^n} \tag{6}$$

If the area density (or sheet weight) S is introduced, the transverse stiffness factor K can be rewritten in terms of area density, bulk density, and modulus.

The transverse stiffness factor in terms of area density, bulk density, and modulus is

$$K = E\frac{S^3}{\rho^3} \tag{7}$$

where

ρ is bulk density, and

S $t * \rho$ is area density (sheet weight per area).

Experimental data in the literature (Hunt 1999) suggest a relationship between the dynamic modulus and the density of wet process fiberboard. The form of the relationship has not yet been established, but the suggested relationship shown in Eq. (8) has the advantage of reducing Eq. (7) to the simplified model of Eq. (9).

The suggested relationship between modulus of elasticity and density is

$$E = C_2 \rho^2 \tag{8}$$

where

C_2 is constant dictated by the fiber furnish properties

We can now rewrite the transverse stiffness factor K in terms of area density, bulk density, and the constant C_2, which will have the SI units of Newton meters^4 divided by kg^2. By writing the transverse stiffness factor in terms of area density and bulk density rather than specimen dimensions, it becomes useful for analytical stiffness analysis as a normalized comparison of bending stiffness.

The transverse stiffness factor model in terms of area density and density is

$$K = \frac{C_2 S^3}{\rho} \tag{9}$$

The stiffness k, from Eq. (6), can now be rewritten by substituting for the transverse stiffness factor, K, in terms of area density and bulk density.

The proposed general stiffness model equation for fiberboard is

$$k = \frac{C_1 C_2 S^3 b}{12 \rho l^n} \tag{10}$$

From this theoretical relationship we can see that for a given area density, S, increasing the bulk density of the panel is expected to result in decreasing stiffness. More importantly, if the relationship proposed in Eq. (8) can be established, a complete family of curves relating the panel stiffness to the sheet weight based on density can be populated based on a single constant, C_2. Correlating the performance property of stiffness to the process variables of area density and bulk density based on a single constant would provide a valuable tool for parametric modeling and product design. Several avenues exist for determining the constant C_2. The direct method of determining the correlation by curve fit is applied here. Indirect methods may be used to determine C_2 for a fiber furnish before any panels are manufactured, or C_2 may be extracted from existing data sets maintained by research and production facilities.

RESULTS AND DISCUSSION

The goal of the analysis presented here is to obtain a useful relationship between the area density, which drives the product cost, and the panel stiffness, a critical factor driving the product value. Panels for this study were not produced at constant area densities; the stiffness factor cannot be measured directly, so the relationship must be developed from the specimen properties including dimensions, mass, and the measured dynamic modulus. The collected modulus data for recycled corrugated fiber are shown in Fig. 1 as plotted versus calculated density.

Effects of resin content

Figure 2 provides the dynamic modulus data for panels produced at a target thickness of 2.54 mm. The six replicate specimens produced at each process condition are grouped and averaged for display. The data reveal that the resin content had no statistically significant effect on the dynamic modulus of the wet process fiberboard at this thickness and moisture content. The same analysis performed on panels produced at 1.27-mm thickness produced similar results. Therefore, the effect of resin content on the stiffness of the panels will be disregarded during the following stiffness analysis.

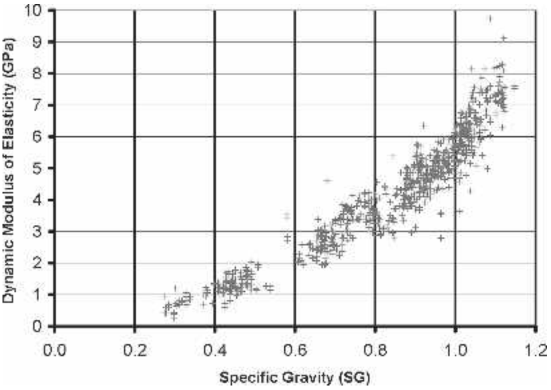

FIG. 1. Dynamic modulus of wet-formed fiberboard made of recycled corrugated fiber plotted versus bulk density.

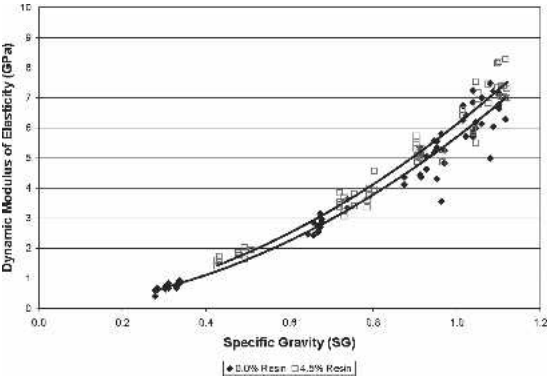

FIG. 2. Average dynamic moduli data plotted versus specific gravity for 0% and 4.5% (minimum and maximum) resin levels.

Density and modulus correlation

To develop a relationship between modulus and density, curve fitting can be performed on the data presented in Fig. 1. A variety of curve-fit functions can be applied to the same data set. Typically, when the relationship between variables is unknown, a polynomial curve fit is used. A general power fit will also provide a good fit to this data set, as shown in Fig. 3. To develop the model, however, a relationship with the form of Eq. (8), a square power fit, was desired. Methods for applying specific forms of equations to curve fitting via a least sum of squared errors fit (John 1998) in Microsoft Excel (Microsoft Corporation, Redmond, WA) were used. To evaluate the quality of each different fit, the sum of squared errors for each curve fit are calculated and presented in Table 1.

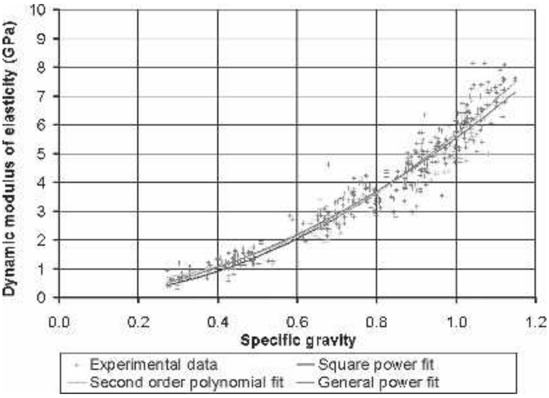

FIG. 3. All experimental dynamic modulus data with curve fits superimposed.

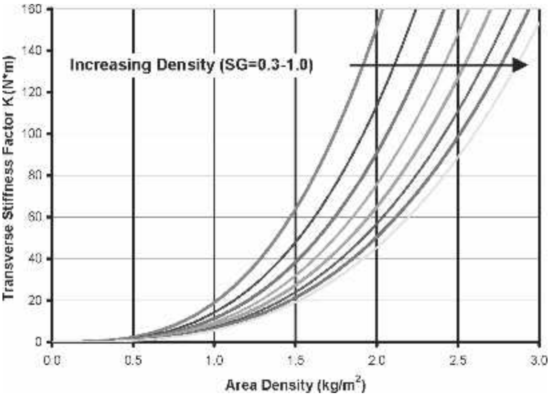

FIG. 4. Transverse bending stiffness model visualized.

TABLE 1. *Fit equations and sum of square error for different curve fits shown in Fig. 3.*

Fit type	Equation	Sum of squared error
Second order polynomial	$E = 6.2416\_^{2} - 1.1189\_ + 0.5361$	108.38
General power	$E = 5.5372\_^{1.8205}$	114.58
Square power	$E = 5.6800\_^{2}$	111.61

Model developed

To obtain an appropriate value for C_2 (Eqs. (8), (9), and (10)), a customized curve fit was performed on the modulus data. By dictating a square power fit with the form of Eq. (8) and iteratively solving for the value of C_2, which resulted in the least sum of squared errors, the best quality fit of that form was obtained. The family of curves shown in Fig. 4 is generated based upon C_2 obtained from this method, as shown in Table 1, with $C_2 = 5.6800$ as the coefficient of the square power fit equation. This curve fit is performed versus bulk density in terms of specific gravity. To convert C_2 to a value in appropriate SI units for a density correlation, it must be multiplied by 1000, resulting in a value of 5680 Newton meters^4 divided by kg^2.

The quality of the square power fit, quantified by the sum of squared errors, is approximately the same as the general power fit or the polynomial fit. However, neither a general polynomial nor a general power fit results in any meaningful reductions of Eqs. (5) or (6). When a curve fit of the form of Eq. (7) is applied to the modulus data, the reduction of terms can be applied in Eq. (8) and a simplified model can be developed. With the relationship between modulus and density described by the square power fit, it is possible to model the effect of area density and bulk density on the panel stiffness factor according to Eq. (8). For visualization, a family of curves can be generated with each curve representing the relationship between the stiffness factor and the area density for a given bulk density.

To evaluate the model, the actual transverse stiffness factor is computed for specimens and then plotted versus area density; the results for selected densities are shown in Fig. 5. A trend becomes clear when the model curves are plotted in comparison to the cubic fits to the experimental data (Fig. 6).

Examination of Figs. 5 and 6 reveals that although the model predicts the correct general behavior of the relationship between area density and stiffness, the experimental stiffness values are underestimated by the model. Furthermore, the magnitude of the error, Δ, appears to be inversely proportional to the density of the panel; that is, the model curves become more accurate at higher densities and approach but do not exceed the experimental values where the bulk density is 0.9. Engineers often call this a conservative model because it will not overestimate the performance of the product. But why

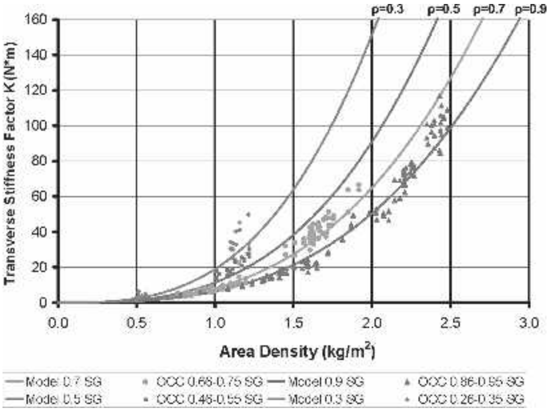

FIG. 5. Overlay of experimental stiffness data and model curves for several density ranges.

does the model predict conservative stiffness values?

Effect of vertical density profile

The developed model assumes a uniform bulk density in the calculation of the specimen stiffness. Assuming uniform bulk density may introduce error into the stiffness calculation if the vertical (through the thickness) density profile is not uniform. Whether the error is conservative or not will depend upon the magnitude and gradient of the actual density profile. Because the faces experience the highest strain in pure bending and therefore carry the highest proportion of the stress, the moduli of the faces contribute more to the panel stiffness than the modulus of the core. If the faces have higher bulk densities than the core of the panel, the stiffness model will be conservative because of the relative increase in modulus at the faces. To model the stiffness of the panel more accurately by accounting for the density profile, a corrected definition of dynamic modulus may be developed for the modulus of elasticity value given in Eq. (8). An effective modulus may then be substituted into Eq. (5) to define a transverse bending stiffness model that accounts for the vertical density profile of the panel. Development of an effective bending modulus or stiffness corrected for the vertical density profile is beyond the scope of this paper; further research is needed to determine how to adequately calculate the effective stiffness of boards with variation in vertical density profile.

Effect of different fibers

As an initial step toward independent validation of the stiffness model for other fiber types, initial results for thermomechanical-derived lodgepole pine fiber are presented (Fig. 7). The lodgepole fibers were processed into fiberboard using the same fiberboard processing methods used with the corrugated fibers. Here, 30 repli-

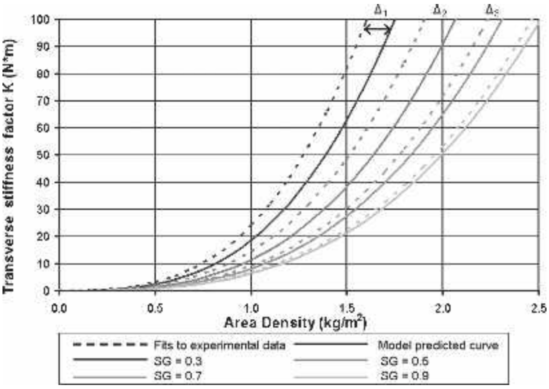

FIG. 6. Model curves overlaid with cubic fits to experimental data.

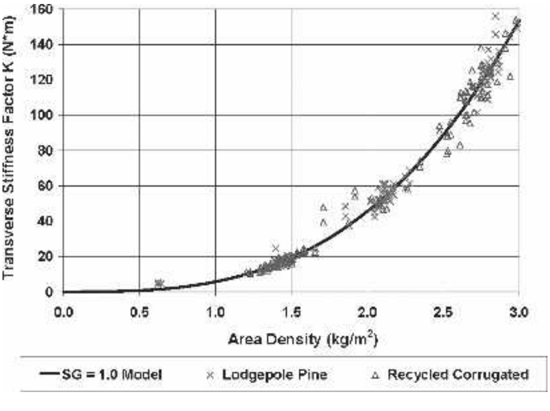

FIG. 7. Experimental stiffness results for panels made from corrugated and lodgepole fibers at 2.54-mm thickness and specific gravity of 1.0 compared to the model.

cate specimens produced at each process condition (six thickness and density replicates at five resin levels) are grouped and shown as averaged data with standard deviation bars overlaid. The initial results show little difference in stiffness between the lodgepole and recycled corrugated panels and good correlation to the model. This appears to indicate that the fibers, which were both themomechanically derived, have similar stiffness characteristics in dynamic bending when conditioned at 50% relative humidity. We expect that as different processing methods and fiber types are used, the bending stiffness will also vary. Further research is necessary to determine the magnitude and behavior of the stiffness variation and its effect on this model.

CONCLUSIONS

The model presented here describes the transverse bending stiffness of wet process fiberboard based upon the area density and bulk density of the fiberboard product. Working from the correlation between dynamic modulus and bulk density, the model was developed as a tool to optimize wood fiber composite manufacturing processes. The primary difficulty in utilizing the model will be the determination of an appropriate value for C_2, the constant describing the correlation between bulk density and modulus of elasticity, for each new type of fiber or processing condition. Further analysis of this constant across many different fiber types would be necessary to determine whether it correlates to any physical or chemical properties of the fiber. Other difficulties in implementing the model include identifying the effects of moisture content and better understanding and accounting for the effects of vertical density profile on the panel stiffness.

This transverse stiffness model has potential use when determining processing methods necessary to obtain a specific bending stiffness performance. In an application requiring a specific bending stiffness, the necessary stiffness factor can be obtained from Eq. (6). Then, based on the desired bulk density, the model can be referenced to determine the necessary area density, which in turn can be used to calculate the optimum thickness for the application. For example, take a specific application where the necessary stiffness factor is computed to be 50 Newton meters, and the desired density is 900kg/m^3, or 0.9 specific gravity. The corresponding area density of 2 kg/m^2 will provide the optimum desired stiffness at a thickness of 2.22 mm. Conversely, if the desired thickness is 2.22 mm and the same stiffness factor of 50 Newton meters is desired, a parametric solution using the definition of area density, S, and Eq. (9) provides the required area density of 2 kg/m^2 and bulk density of 900 kg/m^3. As part of an appropriate set of tools, this model is intended to provide a methodology to determine the optimum manufacturing processes for wood fiber composites over a vast array of potential processing variables. This will allow engineers to tailor products to specific performance requirements based on the fiber characteristics and utilize low-value fibers in more applications by understanding their fundamental performance characteristics.

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