ESTIMATING THE CONCOMITANCE OF LUMBER STRENGTH PROPERTIES[1]

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ABSTRACT

Although lumber bending and tensile strength properties have been extensively studied, their cofunctioning—so important to the performance of structures—is not well understood. A novel application of proofloading is now available to wood scientists who need to estimate the cofunctioning of two strength properties. This technique and a statistical analysis of the data developed from the proofload experiments are presented.

Keywords: Proofload, strength properties, correlation, cofunctioning, concomitance.

INTRODUCTION

Lumber grading, whether by visual or mechanical means, is an effort to sort lumber nondestructively so that the lumber in the resulting grade will perform adequately for all claimed properties. Although lumber bending and tensile strength properties have been studied for many years, adequately modeling the property estimation process has been a difficult task. This estimation process must contend with the probability of occurrence of critical characteristics in the test span and, of course ultimately, with the occurrence of these same characteristics in the loaded span in actual use. Recently, Riberholt and Madsen (1979) addressed this latter statistical question, and the Forest Products Laboratory (FPL) began similar work. The statistical distribution of defects, of course, was one of the bases of the modern laminating process (Freas and Selbo 1954).

Of the six mechanical properties of lumber specified as part of standard wood design practice (NFPA 1977) and related to lumber grading criteria, five are strength properties: bending, tension, compression parallel, shear, compression perpendicular. At least three (bending, tension, compression parallel) cannot be evaluated independently on the same piece of lumber by conventional tests; yet it is evident that some characteristics (for example, edge knots) influence failure in more than one strength mode.

These observations lead to the notion of concomitance, or cofunctioning, of

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properties in a piece. ("Concomitance" is used here in its general context of cofunctioning rather than in its narrower technical statistical sense as a covariate.) Concomitant properties are defined as those properties that function "together" in certain structural uses such as wood trusses. By current design practice (NFPA 1977) these properties are treated as independent but are considered to have joint influence on structural performance. The degree of concomitance can, presumably, influence product performance as defined by the design procedure.

One approach to addressing concomitance is to test actual lumber under combined stresses. Early efforts at this (Senft 1973; Senft and Suddarth 1970) are not very revealing; recent research (Zahn 1981) should provide new insights to combined stress performance. Another approach to concomitance is to recognize the strength prediction methods currently used in the grading of lumber and to use these in an attempt to quantify the degree of concomitance. The most sophisticated of these methods is the use of defect size plus modulus of elasticity (E) to predict strength through regression. An initial attempt using this approach employed proofloading to identify the correlation between regression residuals (Galligan et al. 1980). That research has since been expanded to include further proofloaded data sets.

This report summarizes the current state-of-the-art, combining previous results with analysis of two new data sets. Specifically, it includes the estimation of the correlation among the residuals in the regression of strength properties on stiffness and edge knot size, and the development of an ancillary approach to estimating the correlation between bending and tension or bending and compression directly (i.e., without concern for the regression residuals).

The basis for this research is a mathematical approach (Galligan et al. 1980) that permits estimation of the error correlation in a regression model for strength. This approach, in turn, depends upon censoring a lumber strength distribution using proofloading (Johnson 1980)—a novel application of this latter technique.

Lumber specimens were selected in sufficient quantity to permit several levels of censoring. Censoring was done by proofloading a portion of the pieces to failure either in compression parallel or tension parallel. Pieces passing the proofload then were failed in bending.

MATERIAL SELECTION AND TESTS

Lumber grades and species chosen for tests were No. 2 medium grain KD southern pine (medium grain lumber was at the time of sampling a common grade classification for southern pine) and 1.5E-1650f hem-fir 2 × 4's, 14 feet long. All lumber was randomly selected, the southern pine at a mill in Oklahoma and the hem-fir at a mill in the state of Washington.

In the first experiments, reported in Galligan et al. (1980), 720 pieces of each species/grade combination were chosen and randomly divided into three sets each of 80 specimens and four sets each of 120 specimens. The 80-specimen sets were broken in bending, tension, or compression with no proofload. The 120-specimen sets were proofloaded either in compression or tension and the survivors broken in bending. Proofload targets were set with the intent of achieving 85% and 95% survival. These "low" levels were chosen so that concomitance analysis would relate to data in which design levels are based only on near-minimum specimens. These first tests (Galligan et al. 1980) demonstrated the need to fail a higher

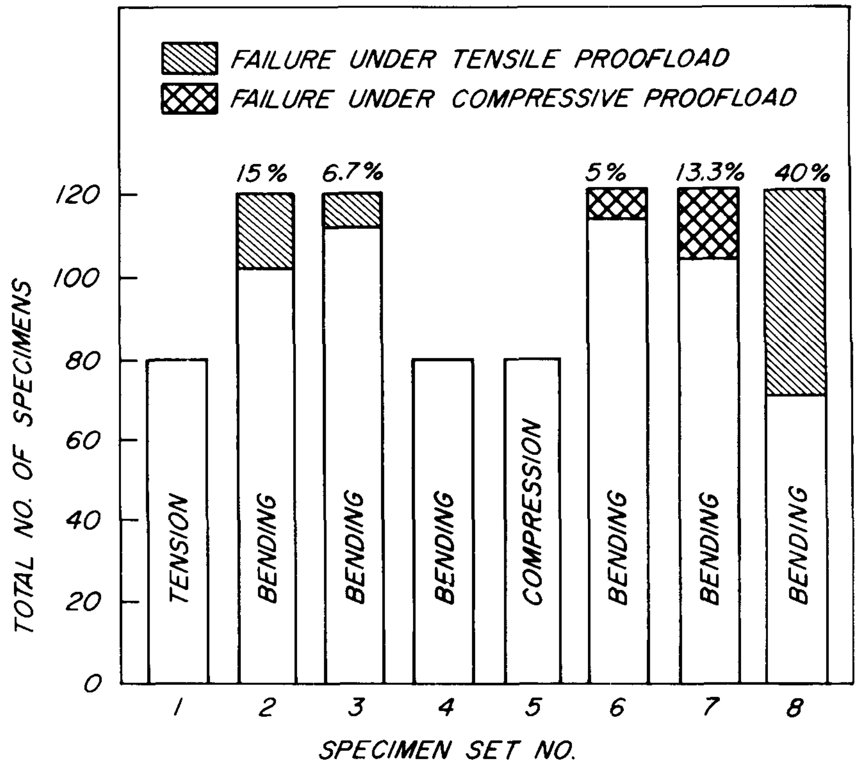

FIG. 1. Diagram of lumber testing plan for southern pine No. 2 KD. Percentages are proportion of pieces that failed by proofloading. A similar series was carried out for 1.5E-1650f hem-fir.

percentage of specimens in proofloading. Consequently one further set of southern pine and one of hem-fir were subsequently tested at the proofload designed to fail an estimated 50% of the specimens.

Data from all sets included E measured on edge according to ASTM D 198 (1970) and by both stress wave and E computer. These E values were measured on each piece. Knot size, strength ratio, and machine stress-rated (MSR) visual edge knot quality levels also were among the data measured to permit multivariate modeling. The lumber was conditioned and tested at Washington State University in accordance with ASTM procedures.

Further details of the testing procedures used have been outlined (Galligan et al. 1982) and the complete data set summarized (Fig. 1). The two compression proofload sets 6SP and 7HF were removed from the analysis because far too few failed the proofload.

MATHEMATICAL APPROACH

Correlation between regression residuals (conditional correlation)

The basic approach depends upon identifying the correlation between residuals in two regressions used to predict two strength properties from the same prediction variables (Galligan et al. 1980). Proofloading gives additional information to facilitate identification of the residual correlation. To illustrate: assume that bending and tension are the two strength properties of interest. The model assumes that the dependent variables—bending strength, b, and tensile strength, t—satisfy

$$\begin{aligned} b &= B_0 + B_1x_1 + B_2x_2 + \ldots + B_kx_k + \epsilon_b \\ t &= T_0 + T_1x_1 + T_2x_2 + \ldots + T_kx_k + \epsilon_t \end{aligned} \tag{1}$$

conditional on k independent variables, x grading variables. Here $B_0, B_1, \ldots, B_k$ and $T_0, T_1, \ldots, T_k$ are the regression coefficients. For example, x_1 may represent E and x_2 knot size. It is further assumed that the additive errors ϵ_b, ϵ_t have means equal to zero, variances σ_b^2, σ_t^2 and covariance $\rho_{bt}\sigma_b\sigma_t$ where ρ_{bt} denotes the correlation between ϵ_b and ϵ_t. ρ_{bt} can also be thought of as the conditional correlation between bending and tension.

Because we cannot simultaneously observe b and t on a single specimen (each is a destructively observed measurement), we consider a strategy in which a sample of size N is first proofloaded in tension to a load L. If a specimen does not fail in tension at loading level L, it is then loaded to its ultimate bending strength. Under this testing scheme we observe either

$$t = \text{tensile strength if } t \leq L$$

or

$$b = \text{bending strength and } t > L$$

for each of the N specimens.

We tentatively assume that, conditional on all of the values of the predictor variables, the errors $(\epsilon_{bj}, \epsilon_{tj})$, $j = 1, 2, \ldots, N$ are independent and have a bivariate normal distribution. The conditional likelihood $\mathscr{L}$ [given $\mathbf{x}_j = (x_{1j}, x_{2j}, \ldots, x_{kj})$, $j = 1, 2, \ldots, N$] then has the form of a product of two types of terms. A tension failure contributes a marginal term, while a bending failure contributes its marginal term times a conditional term expressing that the specimen passed the proofload.

$$\begin{aligned} \mathscr{L} = & \prod_{[j:t_j \leq L]} \frac{1}{\sqrt{2\pi}\sigma_t} e^{-\frac{1}{2\sigma_t^2}(t_j - T_0 - T_1x_{1j} - \ldots - T_kx_{kj})^2} \\ & \cdot \prod_{[j:t_j > L]} \frac{1}{\sqrt{2\pi}\sigma_b} e^{-\frac{1}{2s\sigma_b^2}(b_j - B_0 - B_1x_{1j} - \ldots - B_kx_{kj})^2} \\ & \cdot \int_{a_j}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz \end{aligned} \tag{2}$$

where

$$\begin{aligned} a_j = & [L - T_0 - T_1x_{1j} - \ldots - T_kx_{kj} - \rho_{bt}\frac{\sigma_t}{\sigma_b} \\ & \cdot (b_j - B_0 - B_1x_{1j} - \ldots - B_kx_{kj})]/(\sigma_t\sqrt{1 - \rho^2_{bt}}) \end{aligned}$$

ρ is a correlation and σ a standard deviation whose subscripts denote the strength properties. In this likelihood, expression $(t_j, x_{1j}, \ldots, x_{kj})$ are known for those members that failed the proofload, and $(b_j, x_{1j}, \ldots, x_{kj})$ are known for those members that passed. Thus, the conditional likelihood $\mathscr{L}$ can be maximized over values of the parameters ρ_{bt}, σ_b, σ_t, $(B_0, B_1, \ldots, B_k)$, and $(T_0, T_1, \ldots, T_k)$ to obtain their maximum likelihood estimates.

Several models employing different E measurements and strength ratio and knot size criteria were explored to develop efficient models consistent between the pairs, bending and compression, and bending and tension. The models selected for analysis were of the following forms:

$$\begin{aligned} \text{bending} &= B_0 + B_1(E_{cptr}) + B_2(\text{edge knot}) + \text{error(b)} \\ \text{tension} &= T_0 + T_1(E_{cptr}) + T_2(\text{edge knot}) + \text{error(t)} \\ \text{compression} &= C_0 + C_1(E_{cptr}) + C_2(\text{edge knot}) + \text{error(c)} \end{aligned} \tag{3}$$

where E_{cptr} is an E measured by transverse vibration (E computer) and edge knot is the knot size measured in inches.

Unconditional correlation

One can also directly estimate the correlation between strengths. For a specimen, selected at random, this is the correlation between its bending strength and tensile strength and is different from the conditional correlation ρ_{bt} in Eq. 1. The independent variables such as E, edge knot size, and strength ratios are ignored. The unconditional model is

$$\begin{aligned} b &= \mu_b + \epsilon_b^* \\ t &= \mu_t + \epsilon_t^* \end{aligned} \tag{4}$$

where μ_b is the mean bending strength, μ_t is the mean tensile strength, and the errors $(\epsilon_b^*, \epsilon_t^*)$ are jointly normal with zero means, $\mathrm{Var}(\epsilon_b^*) = \sigma_b^{*2}$, $\mathrm{Var}(\epsilon_t^*) = \sigma_t^{*2}$ and $\mathrm{Cov}(\epsilon_b^*, \epsilon_t^*) = \rho^*\sigma_b^*\sigma_t^*$. That is, the unconditional correlation between bending and tensile strengths is

$$\rho^* = \mathrm{Corr(b,t)} \tag{5}$$

As before, if the j^{th} specimen fails in tension, it contributes a marginal term to the likelihood. If the j^{th} specimen survives the proofload and is broken in bending, it contributes a marginal term times the conditional probability that it did not fail the proofload. The likelihood for a sample of size N, when each specimen undergoes a tension proofload of L and those that survive are loaded in bending to ultimate failure, is then

$$\begin{aligned} \mathscr{L}^* = &\prod_{\{j:t_j \le L\}} \frac{1}{\sqrt{2\pi}\sigma_t^*} e^{-\frac{1}{2}(t_j - \mu_t)^2/\sigma_t^{*2}} \\ &\cdot \prod_{\{j:t_j > L\}} \frac{1}{\sqrt{2\pi}\sigma_b^*} e^{-\frac{1}{2}(b_j - \mu_b)^2/\sigma_b^{*2}} \int_{a_j}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz \end{aligned} \tag{6}$$

where

$$a_j^* = [L - \mu_t - \rho^* \frac{\sigma_t^*}{\sigma_b^*}(b_j - \mu_b)]/\sigma_t^* \sqrt{1 - \rho^{*2}}$$

Relationship between conditional and unconditional correlation

We now determine the relationship between the unconditional correlation $\rho^* = \mathrm{Corr(b,t)}$ in Eq. 4 and the conditional correlation $\rho_{bt} = \mathrm{Corr}(\epsilon_b, \epsilon_t)$ in Eq. 1. In this derivation, we assume that the parameters are all known. By the properties of conditional expectation,

$$\begin{aligned} E(b) = E\{E(b|x_1, \ldots, x_k)\} &= E\{B_0 + B_1X_1 + \ldots + B_kX_k + E(\epsilon_b|x_1, \ldots, x_k)\} \\ &= E\{B_0 + B_1X_1 + \ldots + B_kX_k\} \end{aligned} \tag{7}$$

Similarly

$$E(t) = E\{T_0 + T_1X_1 + \ldots + T_kX_k\} \tag{8}$$

and

$$E(c) = E\{C_0 + C_1X_1 + \ldots + C_kX_k\} \tag{9}$$

Further

$$E(bt) = E\{(B_0 + B_1X_1 + \ldots + B_kX_k)(T_0 + T_1X_1 + \ldots + T_kX_k)\} + \rho_{bt}\sigma_b\sigma_t$$

where $\rho_{bt}\sigma_b\sigma_t$ is the conditional covariance of ϵ_b and ϵ_t in Eq. 1. Setting $\mathbf{B}' = (B_1, \ldots, B_k)$, $\mathbf{T}' = (T_1, \ldots, T_k)$ and $\mathbf{C}' = (C_1, \ldots, C_k)$

$$\mathrm{Cov}(b,t) = \rho_{bt}\sigma_b\sigma_t + \mathbf{B}'\Sigma_X\mathbf{B} \tag{10}$$

where Σ_X is the covariance matrix of $X_1, \ldots, X_k$. A similar calculation for other cases gives

$$\begin{aligned} \mathrm{Var}(b) &= \sigma_b^2 + \mathbf{B}'\Sigma_X\mathbf{T} \\ \mathrm{Var}(t) &= \sigma_t^2 + \mathbf{T}'\Sigma_X\mathbf{T} \end{aligned} \tag{11}$$

Thus,

$$\rho^* = \mathrm{Corr}(b,t) = \frac{\rho_{bt}\sigma_b\sigma_t + \mathbf{B}'\Sigma_X\mathbf{T}}{\sqrt{\sigma_b^2 + \mathbf{B}'\Sigma_X\mathbf{B}}\sqrt{\sigma_t^2 + \mathbf{T}'\Sigma_X\mathbf{T}}} \tag{12}$$

RESULTS

Correlation between regression residuals

Implementation of the numerical computations using the models of Eq. 3 proved difficult. The regression parameter estimates from the specimens that failed under low prooflloading were not adequate because few of the specimens failed at low proofload levels. To surmount this difficulty, we adopted an approach in which all the parameters except the correlation between residuals, ρ, were considered fixed. Numerical values for σ_b and B_0, B_1, and B_2 were obtained by the least squares procedure from the specimen set in which all specimens were broken in bending (specimen set 4 in Fig. 1). Likewise, estimates for σ_t and T_0, T_1, and T_2, and σ_c and C_0, C_1, and C_2 were obtained from sets in which specimens were tested exclusively in tension or compression, respectively.

Table 1 summarizes the resulting regression parameters.[2] It also shows that the value of E_{cptr} and edge knot size as predictors (measured by the coefficient of

[2] Even if one decides to maximize the likelihood, $\mathscr{L}$, over all regression parameters as originally intended, the calculation must be iterative. The numerical values obtained from the "no proofload" destructive tests by least squares can then serve as initial values.

TABLE 1. *Summary of fit and estimates of parameters for the regression model.*

Species[a]	Strength	Regression coefficients[b] Constant	E_{cptr}	Edge knot size	Variance, σ^2	R^2 [c]
Southern pine	Tensile	T_0 = 748.6	T_1 = 2.206	T_2 = −47.97	393,909	0.5081
	Bending	B_0 = 1,342.3	B_1 = 3.496	B_2 = −57.65	1,667,724	.3386
	Compression	C_0 = 2,086.4	C_1 = 1.585	C_2 = −10.55	134,514	.5096
Hem-fir	Tensile	T_0 = 1,290.2	T_1 = 2.461	T_2 = −94.81	1,885,485	.3274
	Bending	B_0 = 1,313.5	B_1 = 2.974	B_2 = −63.86	2,701,140	.2057
	Compression	C_0 = 2,222.2	C_1 = 1.388	C_2 = 0.94	393,246	.2228

[a] SP – No. 2 KD; HF = 1.5E MSR.
[b] As illustrated by model (Eq. 3).
[c] Squared multiple correlation coefficient.

determination, R^2) is lower for the MSR hem-fir (HF) than for the visually graded southern pine (SP), a result often observed for regressions based on samples such as MSR that contain a limited range in the independent variables.

Table 2 summarizes the estimates for ρ in the conditional regression model (Eq. 1) from the eight proofload experiments coded 2SP, 3SP, 7SP, 8SP, 2HF, 3HF, 6HF, 8HF. The estimates from 2HF and 3SP must be viewed with more caution than the others. In each case, only 8 out of 120 specimens failed the proofload, so we conjecture that the data are not sufficiently informative concerning ρ. The hem-fir results are quite consistent with the highest-tension proofload (nearest 50% failure) providing the shortest approximate confidence interval.

The three estimated correlations (ρ) between bending and tension for hem-fir are considerably higher than their counterparts for southern pine (Table 2). We also calculated an approximate large sample confidence interval for ρ, consisting of all ρ such that

$$-2 \ln \mathscr{L}(\rho) \leqslant -2\mathscr{L}(\hat{\rho}) + \chi^2_1(.05) \tag{13}$$

TABLE 2. *Summary of estimates of the correlation of residuals, ρ, in the conditional regression model.*

Species		Failure mode Proofload	Survivor	Proofload level, L	Proportion failing proofload	Estimated residual correlation, $\hat{\rho}$	Approximate 95% confidence interval for ρ
				psi			
Southern pine	3SP	tension	bending	[a]1,339	8/120	−0.1	(−0.45, 0.35)
	2SP	tension	bending	1,657	18/120	0.4	(−0.2, 0.7)
	8SP	tension	bending	2,350	48/120	0.7	(−0.5, 0.9)
	7SP	compression	bending	[b]3,588	16/120	−0.3	(−0.5, 0.1)
Hem-fir	2HF	tension	bending	[c]2,611	8/120	0.6	(−0.05, 0.8)
	3HF	tension	bending	2,857	16/120	⩾0.9	(0.75, 1)
	8HF	tension	bending	4,885	54/105	0.95	(0.8, 1)
	6HF	compression	bending	[d]4,038	28/120	0.6	(0.3, 0.8)

[a] Specimen 108 failed at tension 1,339 psi.
[b] Specimen 84 failed at compression 3,588 psi.
[c] Specimen 103 failed at tension 2,611 psi.
[d] Specimen 76 failed at compression 4,038 psi.

TABLE 3. *Estimates of parameters in the unconditioned model, Eq. 15.*

Species	Data code	Population mean			Standard deviation		
		μ_t	μ_b	μ_c	σ_t^*	σ_b^*	σ_c^*
Southern pine	1SP	2,509.1			883.02		
	4SP		4,147.9			1,567.73	
	5SP			3,780.9			516.96
Hem-fir	1HF	4,898.6			1,652.73		
	4HF		6,183.2			1,820.35	
	5HF			4,564.5			702.16

where $\chi^2_1(0.05)$ is the upper 5% point of the χ^2 distribution with 1 degree of freedom and $\mathscr{L}(\rho)$ is the likelihood (Eq. 2) with ρ as the only unknown parameter. For southern pine, the width of the confidence intervals suggested that zero correlations are plausible. Double precision was required to evaluate the log of the normal integral in $-2 \ln \mathscr{L}(\rho)$.

We modeled strength properties, given E_{cptr} and edge knot size, as having a normal distribution. However, some experimental evidence suggests that a three-parameter Weibull or other distribution may be more appropriate. Unfortunately, any inadequacy in Eq. 3 caused by ignoring important prediction variables or incorrect distributional assumptions in Eq. 2 would likely result in an increased error correlation. In this respect, the normal theory maximum likelihood estimate of ρ may not be very robust if, in fact, the model is misspecified.

Unconditional correlation

Similar to our analysis of the likelihood (Eq. 2) we treat all of the parameters in Eq. 6, except ρ^*, as fixed. The other parameters are estimated from the experimental data obtained from nonproofloading schemes (Table 3).

Our likelihood routine then maximizes L* over ρ^* (Table 4) using the appropriate estimates from Table 3.

The hem-fir tension-bending schemes consistently estimate a high value for ρ^*. The shortest confidence interval comes from the highest proofload. Except for the 3SP set in which only 8 of 120 specimens failed in tension, the high correlations also seem to hold for southern pine.

TABLE 4. *Summary of estimates of ρ^* from the unconditioned model.*

Species	Data code	Failure mode: Proofload	Failure mode: Survivor	Proofload level, L	Estimate, $\hat{\rho}^*$	Approximate 95% confidence interval for ρ^*
				psi		
Southern pine	3SP	tension	bending	1,339	−0.4	(−0.7, 0.1)
	2SP	tension	bending	1,657	≥0.9	(0.4, 1)
	8SP	tension	bending	2,350	≥0.95	(0.8, 1)
	7SP	compression	bending	3,588	0.3	(−0.1, 0.6)
Hem-fir	2HF	tension	bending	2,611	0.4	(−0.8, 0.8)
	3HF	tension	bending	2,857	≥0.9	(0.75, 1)
	8HF	tension	bending	4,885	≥0.95	(0.9, 1)
	6HF	compression	bending	4,038	0.5	(0.2, 0.7)

These results are limited by the assumption that the strength properties are jointly normal. It is well known that for strength the Weibull is usually more appropriate than the normal distribution. Large sample sizes do not alleviate the difficulty in the estimation of ρ^* as they do for estimation of the means.

Relationship between conditional and unconditional correlation

As one check on the conditional model, we compare the direct estimate of $\rho^* = \mathrm{Corr}(b,t)$ with that obtained using the parameter estimates for the conditional model and the relation Eq. 12. In this example the correlation of interest is that between bending and compression in hem-fir. Data set 4HF will be used to estimate $\mathbf{B}$, $\Sigma\mathbf{X}$, σ_b, and σ_c. Thus

$$\begin{aligned}\mathbf{B}'\Sigma_{\mathbf{X}}\mathbf{B} &= \hat{B}_1{}^2\mathrm{Var}(E_{cptr}) + \hat{B}_2{}^2\mathrm{Var}(\text{Edge knot size}) \\ &\quad + 2\hat{B}_1\hat{B}_2\mathrm{Cov}(E_{cptr}, \text{Edge knot size}) \\ &= 681{,}851.9 \end{aligned} \tag{14}$$

Continuing the computation of Eq. 12 using the estimate of ρ_{bc} from data set 6HF, we determine

$$\mathrm{Corr}(c,b) = \frac{681{,}851.9 + 251{,}604.42}{(1{,}839.29)(716.43)} = 0.66. \tag{15}$$

This may be compared to the estimate $\rho_{bc}{}^* = 0.5$ (Table 4). The comparison for the other sets with low proofload levels is not reliable.

Sensitivity analysis

In the applications above, the true values of ρ are unknown. To gain a preliminary indication on the sensitivity of the estimation approach to some of the mechanical method and mathematical assumptions, it was decided to generate some data for two proofloads and two choices of error variances. There were too few runs to draw any substantial conclusions but we did note

(1) There is indication that increasing the proofload may make the likelihood a little more peaked.
(2) Reducing the standard deviation of the errors by a factor of 2 does not seem to help in the determination of ρ.

These conclusions concerning sensitivity are tentative and the major conclusion is that the method should be checked by further runs.

Finally, we note that it is clear from Eq. 2 that the likelihood depends on bending and tensile strengths through

$$Z_{bj} = \frac{b_j - B_0 - B_1x_{1j} - \ldots - B_kx_{kj}}{\sigma_b} = \frac{\epsilon_{bj}}{\sigma_b}$$

or

$$Z_{tj} = \frac{t_j - T_0 - T_1x_{1j} - \ldots - T_kx_{kj}}{\sigma_t} = \frac{\epsilon_{tj}}{\sigma_t}$$

which are standard normal variables. If both (b_j, t_j) were available for every specimen (they are not), the estimation of ρ_{bt} would not be influenced even if σ_b and σ_t were greatly reduced. However, the other quantity of interest in the likelihood is a_j:

$$a_j = \left[\frac{L - T_0 - T_1x_{1j} - \ldots - T_kx_{kj}}{\sigma_t} - \rho_{bt}Z_{bj}\right] \Big/ \sqrt{1 - \rho_{bt}^2}$$

Because $[L - T_0 - T_1x_{1j} - \ldots - T_kx_{kj}]$ is divided by σ_t, the size of σ_t does influence the precision of the estimate of ρ_{bt}.

CONCLUSIONS

(1) Information about ρ can be obtained from proofloading experiments of the kind conducted. However, 120 specimens are not sufficient to determine ρ to within ± 0.1 with high confidence.

(2) The residual correlations for the 1.5E-1650f MSR hem-fir appears consistently higher for each data set than for the comparable data set for visually graded No. 2 KD southern pine. The correlation between tension and bending appears to be significantly higher than that between compression and bending. The confidence intervals, however, are very broad.

(3) More efficient computer analysis programs seem possible but complex. Attention to this objective would permit return to the original approach of estimating regression parameters from the proofload data sets.

REFERENCES

AMERICAN SOCIETY FOR TESTING AND MATERIALS. 1970. Standard methods of static tests of timbers in structural sizes. ASTM Stand. Des. D 198-67. Philadelphia, PA.

FREAS, A. D., AND M. L. SELBO. 1954. Fabrication and design of glued laminated wood structural members. USDA Tech. Bull. No. 1069. Supt. Doc., Gov. Printing Office, Washington, DC.

GALLIGAN, W. L., R. J. HOYLE, R. F. PELLERIN, J. H. HASKELL, AND J. M. TAYLOR. 1982. Characterizing the properties of 2-inch softwood dimension lumber with regression and stochastic distribution. To be published. For. Prod. Lab., Madison, WI.

———, R. A. JOHNSON, AND J. R. TAYLOR. 1980. Examination of the concomitant properties of lumber. Proc. Conf. Performance Metal Plate Wood Truss. For. Prod. Res. Soc., Madison, WI.

JOHNSON, R. A. 1980. Current statistical methods for estimating lumber properties by proof loading. For. Prod. J. 30(1):14–22.

NATIONAL FOREST PRODUCTS ASSOCIATION. 1977. National design specification. NFPA, Washington, DC.

RIBERHOLT, H., AND P. H. MADSEN. 1979. Measured variation of the cross sectional strength of structural lumber. Rep. No. 114. Structural Research Laboratory, Tech. Univ. of Denmark, Copenhagen.

SENFT, J. F. 1973. Further studies in combined bending and tension strength of structural 2 by 4 lumber. For. Prod. J. 23(10):36–41.

———, AND S. K. SUDDARTH. 1970. Strength of structural lumber under combined bending and tension loading. For. Prod. J. 20(7):17–21.

ZAHN, J. J. 1981. Strength of lumber under combined bending and compression. USDA For. Serv. Res. Pap. FPL 391. For. Prod. Lab., Madison, WI.