MASS TRANSFER IN WOOD EVALUATED WITH A COLORIMETRIC TECHNIQUE AND NUMERICAL ANALYSIS

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ABSTRACT

A colorimetric CoCl₂-treated wood technique was used for determining surface moisture content of wood nondestructively and continuously during unsteady state desorption conditions. Utilizing these surface moisture data, diffusion and surface emission coefficients have been determined simultaneously and continuously during drying. The conversion method, which has been developed to facilitate making comparison between mass transfer coefficients in this study, has proven that the boundary layer theory is useful for evaluating the external moisture resistance during wood drying. The moisture profiles simulated by the finite difference method were quite comparable to the actual moisture profiles in real dried wood, which substantiates the high credibility of using the colorimetric technique for determining surface moisture content and mathematical procedure for determining the diffusion and surface emission coefficient.

Keywords: Mass transfer, diffusion, surface emission, boundary layer theory, finite difference method, colorimetric technique.

INTRODUCTION

Moisture movement in wood during drying is constrained by two resistances. The internal resistance due to wood itself can be described by the diffusion coefficient. Second is an external resistance to moisture leaving the wood surface and being transferred to the ambient drying air. This external resistance can be described by the mass transfer coefficient. Solutions of Fick’s law and boundary layer theory have been widely applied to determine the diffusion and mass transfer coefficients, respectively (Crank 1975; Geankoplis 1993). However, a fully reliable method to determine these coefficients for a hygroscopic material simultaneously and continuously over the entire moisture content range has not yet been developed. A primary reason is the difficulty of determining wood surface moisture con-

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centrat~on nondestructively and continuously in unst-ady state drying conditions. This diffi-
culty results in most mathematical ap-
proach?~ for determining the diffusion coef-
ficient using an improper assumption that the
surfact
moisture concentration is at equilib-
rium with the surrounding air as soon as the
drying process starts. Because of this diffi-
culty, theoretically
 calculated mass transfer
coefficients using boundary layer theory have
not been well proved by experimental results
(Siau 1995).

Accurate and continuous measurement of
surface moisture concentration in wood in un-
steady-state drying conditions can increase
credibility of methods for determining the co-
efficients. In this study, color change on wood surface%
s treated with CoCl,
according to hu-
midity has been used for determining surface
moisture content (SMC) nondestructively and
continuously during drying. Utilizing surface
moisture data, diffusion and mass transfer
coefficients have been determined. Also, using
these coefficients and numerical analysis with
the finite difference method, a com-
puter simulation program was developed to predict
two-
dimensional moisture profiles in wood during
drying.

MATERIALS AND METHODS

Determination of diffusion and mas-
transfer coefficients

Completely water-soaked cubes of hard maple
(Acer saccharum Marsh), red oak (Quer-
cus rubra L.), and southern pine (Pinus spp.),
with 20-mm longitudinal, radial, and tangen-
tial di nsions were manufactured. Four side
walls of each cubic specimen were wrapped
with rubber tape (Magic Wrap, Glasgow Mfg.
Ltd.), leaving open the two opposite surfaces
of interest, to provide one dimensional mois-
ture flow for drying. Six replicates, to deter-
mine the mass transfer coefficient based on
moisture concentration at wood surface, also
called the surface emission coefficient, and the
diffus on coefficient in each orientation, lon-
gitudinal, radial, and tangential, were pre-
pared.

Surface and average moisture content.—
Using the recently developed colorimetric
 technique (Yeo et al. 2002) and oven-drying,
surface moisture content (SMC) and average
moisture content (AMC) changes in wood
were determined, respectively, during drying
at 25% RH and both 30 and 50°C in temper-
ature- and humidity-controlled environmental
chambers. Air velocity in these chambers was
1 m/s, measured with a hot wire anemometer,
and airflow was parallel to exposed wood sur-
faces. Saturated CoCl solution, approximately
5 µL, was dropped on the exposed surface
area of three of those six replicates for each
orientation, and spread with the tip of a dis-
spensing micropipette. The surface spectral re-
reflectance was measured with a portable spec-
trophotometer (Microflash 200d, manufac-
tured by DataColor International) during dry-
ing at 25% RH and both 30 and 50°C, and
then CIE L*a*b* color values were calculated
on the basis of CIE (Commission Internation-
ale de l’Eclairage) standards (CIE 1986). Us-


\[
\text{dE}_{\text{mc-\text{eq}}} = \sqrt{\left(\text{L}_{\text{mc}}^* - \text{L}_{\text{eq}}^*\right)^2 + \left(\text{a}_{\text{mc}}^* - \text{a}_{\text{eq}}^*\right)^2 + \left(\text{b}_{\text{mc}}^* - \text{b}_{\text{eq}}^*\right)^2}
\]

where

\[
\text{L}_{\text{mc}}^*, \text{a}_{\text{mc}}^*, \text{b}_{\text{mc}}^*; \quad \text{L}_{\text{eq}}^*, \text{a}_{\text{eq}}^*, \text{b}_{\text{eq}}^*;
\]

\[
\text{L}^*, \text{a}^*, \text{b}^* \text{values on the CoCl}_2-treated
\text{surfaces equilibrated at any %RH}
\text{L}_{\text{mc}}^*, \text{a}_{\text{mc}}^*, \text{b}_{\text{mc}}^*;
\text{L}_{\text{eq}}^*, \text{a}_{\text{eq}}^*, \text{b}_{\text{eq}}^*;
\]

\[
\text{L}^*, \text{a}^*, \text{b}^* \text{values on the CoCl}_2-treated
\text{surfaces equilibrated at 25% RH}
\]

Also, three untreated specimens were used
as controls to prove the drying rate of treated
wood to be similar to that of untreated wood.

Specific gravity.—Specific gravities of
moist wood (G_{mc}) were not measured directly
during drying because of the excessive time
required to make many measurements. This likely would have decreased the efficiency of the method for determining the diffusion and surface emission coefficients. Instead of direct measurement, $G_{MC}$ was approximated by linear interpolation between oven-dry specific gravity and green specific gravity.

**Diffusion coefficient.**—A new analytical solution was derived to determine the diffusion coefficient when surface moisture contents were varied. The experimentally measured SMC, AMC, the half thickness of specimen ($L$), drying time ($t$), initial moisture content in each drying time interval ($IMC_i$), and specific gravity of moist wood were used for determining the three directional diffusion coefficients. The analytical solution of the unsteady state diffusion equation was obtained by separation of variables for the constant diffusion coefficient case with the following initial and boundary conditions.

**Initial condition:**
\[
C = C(x, 0) \quad \text{at } x = x \text{ and } t = 0
\]

**1st boundary condition:**
\[
C = C(0, t) = Cs(t) \quad \text{at } x = 0 \text{ and } t = t
\]

**2nd boundary condition:**
\[
C = C(2L, t) = Cs(t) \quad \text{at } x = 2L \text{ and } t = t
\]

where $C$ = moisture concentration, kg/m$^3$, $C(x, t)$ = concentration in any location and at any time, kg/m$^3$, $Cs(t)$ = surface moisture concentration at any time, kg/m$^3$, $L$ = half thickness of specimen, m, $t$ = drying time, s, $x$ = a space of the geometry, m.

Assumptions used in deriving the solution are that dimensional change in the moisture movement direction does not occur, and that the diffusion coefficient is constant in each drying time interval. By redefining the fractional change in moisture concentration ($E$) as
\[
E = \frac{C(x, t) - Cs(t)}{C(x, 0) - Cs(t)}
\]

By integration of Eq. (2) over the half thickness from 0 to $L$, the fractional change in the average moisture concentration ($E$) becomes
\[
\bar{E} = \frac{C_{AVG}(t) - Cs(t)}{C_{IMC_i} - Cs(t)}
\]

where $C_{AVG}(t) = \text{average moisture concentration in wood, kg/m}^3$, and $C_{IMC_i} = \text{initial moisture concentration in wood in each drying time interval, kg/m}^3$. Because water concentration in wood can be presented by
\[
C(x, t) = G_{MC} \rho_w MC(x, t)/100,
\]
where $G_{MC} = \text{specific gravity of moist wood, } \rho_w = \text{water density, kg/m}^3$, and $MC = \%\text{moisture content,}$ the diffusion coefficient ($D$) in each drying time interval can be calculated as follows:
\[
D = \frac{4 \cdot L^2}{\pi^2} \left( \frac{G_{i} IMC_i - G_{S} SMC(t)}{G_{A} AMC(t) - G_{S} SMC(t)} \right)
\]

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\]

where $IMC_i = \%\text{initial moisture content in each drying time interval, SMC and AMC = } \%\text{surface and } \%\text{average moisture content, } G_i, G_s, \text{ and } G_A = \text{specific gravity of wood at IMC}_i, \text{ SMC, and AMC.}$

**Mass transfer coefficients.**—Mass transfer coefficient can be expressed in several ways, depending upon the potential of the gradient.

**Surface emission coefficient.**—The surface
emission coefficients, mass transfer coefficient based on moisture concentration in wood, were experimentally determined by

\[ S_{\text{exp}} = \frac{J}{(C_{\text{water}} - C_{\text{wood}})} = \frac{\Delta W/(A \cdot \Delta t)}{G_s \cdot \rho_v \cdot \text{SMC}/100 - G_{\text{E}} \cdot \rho_v \cdot \text{EMC}/100} \]

where \( S_{\text{exp}} = \) experimentally determined surface emission coefficient, \( m/s, J = \) moisture flux from surface to air, \( kg/s \ m^2, C_{\text{water}} = \) moisture concentration in wood, \( kg/m^3, C_{\text{wood}} = \) moisture concentration in wood at equilibrium with surrounding conditions, \( kg/m^3, W = \) mass of evaporated moisture, \( kg, A = \) surface area of specimen, \( m^2, \text{EMC} = \% \text{equilibrium moisture content}, \) and \( G_{\text{E}} = \) specific gravity of wood at EMC. The \( S_{\text{exp}} \) were compared with mass transfer coefficients calculated by boundary layer theory (\( h_{\text{air}} \)).

Mass transfer coefficient calculated by boundary layer theory (\( h_{\text{air}} \)),

\[ h_{\text{air}} = \frac{0.66 \cdot D_{H_2O,\text{air}} \cdot \text{Re}^{1/2} \cdot \text{Sc}^{1/3}}{L_s} \]

(for laminar flow, Geankoplis 1993; Siau 1995) where \( h_{\text{air}} = \) mass transfer coefficient based on moisture concentration in air, \( m/s, D_{H_2O,\text{air}} = \) diffusion coefficient of water vapor in air \( = 2.2 \cdot 10^{-5}(1.013 \cdot 10^5/P) \cdot (T/273)^{1.75}, \) \( m^2/s, P = \) total pressure of air and water vapor, \( Pa, T = \) temperature, \( K, \text{Re} = \) Reynolds number \( = (L \cdot v \cdot \rho_v)/\mu, \text{Sc} = \) Schmidt numbers \( = \mu/(\rho_v \cdot D_{H_2O,\text{air}}), L_s = \) length of surface along which convection occurs, \( m, v = \) air velocity, \( m/s, \rho_v = \) density of air, \( kg/m^3, \) and \( \mu = \) dynamic viscosity of air, \( Pa \cdot s. \)

Conversion between mass transfer coefficients.—Because \( S_{\text{exp}} \) is based on the moisture concentration in wood, and \( h_{\text{air}} \) is based on the moisture concentration in air, a conversion method from \( h_{\text{air}} \) to surface emission coefficient is required to facilitate making comparisons to each other. Siau (1995) believed that water vapor pressure on the surface of wood (\( p_s \)) in the hygroscopic range could be determined by multiplying the saturated vapor pressure of water at dry bulb temperature (\( p_{\text{dry}} \)) by relative humidity at the surface (\( RH_s \)). This calculation, however, resulted in large deviations between experimental and theoretical surface emission coefficients. Siau’s method overestimates the \( p_s \) because when very wet wood surfaces are being dried to the fiber saturation point (FSP), the \( p_s \) is close to the saturated vapor pressure at wet bulb temperature \( p_{\text{wet}} \) not \( p_{\text{dry}} \). Correspondingly, when the surface is being dried from FSP to EMC in the hygroscopic range, the \( p_s \) might decrease from \( p_{\text{wet}} \) to vapor pressure equilibrated to ambient air \( (p_a) \), which can be determined by multiplying \( p_{\text{dry}} \) by environmental relative humidity \( (RH_a) \). Using this concept, a new conversion method between mass transfer coefficients has been developed (Yeo 2001). \( p_s \) can be determined, using the relationship of \( (p_{\text{wet}} - p_a) / (p_{\text{wet}} - p_d) = (100 - RH_a)/(100 - RH_s) \), as

\[ p_s = p_{\text{wet}} - \left( \frac{p_{\text{wet}} - p_a}{100 - RH_a} \right) \cdot (100 - RH_s) \]

\[ = p_{\text{wet}} - \left( \frac{p_{\text{wet}} - p_{\text{dry}} \cdot \left( \frac{RH_a}{100} \right)}{100 - RH_a} \right) \times (100 - RH_s) \]

where \( p_d \) and \( p_a \) = water vapor pressure in air adjacent to surface of wood and in ambient air, \( Pa, p_{\text{wet}} \) and \( p_{\text{dry}} = \) saturated water vapor pressure at wet bulb and at dry bulb temperature, \( Pa, \) and \( RH_a \) and \( RH_s = \% \) relative humidity of air adjacent to surface of wood and of ambient air.

Moisture concentration in air adjacent to a wood surface \( (C_{\text{air}}) \) and moisture concentration in ambient air \( (C_{\text{water}}) \) can be calculated by the gas law, \( C_{\text{air}} = (M_{H_2O} \cdot p_a)/(R \cdot T_a) \), and \( C_{\text{water}} = (M_{H_2O} \cdot p_a)/(R \cdot T_d) \), where \( M_{H_2O} = \) water molecular weight, 18 kg/kmol, \( R = \) gas law constant, 8,314 m³Pa/kmol K, and \( T_a \) and \( T_d = \)
Moisture content profiles in wood

Actual MC profiles.—Fully saturated maple, oak, and pine specimens with 200 (longitudinal) × 50 (radial) × 50 (tangential)-mm dimensions were wrapped in plastic at 30 and 50°C conditions prior to drying. Also, prior to each drying experiment, the specimens were end coated with a fibered asphalt compound (St. Louis Paint Mfg. Co.) to provide two-dimensional only, radial and tangential, moisture flow. MC profiles in each specimen were measured by the slicing method, with 7 sections each in the radial and tangential directions cut and MCs of those sections determined by oven-drying.

Simulated MC profiles.—Two-dimensional MC profiles in wood during drying were simulated by the finite difference method (FDM). The computer simulation program utilized DELPHI updated PASCAL program language. Moisture concentration equations in this program were solved by implicit formulation and Gauss-Seidel iteration.

RESULTS

Colorimetric technique

Surfaces of CoCl₂-treated wood exhibit a light yellow-red color at high MC, which changes to a dark green-blue color at low MC. Spectral reflectance from the surfaces was measured with the spectrophotometer. Surfaces of treated wood at higher MC reflected particularly well longer wavelength light. Using reflectance data and wood hygroscopicity, color value change on the CoCl₂-treated moist wood surface could be plotted in CIE L*a*b* color space. L*, a*, and b* values continuously decrease over the high to low MC range.

Surface and average MC and specific gravity

Using the colorimetric technique and oven drying, SMC and AMC were determined, respectively, during drying at 25% RH and both 30 and 50°C (Fig. 1). Green specific gravity of hard maple, red oak, and southern pine were 0.47, 0.56, and 0.44, and their oven-dry specific gravities were 0.53, 0.64, and 0.50, respectively. Specific gravity of moist wood that was approximated by linear interpolation between green and oven-dry specific gravity has been used to determine diffusion and surface emission coefficients. From Fig. 1, it appears that faster surface drying takes place in the tangential and radial direction than longitudinal. This seems to be because of low permeability in the tangential orientation. The continuity of liquid water in the tangential direction was more likely to be broken at the very beginning of the drying process. This may make it look as though faster drying takes place in the tangential direction. But broken
Longitudinal Water Movement

- AMC (30°C)
- SMC (30°C)
- AMC (50°C)
- SMC (50°C)

Radial Water Movement

- AMC (30°C)
- SMC (30°C)
- AMC (50°C)
- SMC (50°C)

Tangential Water Movement

- AMC (30°C)
- SMC (30°C)
- AMC (50°C)
- SMC (50°C)

Drying Time (Hour \( \frac{1}{2} \))

Fig. 1. Average and surface moisture contents of hard maple whose moisture movement is constrained to either longitudinal, radial, or tangential directions during drying at 25% RH and 30°C or 50°C.

During drying at 30°C and 25% RH, the diffusion coefficients in the longitudinal direction of each species remained constant in the range from maximum MC to 30% AMC, and then decreased to EMC (Fig. 2). These results might indicate that continuity of liquid water exists in the longitudinal direction from initial moisture content to 30% AMC, and that the longitudinal movement of moisture in that range is by capillary action. The diffusion coefficient in the transverse direction for each species, except the radial direction with hard maple, decreased from over 100% AMC to EMC. This could indicate that continuity of liquid water in the radial direction of red oak and southern pine and the tangential direction of each species was broken at the very beginning of the drying process. Major resistance to water movement in this case might be evaporation and condensation of moisture in wood and so governed by water vapor pressure and bound water gradients. The diffusion coefficient in the radial direction of hard maple remained constant to about 60% AMC, and then decreased to EMC. In this case continuity of liquid water exists in the radial direction to about 60% AMC. This result might be due to the anatomical structure of hard maple where ray cells may support continuous free water movement relatively well in the radial direction. Over the entire AMC range, diffusion coefficients decreased during drying, which means that internal resistance increased. Although the data are not presented, diffusion coefficients determined during drying at 50°C and 25% RH condition were twice as high as those determined during drying at 30°C and 25% RH.

**Mass transfer coefficient**

Experimentally determined surface emission coefficients \( (S_{em}) \) — Relationships between S_{em} and SMC of maple, oak, and pine were observed while drying at 30°C and 50°C with 25% RH (Fig. 3). As drying progressed, the coefficients slightly increased when above the FSP range and then rapidly increased from FSP to EMC. Also, a positive effect of temperature on the surface emission coefficient was found, as the coefficient values at 50°C were greater than those at 30°C. These results show that surface emission coefficients were strongly dependent upon the SMC and air temperature as well as air velocity, which is known as a critical factor.

Mass transfer coefficient calculated by boundary layer theory \( (h_{wb}) \) — After the physical properties of air, such as velocity, density,
and viscosity, were measured and surveyed, mass transfer coefficients were calculated by boundary layer theory ($h_{\text{air}}$). Air velocities in the environmental chambers were 1 m/s. The surface lengths of specimens along which convection occurs were 0.02 m. Air densities at 30°C (303K) and 50°C (323K) were 1.167 and 1.096 kg/m³, respectively. Air viscosities at 30°C and 50°C were $1.87 \times 10^{-5}$ and $1.96 \times 10^{-5}$ Pa·s, respectively. Based on those data, diffusion coefficients of water vapor in air ($D_{\text{H}_2\text{O,air}}$) were $2.64 \times 10^{-5}$ m²/s at 30°C and $2.95 \times 10^{-5}$ m²/s at 50°C. Reynolds number ($Re$, 1,248 at 30°C and 1,118 at 50°C) and Schmidt number ($Sc$, 0.6069 at 30°C and 0.6062 at 50°C) were determined. Using those values, mass transfer coefficients ($h_{\text{air}}$) were determined to be 0.026 and 0.028 m/s at 30 and 50°C, respectively. Because the $D_{\text{H}_2\text{O,air}}$ used to calculate the $h_{\text{air}}$ are constant coefficients for interdiffusion of water vapor in bulk air, and the $Re$ and $Sc$ numbers are also constant, the $h_{\text{air}}$ values become constant. This helps explain the external moisture-evaporating rate from very wet wood surfaces and might lead one to conclude that the external resistance is constant at very high SMC condition. However, because there is no consideration of water vapor pressure change with surface moisture content in the hygroscopic range, the use of constant $h_{\text{air}}$ is not good for explaining the resistance of moisture-evaporating rate from the surfaces when SMC is low, especially below FSP.

The previously developed mass transfer coefficient conversion method (Eq. 7) considers water vapor pressure change with surface moisture content in the hygroscopic range. To determine whether boundary layer theory is useful for evaluating the external moisture evaporation...
sistance during wood drying or not, the $S_{\text{beo}}$ converted from $h_{\text{aw}}$ with the mass transfer coefficient conversion method has been compared with $S_{\text{exp}}$.

Surface emission coefficient theoretically converted from $h_{\text{aw}}$ ($S_{\text{beo}}$).—For example, with hard maple dried at 30°C and 25% RH, the wet bulb temperature was 16.8°C (290K); 140%, 29%, and 6.2% MC were used for initial surface moisture content, fiber saturation point, and equilibrium moisture content; 0.47 and 0.33 were used for green and oven-dry specific gravity; and 1,923 Pa ($p_{\text{wet}}$) and 4,249 Pa ($p_{\text{dry}}$) were used for the saturated vapor pressure of water in air at wet bulb temperature 16.8°C and dry bulb temperature 30°C, respectively. Since RH was 25% RH, water vapor pressure in ambient air ($p_a$) was 1,062 Pa = 4,249(25/100). Using those values, $h_{\text{aw}}$ were converted to surface emission coefficients. These theoretically converted surface emission coefficients ($S_{\text{beo}}$) were quite close to $S_{\text{exp}}$ (Fig. 3). This proves that boundary layer theory with conversion method is useful for evaluating the external moisture resistance during wood drying.

Relationships between surface emission coefficient and SMC have been rarely observed in the past. While some papers have addressed the relationship between surface emission coefficient and AMC, it is doubtful whether AMC has a critical effect on external resistance, which includes surface resistance. Yeo has compared some experimentally determined constant surface emission coefficients in certain MC ranges, by Choong and Skaar (1969 and 1972), with theoretically converted surface emission coefficients (Yeo 2001). Results of this comparison have shown that surface emission coefficient can be determined theoretically with boundary layer theory and appropriate conversion method. More comparisons like this might be performed in further studies.

**MC profiles in wood**

Overall, the moisture profiles simulated by finite difference method were quite comparable to the actual moisture profiles in real dried wood. Results for maple at 30°C are illustrated in Fig. 4. This substantiates the high credibility of using the colorimetric technique for determining surface moisture content and mathematical procedure for determining the diffusion and surface emission coefficient.

**CONCLUSIONS**

Use of the colorimetric CoCl$_2$-treated wood technique for determining surface moisture content of wood nondestructively and continuously in unsteady state drying conditions could increase the credibility of methods for calculating the internal and external resistances for moisture movement in, and surface evaporation from, wood.

Experimental results showed that diffusion coefficients were dependent upon the temperature and average moisture content in wood.
Also, the surface emission coefficients were shown to strongly depend on the surface moisture content of wood and air temperature as well as air velocity, which has been known as a critical factor.

The mass transfer coefficient conversion method developed using surface moisture data in this study has proven that the boundary layer theory is useful for evaluating the external resistance during wood drying. Because of this proof, the external resistance for wooden surfaces can be theoretically described in dynamic drying situations.

The moisture profiles simulated by finite difference method were quite comparable to the actual moisture profiles in real dried wood, which substantiates the credibility of using the colorimetric technique for determining surface moisture content and mathematical procedure for determining the diffusion and surface emission coefficient.

REFERENCES


