ESTIMATING LOCAL COMPLIANCE IN A BEAM FROM BENDING MEASUREMENTS PART II. OPTIMAL ESTIMATION OF LOCAL COMPLIANCE

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ABSTRACT

We present a method for optimally estimating local elasticity properties along a beam where each estimate is specific to an increment in a subdivision of the beam's length. Previous research indicates that knowledge of localized elasticity values can improve the estimation of strength. Immediate application is expected in the machine stress-rated (MSR) lumber production process. A sequence of bending measurements on overlapping bending spans, as commonly obtained in the MSR process, serves as input to the estimation method.

The sequence of bending measurements is modeled as an autoregressive moving average (ARMA) random process. Autoregression coefficients are estimated from *a priori* information and refined as additional data are obtained. Moving average weight coefficients come from span functions computed by methods in Part I. A Kalman filter, defined from coefficients of the ARMA process, is applied to the measurements, and local estimates are obtained.

Estimated local elasticity results are presented for both a simulated and a real wood beam. One set of experiments shows that as a modeled correlation coefficient is decreased from an artificially high value, the result evolves from local elasticity estimates that appear much the same as measured elasticity, but without an obvious noise component, to local estimates having more detail. This leads naturally to a suggestion for a practical, non-disruptive introduction of the estimation method to a MSR production line. Grade yield improvement is likely an immediate benefit along with a capability for further research into the estimation method and grading algorithms.

Keywords: Local E, compliance, bending, Kalman, estimation, optimum, ARMA, state-space.

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INTRODUCTION

Our method of estimating local elastic properties in a beam requires use of span functions as defined and computed in Part I. As part of that development, measured compliance at a mea-

surement point w along a beam was written as a convolution integral involving local compliance and a span function. Here, where sampled data are used instead of continuous data, use is made of the discrete form of the convolution:

$$C_m(j) = \sum_i C(j - i)\, h_j(i) \qquad (1)$$

where the definitions of local C and measured C_m compliances are the same as in Part I but in discrete form. In this discrete convolution, the beam length has been considered subdivided into N equal length increments and the bending span into p equal length increments each identified by a discrete index. For example, $C_m(j)$ identifies measured compliance for a measurement point at the j^{th} increment of the beam, and C(u) represents the compliance of the u^{th} increment of the beam, both of these integer indices being beam coordinates increasing to the left from the beam's leading end. The weight coefficient $h_j(i)$ is given by the integral of the j^{th} span function over the i^{th} incremental length of its domain, where integer i is a span coordinate increasing to the right from a span reference.

$$h_j(i) = \int_{i^{th}\,\text{increment}} {}_c h_j(v)\, dv \qquad (2)$$

The leading subscript c in the integrand function ${}_c h_j$ indicates that this span function has a continuous argument as in Part I. Otherwise, here in Part II, discrete values such as $h_j(i)$ are the result of an integration as in Eq. (2). The subscript index j is introduced into the span function to designate that the span function corresponding to the j^{th} measurement point is used. The applicable span function depends on the bending span support configuration, and that can depend on the measurement point. In cases of interest, the extent of the span function domain changes, and it is convenient to define it over the largest domain used, letting the increment weights computed from Eq. (2) be zero where appropriate. When the span reference is aligned with the j^{th} increment along the beam, the i^{th} weight $h_j(i)$ is applied to the local compliance $C(j - i)$. As is true in a convolution integral with the dummy variable of integration, the dummy index of summation for one component of the convolution sum in Eq. (1) runs in the opposite direction from the other.

Others (Foschi 1987, Lam et al. 1993, and Pope and Matthews 1995), applied a Fourier transform method previously identified (Bechtel 1985) to obtain solutions for local elasticity values using the convolution relationship of Eq. (1). Frequency truncation was used to reduce unwanted high frequency noise amplified by this method. Pope and Matthews (1995) concluded that the computed local values were only marginally better than measured values for estimating bending strength. But, while frequency truncation is useful in reducing noise, it is suboptimal. An optimal constrained deconvolution method (Rosenfeld and Kak 1982) using Discrete Fourier transforms and a Wiener filter has also been contemplated (Bechtel et al. 2000). However, any of these Fourier transform methods introduces other issues, one of which is Gibb's phenomenon (Guillemin 1949), a discrepancy that occurs near discontinuities such as at the ends of data having finite extent, as in the present case of finite beam length.

KALMAN FILTER METHOD

One of last century's classic papers (Kalman 1960) described an estimation method that has been applied in many different fields. Kalman's derivation may be found in his original paper, and some details elsewhere (Eubank 2006, Bechtel 2005, and Kailath et al. 2000). The problem of estimating local compliance can be put into a framework for which Kalman's work is applicable.

The approach consists of representing local compliance values along a beam as samples of an autoregressive (AR) random process. This introduces structure that statistically relates local compliance values along a beam to one another. Sequential measurements of beam compliance taken as a measured beam moves relative to a bending span are represented as weighted mov-

ing averages (MA) of the local values. The MA weights are obtained from Eq. (2) using applicable span functions as defined in Part I. The measurement sequence is therefore an autoregressive moving average (ARMA) random process plus measurement noise, and it can be modeled as the output of a linear dynamic system represented in state-space format (Schwarz and Friedland 1965, Ogata 1987). Components of a state vector correspond with local compliance values. In state-space format, the model is ready for application of a Kalman filter. The Kalman filter recursively uses a previous state vector estimate and a new compliance measurement to compute an updated state vector estimate along with its covariance matrix. Because local compliance values are in correspondence with state vector components, local compliance estimates and their variances are obtained.

LOCAL COMPLIANCE VALUES AS SAMPLES OF AN AUTOREGRESSIVE RANDOM PROCESS

Referring to Fig. 1, an uncorrelated random noise source u(k) is the input to the upper block diagram. The blocks labeled z^{-1} are delay elements so that after one unit of delay, the signal at the left of each of these blocks appears at its right. A beam marked **2** in Fig. 1 is shown with oppositely hatched increments in a subdivision of its length. It moves to the right by an amount equal to an increment length in a time equal to one unit of delay. Thus, compliance $c(j + 2)$ for a length increment in correspondence with signal $s_2(k)$ at measurement stage k is in correspondence with signal $s_1(k + 1)$ at measurement stage $k + 1$, one unit of delay later. A part of the signal after each delay is added (with negative sign) to the input. This autoregressive system is shown at the k^{th} stage where noise input u(k) at sampling time k is just entering the system. At that moment, the signal at point **11** in Fig. 1 is given by $s_p(k + 1) = u(k) - a_1 s_p(k) - a_2 s_{p-1}(k) - \cdots - a_p s_1(k)$. Thus, the system evolves according to the matrix equation:

$$s(k + 1) = \phi(k)s(k) + U(k) \qquad (3)$$

where:

$$s(k) = \begin{bmatrix} s_1(k) \\ s_2(k) \\ \vdots \\ s_{p-1}(k) \\ s_p(k) \end{bmatrix},$$

$$\phi = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ -a_p & -a_{p-1} & \cdots & -a_2 & -a_1 \end{bmatrix},$$

$$U(k) = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ u(k) \end{bmatrix} \qquad (4)$$

At the k^{th} stage, s(k) is a state vector, with components called state variables, and U(k) is an input vector with only the last component being nonzero. Here, the state matrix ϕ in Eqs. (4) is not a function of measurement stage k, but it could be, as indicated in Eq. (3). Figure 1 shows that the compliance in each increment of the beam subdivision within the bending span is in correspondence with a component of the state vector. As the beam moves to the right by one increment, each of these local compliances shifts its correspondence to the next lower indexed component of the state vector according to the state matrix ϕ and Eq. (3) and illustrated in Fig. 1. The state vector s(k) may be considered a sample function of an autoregressive random process at the k^{th} measurement time. By adjusting the regression coefficients a_i, the statistical properties of this autoregressive random process can be made to agree with statistics describing the local compliance structure for a population of beams.

COMPLIANCE MEASUREMENT AS A MOVING AVERAGE

In the lower part of Fig. 1, the extent **14** of the bending span between first and last supports is

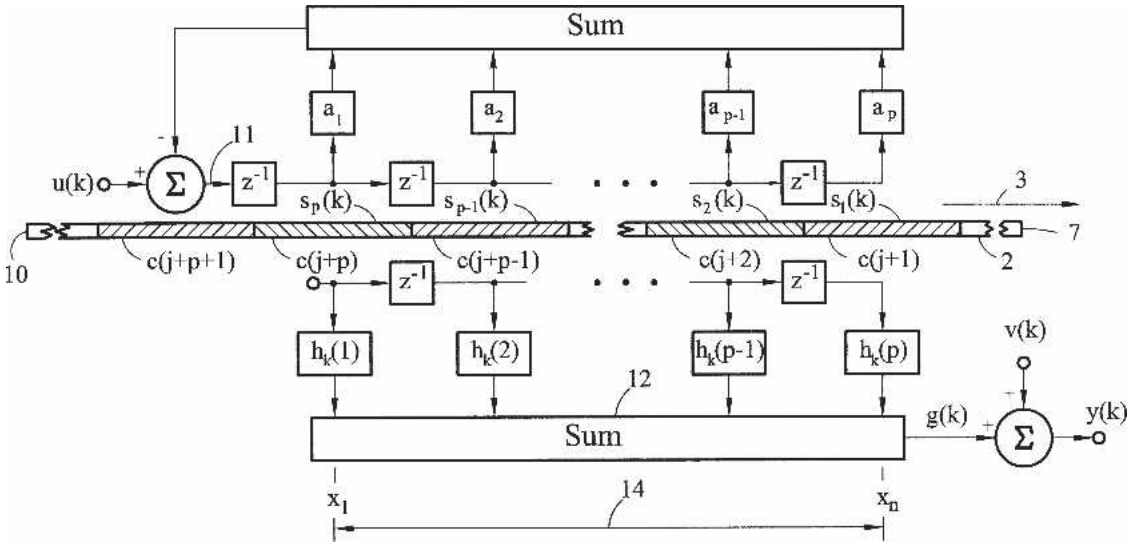

FIG. 1. Upper block diagram illustrates model of local compliance as an autoregressive random process. Lower block diagram illustrates measured output as a moving average of local compliance values plus measurement noise. Subdivided beam in center illustrates correspondence between state variables and local compliances identified with oppositely hatched increments of the beam subdivision.

shown. The lower block diagram of Fig. 1 shows state variables (local compliances) weighted and summed to give a moving average that is summed with measurement noise v(k) to yield the measurement sequence y(k). In state-space jargon, this is the output equation:

$$y(k) = H_k s(k) + v(k) \tag{5}$$

The output matrix H_k is a single row matrix consisting of weight coefficients obtained from the span function by Eq. (2). The $H_k s(k)$ term should be interpreted as equivalent to Eq. (1) recognizing the correspondence between state variables and local compliances as illustrated in Fig. 1. The components of H_k are in inverse order from the order identified by the index i of Eq. (2), and the indexing is shifted so that indices are positive starting with one (a bookkeeping detail). For example, in $H_k = [h_k(p), \cdots, h_k(1)]$, the first component $h_k(1)$ computed from the span function ${}_c h_k$ is last in the list. As the beam moves to the right (corresponding to a different measurement point), a different overlapping set of local compliance values is identified with the state variables, and if a different span function is applicable, a different output matrix is used; hence, the k in H_k. The two block diagrams of Fig. 1 can be combined to give the block diagram of Fig. 2, which illustrates the output as an ARMA random process. Equations (3) and (5) define the state-space system used in setting up the Kalman filter for optimally estimating local compliance.

KALMAN FILTER

The Kalman filter output, at any measurement stage, is the optimal estimate of the state vector in a linear dynamic system model. Kalman (1960) discusses the sense in which the estimate is optimal. The Kalman filter estimator is the minimum-variance, unbiased, linear estimator given the data. The variance of the residual error for every component of the state vector is minimized. The Kalman filter begins its recursive process with a present best estimate of a state vector based on available data, predicts based on the state-space model defining the system dynamics what the state vector will be at the next stage, and then uses new data to correct that prediction.

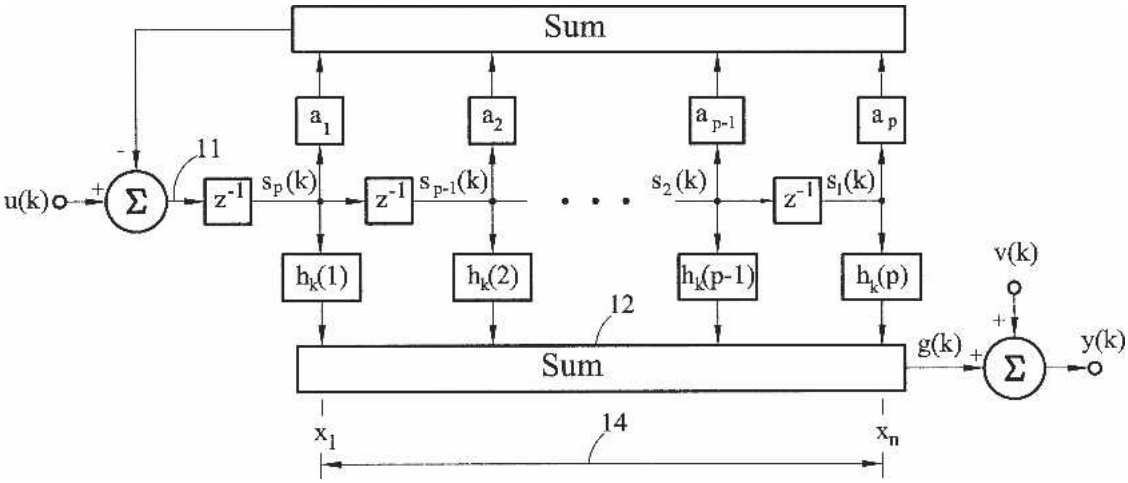

FIG. 2. The block diagrams of Fig. 1 combined to show the output as an autoregressive moving average random process.

The output Eq. (5) models each compliance measurement as a noisy linear combination of the state variables. The measurement is used to update the estimated state vector at each measurement stage. So long as a local compliance is identified with one of the state variables, it continues to contribute to the output and hence to the estimate at that stage. When the beam progresses far enough to the right so that a particular compliance will no longer contribute (e.g. in Fig. 1, a local increment is ready to exit the bending span at the right), then the corresponding component of the estimated state vector is taken as the local compliance estimate for that increment of the beam. Usually, this will be the first component of the state vector. However, when the trailing end of the beam is ready to clear the leftmost support for which a compliance measurement can be made, then the remaining local compliances are estimated through their correspondence with remaining components of the state vector.

The Kalman filter is derived for zero mean random processes, but compliance values for real beams are not zero mean. Consequently, a preliminary step removes the mean from the data before invoking the Kalman filter. It was found that the best mean estimate for a particular beam is its first measured value. Hence, each measured compliance datum entering the Kalman filter first has the initial datum subtracted from it. After the Kalman estimates are obtained, the initial datum is added back in to give the local compliance estimates. In the notation a small c denotes local compliance minus the initial datum and capital C denotes local compliance including the initial datum. Estimated local values are denoted with a superscript asterisk; for example C* is the estimate of compliance value C.

Kalman requires an initial estimate for the covariance of the state vector. State vector components are in correspondence with local compliances; hence, estimates of the state vector covariance will be based on known or measured compliance data. Because the mean of the state vector is zero, the covariance denoted P(k) at measurement stage k, using Eq. (3), is given by:

$$\begin{aligned} P(k) &= E[s(k)s^T(k)] \\ &= E[(\phi s(k-1) + U(k-1)) \\ &\quad (\phi s(k-1) + U(k-1))^T] \\ &= \phi E[s(k-1)s^T(k-1)]\phi^T \\ &\quad + E[U(k-1)U^T(k-1)] \\ &= \phi P(k-1)\phi^T + Q(k-1) \end{aligned} \tag{6}$$

In Eq. (6), E is the expectation operator, and use is made of the fact that the input noise u(k) is white noise. Therefore at stage k, u(k) is uncorrelated with the state vector s(k) because from

Eq. (3), s(k) is a combination of previous values u(k − 1), u(k − 2), and so on. Hence, cross terms in Eq. (6) are zero. Q(k) is the covariance matrix of the input noise vector U(k) and is written:

$$Q(k) = E[U(k)U^T(k)] = \begin{bmatrix} 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & \text{var}(u(k)) \end{bmatrix} \quad (7)$$

Assuming stationary processes for the present initialization purposes, Eq. (6) may be written:

$$P = \phi P \phi^T + Q \quad (8)$$

which has been called a discrete Lyapunov equation (Kailath et al. 2000). The definitions of ϕ, P, and Q are called consistent if they satisfy Eq. (8). For example, it may be verified that Eq. (8) is satisfied in a simple model where all of the a_i in ϕ are set to zero except for $a_1 = -\rho$, and P is given by:

$$P = \sigma^2 T \quad \text{where} \quad \sigma^2 = \frac{\text{var}(u)}{1 - \rho^2}, \quad \text{and}$$

$$T = \begin{bmatrix} 1 & \rho & \cdots & \rho^{p-1} \\ \rho & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \rho \\ \rho^{p-1} & \cdots & \rho & 1 \end{bmatrix} \quad (9)$$

In more complicated models, the discrete Lyapunov equation can be solved by use of programs supplied by The Mathworks, Inc. in Natick, MA, to give consistent definitions of ϕ, P, and Q. It is noted that ρ is the correlation coefficient of compliances in adjacent increments.

Initialization steps include determining variance var(v(k)) of the measurement noise, variance σ^2 for the state variables and the coefficients a_i for the autoregression model. In the simple model above, the variance of the input noise source u(k) may be found from the second of Eqs. (9) in terms of other quantities. The estimation of coefficient $\rho = -a_1$ may be guided by previous work involving correlation between adjacent beam segments (Kline et al. 1986; Richburg and Bender 1992; Taylor and Bender 1989 and 1991; Hernandez et al. 1992; Taylor et al. 1992). Alternatively, there is a method of finding the autoregression coefficients from measured compliance data. First, the autoregression function of measured compliance is obtained from tests on beams from a similar population, and then a system of equations similar to the Yule-Walker equations (Papoulis 1991) is used to solve for the a_i coefficients. Details are described in a United States patent (Bechtel et al. 2006). Variance of local compliances σ^2 may be estimated as the variance of measured compliance in a beam population. A guess based on the coefficient of variation of measured modulus of elasticity may be a reasonable starting point. The variance of measurement noise is based on experience with the equipment involved.

Recursive steps in the Kalman filter

The Kalman filter is recursive with initialization and then iterative computations.

Initialization.—Define the initial measurement stage index k_i (a bookkeeping detail). Set initial state vector estimate to zero: $s^*(k_i) = 0$. Compute Q and the initial $P(k_i)$ to satisfy Eq. (8). Take first measurement $C_m(k_i)$. Set $k = k_i$.

Iterative computations.—Set $k = k + 1$. Compute covariance matrix $P^*(k) = \phi P(k-1)\phi^T + Q$. Compute Kalman Gain $K(k) = P^*(k)H_k^T[H_k P^*(k) H_k^T + \text{var}(v)]^{-1}$. Compute matrix factor $F(k) = I_p - K(k)H_k$, where I_p is a p-dimensional identity matrix. Take measurement $C_m(k)$, and compute $y(k) = c_m(k) = C_m(k) - C_m(k_i)$. Compute state estimate. $s^*(k) = F(k)\phi s^*(k-1) + K(k)y(k)$. Compute covariance matrix $P(k) = F(k)P^*(k)$. If $k = k_f$, where $C_m(k_f)$ is the last valid measurement, stop; else, repeat the above iterative steps.

Kalman filter details

There is a subtle distinction between the covariance matrices P(k) and P*(k) used in the Kalman recursion. P(k) is the covariance of the

residual error between the state vector s(k) and the optimal estimator s*(k) for it at the k^{th} stage. The optimal estimator uses all the data up through the measurement y(k). The notation s*(k|k) would be more consistent with Kalman's paper, (Kalman 1960). P*(k) is the covariance of the residual error between the state vector s(k) and another optimal estimator s*(k|k − 1) for it at the k^{th} stage, but this second estimator uses measurements only up through y(k − 1) not including y(k). The estimate is predicted by the dynamic system but not improved by information in the measurement y(k). Details are explained more fully elsewhere (Bechtel 2005). Both P(k) and P*(k) are used in the recursion. The diagonal elements of P(k) are the variances of the corresponding components of the state vector at the k^{th} measurement stage. They are used to obtain the residual error variances of the estimators used for local compliances.

Optimal local compliance estimates are obtained through their correspondence with state variables as illustrated in Fig. 1. Table 1 illustrates practical bookkeeping details for one example.

The local compliance estimate C*(j) for the j^{th} increment of the beam subdivision may be used to provide a local estimate for the modulus of elasticity E*(j) of that increment according to:

$$E^*(j) \cong [1 + COV_C{}^2(j)]/C^*(j) \qquad (10)$$

where $COV_C(j) = (var(C(j)))^{1/2}/C^*(j)$ is the coefficient of variation of the estimator for C(j).

TABLE 1. *Bookkeeping example.*

k	Output matrix	Compliance measurement	Kalman filter input	Local compliance estimate	Residual error variance
12	H1	$C_m(12)$	$c_m(12) = 0$		
13	H1	$C_m(13)$	$c_m(13) = C_m(13) - C_m(12)$		
14	H1	$C_m(14)$	$c_m(14) = C_m(14) - C_m(12)$		
15	H1	$C_m(15)$	$c_m(15) = C_m(15) - C_m(12)$		
16	H2	$C_m(16)$	$c_m(16) = C_m(16) - C_m(12)$		
17	H2	$C_m(17)$	$c_m(17) = C_m(17) - C_m(12)$		
18	H2	$C_m(18)$	$c_m(18) = C_m(18) - C_m(12)$		
19	H3	$C_m(19)$	$c_m(19) = C_m(19) - C_m(12)$	$C^*(1) = s_1^*(19) + C_m(12)$	$P_{1,1}(19)$
20	H3	$C_m(20)$	$c_m(20) = C_m(20) - C_m(12)$	$C^*(2) = s_1^*(20) + C_m(12)$	$P_{1,1}(20)$
⋮	⋮	⋮	⋮	⋮	⋮
k	H3	$C_m(k)$	$c_m(k) = C_m(k) - C_m(12)$	$C^*(k - 18) = s_1^*(k) + C_m(12)$	$P_{1,1}(k)$
⋮	⋮	⋮	⋮	⋮	⋮
$k_f - 7$	H3	$C_m(k_f - 7)$	$c_m(k_f - 7) = C_m(k_f - 7) - C_m(12)$	$C^*(k_f - 25) = s_1^*(k_f - 7) + C_m(12)$	$P_{1,1}(k_f - 7)$
$k_f - 6$	H4	$C_m(k_f - 6)$	$c_m(k_f - 6) = C_m(k_f - 6) - C_m(12)$	$C^*(k_f - 24) = s_1^*(k_f - 6) + C_m(12)$	$P_{1,1}(k_f - 6)$
$k_f - 5$	H4	$C_m(k_f - 5)$	$c_m(k_f - 5) = C_m(k_f - 5) - C_m(12)$	$C^*(k_f - 23) = s_1^*(k_f - 5) + C_m(12)$	$P_{1,1}(k_f - 5)$
$k_f - 4$	H4	$C_m(k_f - 4)$	$c_m(k_f - 4) = C_m(k_f - 4) - C_m(12)$	$C^*(k_f - 22) = s_1^*(k_f - 4) + C_m(12)$	$P_{1,1}(k_f - 4)$
$k_f - 3$	H5	$C_m(k_f - 3)$	$c_m(k_f - 3) = C_m(k_f - 3) - C_m(12)$	$C^*(k_f - 21) = s_1^*(k_f - 3) + C_m(12)$	$P_{1,1}(k_f - 3)$
$k_f - 2$	H5	$C_m(k_f - 2)$	$c_m(k_f - 2) = C_m(k_f - 2) - C_m(12)$	$C^*(k_f - 20) = s_1^*(k_f - 2) + C_m(12)$	$P_{1,1}(k_f - 2)$
$k_f - 1$	H5	$C_m(k_f - 1)$	$c_m(k_f - 1) = C_m(k_f - 1) - C_m(12)$	$C^*(k_f - 19) = s_1^*(k_f - 1) + C_m(12)$	$P_{1,1}(k_f - 1)$
k_f	H5	$C_m(k_f)$	$c_m(k_f) = C_m(k_f) - C_m(12)$	$C^*(k_f - 18) = s_1^*(k_f) + C_m(12)$	$P_{1,1}(k_f)$
				$C^*(k_f - 17) = s_2^*(k_f) + C_m(12)$	$P_{2,2}(k_f)$
				⋮	⋮
				$C^*(k_f + 11) = s_{30}^*(k_f) + C_m(12)$	$P_{30,30}(k_f)$

The initial measurement index is $k = k_i = 12$. The first measurement point on the beam is $w = d(k - 1) + d/2 = 639$ mm (25.1 inch) from its leading end if $d = 55.54$ mm (2.19 inch), see Fig. 3 of Part I. The output matrix changes from H1 through H5 as the span function changes. The Kalman filter input shows subtraction of the initial compliance measurement from each measurement. Local compliance estimate and estimator variance come from the first state variable estimate s_1^* and first diagonal component of the covariance matrix P until the end of the beam where compliances and variances are associated with the other state variables. Other bookkeeping arrangements are possible; e.g. here, the first local compliance estimate is taken when the beam's leading end first contacts the last support at x_7 which doesn't occur until measurement index $k = 19$. From Fig. 4 of Part I, the span functions are trivially small for the first local compliance after the leading end passes support x_5, and the first estimate could have been taken as $s_1^*(12) + C_m(12)$ at index $k = 12$ instead of waiting for 19. In that case the number of state variables would be correspondingly reduced and processing would occur more rapidly.

Similarly, the coefficient of variation for E*(j) is obtained from:

$$COV_E(j) \cong COV_C(j)/[1 + COV_C{}^2(j)] \quad (11)$$

The correction factor $[1 + COV_C{}^2(j)]$ appears in Eqs. (10) and (11) because the mean of a reciprocal distribution is **not equal** to the reciprocal of the mean (Papoulis 1991).

Estimates including error variances are available for leading end increments of a beam while it is being sequentially processed. It is not necessary to wait until all measurements of a beam are completed. As soon as an increment no longer contributes to a measurement, an optimal estimate for its compliance can be obtained.

The sampling interval for the machine yielding the following test results was (13.9 mm) 0.547 inch. To reduce the processing time required, a decimation preprocessing step was performed to effectively increase the sampling interval by a factor of 4 to 55.5 mm (2.19 inch). A study of power spectral density data from this machine (Bechtel et al. 2000) showed a data bandwidth small enough that the Nyquist Sampling Theorem (Oppenheim and Schafer 1989) is well satisfied by this reduced sampling rate. Using a relatively slow microcomputer (400 MHz) and without code optimization effort, processing speeds were shown sufficient to easily keep up with 6 m/sec (20 ft/sec) line speeds. With a faster computer and some code optimization, much higher line speeds will be possible. There are identified code optimization steps that can be taken. For example, the measured data are used to estimate the state vector, but they are not used to update the Kalman gain K(k), matrix factor F(k), output matrix H_k or covariance matrices P(k) and P*(k). Computations involving these quantities at each iteration could be preprocessed, stored and results read from memory as required.

TEST RESULTS

Tests were performed with simulated and real data. A reduced model was used where the regression coefficient $a_1 = -\rho$, and all others were set to zero. The simulated data were used to test the robustness of the process by generating the data with a specified correlation coefficient ρ and using another for the Kalman filter processing. The experimentation showed that the process performs well if analysis uses a value of ρ different from that used to generate the data.

In Part I, compliance was defined as reciprocal of the EI product, and it was recognized there that this was to generalize the work. Here, with test data and results, moment of inertia I is assumed constant and absorbed into other constants, and compliance becomes the reciprocal of modulus of elasticity alone. Thus, local $C = 1/E$ and measured $C_m = 1/E_m$.

Figure 3 illustrates, in its upper part, a simu-

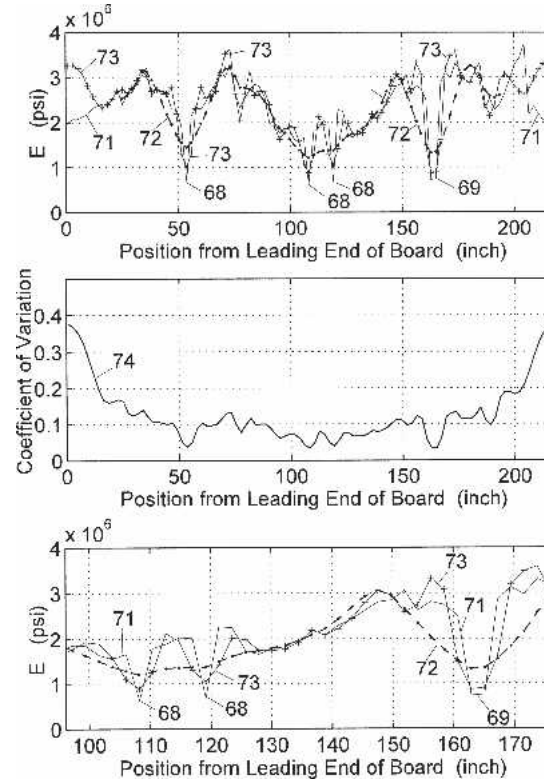

FIG. 3. Kalman filter applied to a simulated beam. Simulated local E is curve **71**, simulated measured E is curve **72**, and Kalman estimated E is curve **73**. Coefficient of variation of the estimator is curve **74**. Note that the estimated local E **73** follows the dips in the simulated local E **71** better than measured E **72**. Model of Fig. 2 was used, but with all AR coefficients set to zero except a_1.

lated local E function labeled **71** generated as the reciprocal of an autoregressive local compliance function outcome. Coefficients used to generate the local compliance function were mean = $(13.8\ \text{GPa})^{-1} = (2.00\ \text{Mpsi})^{-1}$ and variance $\sigma^2 = (34.5\ \text{GPa})^{-2} = (5.00\ \text{Mpsi})^{-2}$, the variance obtained assuming a compliance coefficient of variation of 0.4. Variance of the zero mean white noise random process u = u(k) in Fig. 2 is given by $\text{var}(u) = \sigma^2(1 - \rho^2)$ where $\rho = -a_1 = 0.97$ in Fig. 2 with all other autoregression coefficients set to zero. Thus $\text{var}(u) = (142\ \text{GPa})^{-2} = (20.6\ \text{Mpsi})^{-2}$. The zero mean white noise variates u(k) were generated from a lognormal pseudo-random number generator adjusted to have zero mean and variance as specified. The lognormal distribution was used because it provides a lower limit of generated variates. Then, when the mean is added to provide simulated local compliance values C, the reciprocal E = 1/C values are neither extraordinarily large nor negative. In an effort to better simulate knots, compliance pulses were added at defined points on the beam. Three compliance pulses having amplitude equal to twice the mean compliance value were added to the generated local compliance function sample values at 53.6, 108.2 and 119.2 inch from the leading end of the simulated beam. Also, a compliance pulse with twice the mean compliance amplitude was added to adjacent generated samples of the local compliance function at 162.9 and 165.1 inch, giving a pulse two samples wide.

The resulting local E = 1/C function is shown labeled **71** in Fig. 3. Relative minima associated with the single sample pulses are labeled **68** and the double sample wide pulse **69**. The lower part of Fig. 3 is an expanded version of the same plot. To simulate bending measurement by the bending span of the configuration in Part I, the simulated local compliance function was convolved with weights corresponding to the appropriate succession of span functions, Part I, Fig. 4. Simulated "measured" compliance C_m was obtained by adding pseudo-random white Gaussian noise having zero mean and variance given by $\text{var}(v) = (2180\ \text{GPa})^{-2} = (316\ \text{Mpsi})^{-2}$ to simulate measurement noise from this system.

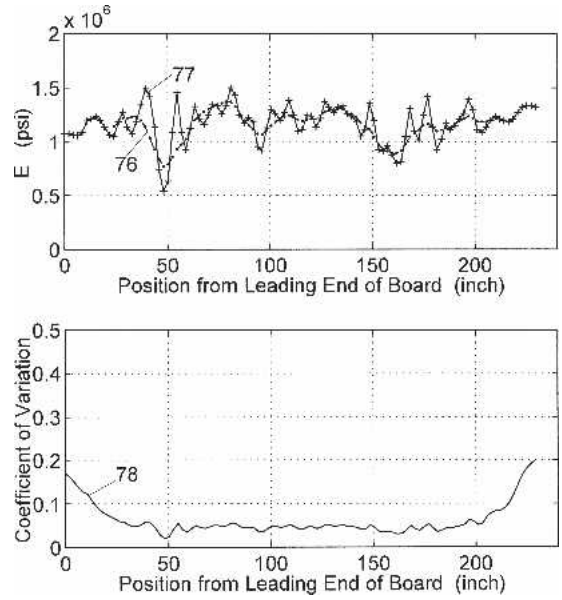

FIG. 4. Kalman filter applied to measured E of curve **76**, for a wood beam. Estimated local E is curve **77** and coefficient of variation is curve

The reciprocal $E_m = 1/C_m$ is plotted in Fig. 3 and labeled **72**. The curve **72** is observed to be a smoothed version of **71** and misses much of the detail of **71** as expected with a bending measurement.

A Kalman filter was applied, but instead of using coefficient $\rho = 0.97$, which was applied to generate the local compliance function, a value $\rho = 0.90$ was used to define the required matrices for the Kalman filter. It is not expected that one would know precisely the coefficients describing the statistics of the local compliance functions. If the 0.97 value had been known, there would be justification in this case for reducing it in the filter because of the added pulses of compliance. Experimentation with different values of ρ in the Kalman filter showed that smaller values give results with more variation thereby allowing better tracking of large variations. Thus, the Kalman filter, with $\rho = 0.90$ and $\sigma^2 = (34.5\ \text{GPa})^{-2} = (5.00\ \text{Mpsi})^{-2}$, used $\text{var}(u) = \sigma^2(1 - \rho^2) = (79.1\ \text{Gpa})^{-2} = (11.5\ \text{Mpsi})^{-2}$. The estimated local E obtained from the Kalman filter is illustrated in Fig. 3 as the curve labeled **73**. It follows the local E function

71 more closely than the measured result **72**. The coefficient of variation of the estimator at each point along the beam is illustrated by the center part of Fig. 3. It is clear that estimation quality suffers near the ends of the beam as a result of both reduced span function weighting and a reduced number of measurements having contributions from them. The estimated local E function and coefficient of variation were obtained from local compliance quantities by using Eqs. (10) and (11).

Figure 4 shows results of testing a wood beam with bending apparatus as in Fig. 3 of Part I. The beam was a 2 × 6 from a spruce-pine-fir population. Its cross-section and length were 38 × 140 mm (1.5 × 5.5 inch) and 6.1 m (20 ft). The actual local E analogous to simulated curve **71** of Fig. 3 is, of course, unknown. The measured E_m is given by curve **76** in Fig. 4, and the estimated local E result from the Kalman filter is given by curve **77**. Coefficient of variation of the estimator is given by curve **78**. In the Kalman filter, coefficients were set the same as for the filter resulting in the estimated local E of Fig. 3.

IMPLEMENTATION AND ADDITIONAL RESEARCH

It is interesting that if the correlation is set artificially high, e.g. $\rho = 0.999$, the estimated local E closely tracks the measured E_m, but without the vibration induced noise that often appears on the measured signal. This leads naturally to a suggested practical implementation of the estimation method into a machine stress-rated (MSR) lumber mill operating with modern equipment under North American rules. The primary modification to existing equipment would be new software for data processing. The implementation would use a correlation coefficient of 0.999 with little if any change in the grading process, except that the consequent reduced noise should result in some increase of high grade yields. Then, the correlation coefficient would be reduced in small steps along with allowable small reductions in machine grade thresholds. Grade yields and lumber quality would be monitored carefully using the existing off-line quality control procedures and equipment to determine if the local E estimates increase high grade yields and profits as expected. Simultaneously, data may be gathered for help in determining coefficients best suited for use in the autoregression model. Whether one or more regression coefficients should be used in the model would be part of this effort.

RAMIFICATIONS

The output equation of the state-space representation of the foregoing linear dynamic system produces a scalar (measured C_m) as its output. However, the Kalman filter has no such restriction. In fact, the output may be a vector output in which case the number of rows in the output matrix is equal to the number of components of the vector output. Thus the linear dynamic system model can have another linear combination of the state vector components, i.e. compliances, to give a different measured output. That output can be used along with other such outputs for processing by a Kalman filter.

CONCLUSIONS

A Kalman filter is practical, and likely useful and profitable, in the production-line to determine local elasticity values along the length of a beam. Input typically consists of a sequence of measured elasticity values from a sequence of overlapping bending spans along the beam. These are processed by the Kalman filter to give optimal local estimates for each increment in a subdivision of the beam length. The Kalman filter uses an autoregression model of the local compliance function appropriate for the tested population of beams. The autoregression coefficients may be estimated from prior research or learned from autocorrelation of measured compliances obtained from tests on beams from a similar population. Also required is a matrix of coefficients defining each measurement as a weighted moving average of local compliance values. These coefficients are obtained from span functions computed by the general method of Part I. Tests with simulated and actual data indicate beneficial possibilities of the method

particularly for use with machine grading of lumber. A suggested implementation would seamlessly introduce the method by first using it with an assumed artificially large correlation coefficient between adjacent local compliances. This has the observed effect of reducing noise on the measured signal without affecting it otherwise. For this reason alone, it is likely that implementation will cause small improvement in grade yields and profits. The next step would reduce the correlation coefficient used in the Kalman filter, as well as grade thresholds in small increments according to existing rules. Off-line quality control procedures would ensure that grade standards are met. At the same time, measurement data can be gathered for determining autocorrelations and better determination of the autoregression model used in the Kalman filter. With this implementation, required research can be performed to economically determine profitability of the local estimation method.

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