THE CONTRIBUTION OF EARLYWOOD AND LATEWOOD SPECIFIC GRAVITIES TO OVERALL WOOD SPECIFIC GRAVITY

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ABSTRACT

A detailed analysis of the mathematical identity relating overall wood specific gravity to earlywood and latewood specific gravities and proportions is presented. The exact formula for expressing the variance of the overall specific gravity in terms of the means, variances, and covariances of these variables is developed. A small but typical data set is used to illustrate the formula. In this example the major contributor (34%) to the overall variance is the variance of the proportion of latewood weighted by the square of the difference between the mean latewood and earlywood specific gravities. The covariance between the earlywood specific gravity and the proportion of latewood, again appropriately weighted, also makes an important contribution (18%) to the overall variance. These results are supported by analysis of an independent data set.

As a byproduct of the theory, it is possible to predict the behavior of linear regressions of overall specific gravity on any combination of the above components.

Keywords: Variance components, density, regression analyses, multiplicative models, coniferous species, *Pseudotsuga menziesii*.

INTRODUCTION

It is a common belief, typified by Smith et al. (1976), that the proportion of latewood plays a dominant role in determining the value of the overall specific gravity of wood of coniferous trees. On the other hand, as will be shown below, simplistic analysis of the mathematical identity relating the overall specific gravity to earlywood and latewood specific gravities and proportions suggests that the overall specific gravity is affected primarily by earlywood specific gravity. This interpretation is dependent, however, on the validity of the assumption of mutual independence of the components. To clarify the situation, the exact formula for expressing the variance of the overall specific gravity in terms of the means, variances, and covariances of the earlywood and latewood specific gravities and proportions will be developed. The formula will be applied to some typical data to determine which, at least in this case, are the more important components.

The methodology will also permit the study of the linear regression of the overall specific gravity on the earlywood and latewood specific gravities and the proportion of latewood and thus, perhaps, throw further light on why workers have emphasized the proportion of latewood as a predictor variable (e.g. Diana Smith 1956; Smith et al. 1976).

THE BASIC RELATIONSHIP

The starting point is the identity

$$z \equiv x(1 - P) + yP$$

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where z represents the overall specific gravity; x, y the earlywood and latewood specific gravities, respectively, and P the proportion of latewood. This relationship is exact, and any observed deviation from it can arise solely from the normal limitations of measurement accuracy (see, e.g. Diana Smith 1955) and can be ignored for the purpose of this study.

The rate of change of overall specific gravity with respect to any one of the components, with the others assumed fixed, can be obtained by partial differentiation, thus:

$$\frac{\partial z}{\partial x} = 1 - P$$

$$\frac{\partial z}{\partial y} = P$$

$$\frac{\partial z}{\partial P} = y - x$$

A small set of actual data is given in Table 1. At the mean values of the components, the rate of change of overall specific gravity with respect to earlywood specific gravity is $1 - P \simeq 0.69$; with respect to latewood specific gravity it is $P \simeq 0.31$ and with respect to the proportion of latewood it is $y - x \simeq 0.40$. Seemingly, therefore, the overall specific gravity is most strongly affected by the earlywood specific gravity. Indeed, this effect appears to be more than double that of the latewood specific gravity and almost 75% more than that of the proportion of latewood. In other words, other things being equal, a change of one point in earlywood specific gravity has about twice the effect on overall specific gravity as a change of one point in either latewood specific gravity or latewood percentage.

The above statements, although mathematically valid, are not necessarily realistic. In the real world, several factors, climatic, genetic and others, including silvicultural practices such as irrigation, fertilization, spacing and thinning, will simultaneously affect all components (see e.g. Smith 1976; Kennedy 1970; Parker et al. 1976). Thus, although one can say, for example, what would be the effect of increasing earlywood specific gravity, with latewood specific gravity and percentage held fixed; it is unlikely, if not impossible, that this could ever be observed.

VARIABILITY COMPONENTS OF OVERALL SPECIFIC GRAVITY

An alternative approach to the above would be to express the variance of the overall specific gravity in terms of the variances and covariances of its components, i.e. the proportion of latewood to earlywood and their specific gravities. One may then see which are the major contributors to the overall variation by the substitution of values from typical data sets.

Let the mean value of the earlywood specific gravity be denoted by $E(x) = \mu_{100}$; likewise the mean values of the latewood specific gravity and the proportion of latewood are denoted by $E(y) = \mu_{010}$ and $E(P) = \mu_{001}$, respectively. The higher central moments are then defined, in general as

$$E\{[x - E(x)]^i[y - E(y)]^j[P - E(P)]^k\} = \mu_{ijk}$$

where $i + j + k \geqslant 2$. For example, the variance of the earlywood specific gravity is written as:

TABLE 1. *Basic data of illustrative example.*

	Earlywood specific gravity (x)	Latewood specific gravity (y)	Proportion latewood (*P*)	Overall specific gravity (z)
	0.35	0.71	0.31	0.4616
	0.33	0.74	0.25	0.4325
	0.34	0.70	0.35	0.4660
	0.39	0.78	0.40	0.5460
	0.39	0.71	0.27	0.4764
	0.37	0.84	0.33	0.5251
	0.36	0.78	0.32	0.4944
	0.34	0.84	0.36	0.5200
	0.34	0.74	0.24	0.4360
	0.31	0.82	0.33	0.4783
	0.33	0.69	0.29	0.4344
	0.38	0.74	0.34	0.5024
	0.30	0.69	0.25	0.3975
	0.34	0.71	0.39	0.4843
	0.36	0.71	0.30	0.4650
	0.33	0.72	0.44	0.5016
	0.36	0.69	0.40	0.4920
	0.33	0.74	0.27	0.4407
	0.36	0.71	0.27	0.4545
	0.32	0.73	0.28	0.4348
	0.30	0.69	0.24	0.3936
	0.34	0.80	0.29	0.4734
	0.32	0.77	0.28	0.4460
	0.32	0.79	0.32	0.4704
Av.	$0.34208\dot{3}$	$0.74\dot{3}$	$0.31\dot{3}$	0.4677875

$$E\{[x - E(x)]^2\} = \mu_{200}$$

and the covariance of earlywood specific gravity and the proportion of latewood as:

$$E\{[x - E(x)][P - E(P)]\} = \mu_{101}$$

It can be shown that the mean value of the overall specific gravity is given by:

$$\begin{aligned} E(z) &= E(x)[1 - E(P)] + E(y)E(P) + \mathrm{Cov}(y, P) - \mathrm{Cov}(x, P) \\ &= \mu_{100}(1 - \mu_{001}) + \mu_{010}\mu_{001} + \mu_{011} - \mu_{101}. \end{aligned}$$

This can be readily checked numerically from the data of Table 1. The mean overall specific gravity is 0.4677875[1] and $\mu_{100} = 0.34208\dot{3}$,

$\mu_{010} = 0.74\dot{3}$, $\mu_{001} = 0.31\dot{3}$, $\mu_{011} = 0.0004\dot{2}$ and $\mu_{101} = 0.000430\dot{5}$.[2]

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[1] The presentation of mean densities etc. to a large number of decimals is to demonstrate the exactness of the mathematical formulae and does not imply that these quantities can be meaningfully measured to more than two or three figures.

[2] The dot over a digit denotes a recurring decimal, e.g. ⅓ = $0.\dot{3}$.

TABLE 2. *Moment values of illustrative example.*

$\mu_{100} = 0.34208\dot{3}$	$\mu_{010} = 0.74\dot{3}$	$\mu_{001} = 0.31\dot{3}$
$\mu_{200} = 0.0006164930\dot{5}$	$\mu_{020} = 0.00220\dot{5}$	$\mu_{002} = 0.00290\dot{5}$
$\mu_{110} = 0.00010\dot{5}$	$\mu_{011} = 0.0004\dot{2}$	$\mu_{101} = 0.0004430\dot{5}$
$\mu_{201} = -0.000003832175925$	$\mu_{021} = 0.000025962962965$	
$\mu_{102} = -0.000004196759258$		$\mu_{012} = -0.000047787037035$
	$\mu_{111} = -0.000004481481481$	
$\mu_{202} = 0.000001968718171$		$\mu_{022} = 0.00000456231483$
	$\mu_{112} = 0.000001152951390$	

Then $\mu_{100}(1 - \mu_{001}) + \mu_{010}\mu_{001} + \mu_{011} - \mu_{101}$

$$= 0.34208\dot{3} \times 0.68\dot{6} + 0.74\dot{3} \times 0.31\dot{3} + 0.0004\dot{2} - 0.0004430\dot{5}$$

$$= 0.4677875.$$

[NOTE: For the purpose of these calculations, the data are treated as a population and not as a sample. Thus the divisor in the calculation of the higher moments is n = 24 and not n − 1, see e.g. Seber (1973). For example μ_{101} is calculated as $\Sigma(x - \bar{x})(P - \bar{P})/24 = [\Sigma\, xP - \Sigma\, x\, \Sigma\, P/24]/24$ etc.].

Likewise, it can be shown that:

$$\begin{aligned}\mathrm{Var}(z) = {} & \mu_{200}(1 - \mu_{001})^2 + \mu_{020}\mu_{001}^2 + \mu_{002}(\mu_{010} - \mu_{100})^2 + 2\mu_{110}\mu_{001}(1 - \mu_{001}) \\ & + 2\mu_{101}(\mu_{010} - \mu_{100})(1 - \mu_{001}) + 2\mu_{011}(\mu_{010} - \mu_{100})\mu_{001} - (\mu_{101} - \mu_{011})^2 \\ & - 2\mu_{201}(1 - \mu_{001}) + 2\mu_{021}\mu_{001} - 2\mu_{102}(\mu_{010} - \mu_{100}) + 2\mu_{012}(\mu_{010} - \mu_{100}) \\ & + 2\mu_{111}(1 - 2\mu_{001}) + \mu_{202} + \mu_{022} - 2\mu_{112}.\end{aligned}$$

The moments calculated from the data of Table 1 are given in Table 2, and the calculation of Var(z) is presented in Table 3. The sum of the component values can be compared with the value obtained by direct calculation (the difference of 7 in the 15th decimal is simply the result of rounding on a desk calculator).

It will be seen that 34.4% of the variance in the overall specific gravity can be ascribed to the term $\mu_{002}(\mu_{010} - \mu_{100})^2$, which can be interpreted as the variance of the proportion of latewood weighted by the square of the difference between latewood and earlywood specific gravities. Also, 21.4% of the overall variance can be ascribed to the term $\mu_{200}(1 - \mu_{001})^2$, i.e. to the variance of the earlywood specific gravity weighted by the square of the proportion of earlywood, and 15.9% of the overall variance can be ascribed to the term $\mu_{020}\mu^2{}_{001}$, i.e. to the variance of the latewood specific gravity weighted by the square of the proportion of latewood.

These three terms account for 34.4% + 21.4% + 15.9% = 71.7% of the overall variance. Of the remaining 28.3%, 18.0% can be ascribed to $2\mu_{101}(\mu_{010} - \mu_{100})(1 - \mu_{001})$, i.e. to the covariance of the earlywood specific gravity and the proportion of latewood, weighted by twice the product of the difference between latewood and earlywood specific gravities and the proportion of earlywood. Also 7.8% can be ascribed to $2\mu_{011}(\mu_{010} - \mu_{100})\mu_{001}$, i.e. to the covariance of the latewood specific gravity and proportion of latewood, weighted by twice the product of the

TABLE 3. *Components of overall specific gravity variation.*

$\mathrm{Var}(x)(1 - E(P))^2$	$= \mu_{200}(1 - \mu_{001})^2$	$= 0.000290683325617$	21.4%	
$\mathrm{Var}(y)E^2(P)$	$= \mu_{020}\mu_{001}^2$	$= 0.000216536543209$	15.9%	
$\mathrm{Var}(P)(E(y) - E(x))^2$	$= \mu_{002}(\mu_{010} - \mu_{100})^2$	$= 0.000467798984375$	34.4%	
$2\mathrm{Cov}(x,y)E(P)(1 - E(P))$	$= 2\mu_{110}\mu_{001}(1 - \mu_{001})$	$= 0.000045421728394$	3.3%	100.8%
$2\mathrm{Cov}(x,P)(E(y) - E(x))(1 - E(P))$	$= 2\mu_{101}(\mu_{010} - \mu_{100})(1 - \mu_{001})$	$= 0.000244145763888$	18.0%	
$2\mathrm{Cov}(y,P)(E(y) - E(x))E(P)$	$= 2\mu_{011}(\mu_{010} - \mu_{100})\mu_{001}$	$= 0.000106167777776$	7.8%	
$-(\mathrm{Cov}(x,P) - \mathrm{Cov}(y,P))^2$	$= -(\mu_{101} - \mu_{011})^2$	$= -0.000000000434027$		
	$-2\mu_{201}(1 - \mu_{001})$	$= 0.000005262854936$		
	$2\mu_{021}\mu_{001}$	$= 0.000016270173458$		
	$-2\mu_{102}(\mu_{010} - \mu_{100})$	$= 0.000003367899304$		
	$2\mu_{012}(\mu_{010} - \mu_{100})$	$= -0.000038349097220$		
	$2\mu_{111}(1 - 2\mu_{001})$	$= -0.000003346172838$		
	μ_{202}	$= 0.000001968718171$		
	μ_{022}	$= 0.000004562314813$		
	$-2\mu_{112}$	$= -0.000002305902780$		
		0.001358184427076		
Var(z) calculated directly		0.001358184427083		

TABLE 4. *Components of overall specific gravity variation, independent example.*

$\mu_{200}(1 - \mu_{001})^2$	$= 0.00047894$	14.5%	
$\mu_{020}\mu_{001}^2$	$= 0.00042619$	12.9%	
$\mu_{002}(\mu_{010} - \mu_{100})^2$	$= 0.00148891$	45.24%	
$2\mu_{110}\mu_{001}(1 - \mu_{001})$	$= 0.00003024$	0.9%	107%
$2\mu_{101}(\mu_{010} - \mu_{100})(1 - \mu_{001})$	$= 0.00097030$	29.4%	
$2\mu_{011}(\mu_{010} - \mu_{100})\mu_{001}$	$= 0.00013835$	4.2%	
$-(\mu_{101} - \mu_{011})^2$	$= -0.00000193$	—	
$-2\mu_{201}(1 - \mu_{001})$	$= -0.00006240$		
$2\mu_{021}\mu_{001}$	$= 0.00002607$		
$-2\mu_{102}(\mu_{010} - \mu_{100})$	$= -0.00014091$		
$2\mu_{012}(\mu_{010} - \mu_{100})$	$= -0.00009884$		
$2\mu_{111}(1 - 2\mu_{001})$	$= -0.00001473$		
μ_{202}	$= 0.00002307$		
μ_{022}	$= 0.00002144$		
$-2\mu_{112}$	$= 0.00000956$		
	0.00329426		

difference between the latewood and earlywood specific gravities and the proportion of latewood, while 3.3% can be ascribed to $2\mu_{110}\mu_{001}(1 - \mu_{001})$, i.e. to the covariance of the earlywood and latewood specific gravities weighted by twice the product of the earlywood and latewood proportions.

These six terms ostensibly account for more than 100% (actually 100.8%) of the overall variance. This is compensated for by small negative contributions from some of the higher order moments.

It should be emphasized that this situation, where the variables are multiplicative, is very different from the usual variance components model in which some quantity is assumed to be the sum of independent random variables, and where the total variation can be apportioned in a unique and meaningful manner (see e.g. Snedecor and Cochran 1967).

Although it seems fair to say that the variance of the proportion of latewood plays a major role in the overall variation, its contribution is dependent on the latewood-earlywood specific gravity difference. The *covariance* between earlywood specific gravity and the proportion of latewood seems to have roughly the same importance as the earlywood and latewood specific gravity variances, but again its contribution is dependent on, not only the latewood-earlywood specific gravity difference, but also on the proportion of latewood, while the latter two are dependent on the earlywood and latewood proportions, respectively.

An analysis of an independent set of data, although carried out with somewhat less precision (Table 4), shows essentially the same pattern.

REGRESSION MODELS

Another approach that has been tried is that of multiple linear regression. Notwithstanding her recognition of the identity $z \equiv x(1 - P) + yP$ in her 1955 paper, Diana Smith (1956) observed that, for her particular data set, multiple linear regression on earlywood and latewood specific gravity and the proportion of latewood accounted for 96.90% of the variation of the overall specific gravity, with 88.94% being attributable to the proportion of latewood. It would appear that multiple linear regression is such a popular tool that, in spite of the existence of the above identity, there remains a temptation to fit models of the form

$$z = a + bx + cy + dP + e.$$

Such regressions might be justified, if measurements on one or two of the "independent" variables (x, y, P) were not available.

Since it is known that

$$z \equiv x(1 - P) + yP$$

it is a straightforward matter to derive the covariance of z with each of x, y, and P and hence the full correlation matrix. Specifically,

$$\mathrm{Cov}(z, x) = \mu_{200}(1 - \mu_{001}) + \mu_{101}(\mu_{010} - \mu_{100}) + \mu_{110}\mu_{001} - \mu_{201} + \mu_{111}$$

$$\mathrm{Cov}(z, y) = \mu_{110}(1 - \mu_{001}) + \mu_{011}(\mu_{010} - \mu_{100}) + \mu_{020}\mu_{001} + \mu_{021} - \mu_{111}$$

$$\mathrm{Cov}(z, P) = \mu_{101}(1 - \mu_{001}) + \mu_{002}(\mu_{010} - \mu_{100}) + \mu_{011}\mu_{001} - \mu_{102} + \mu_{012}$$

For the example of the previous section the values are:

$$\mathrm{Cov}(z, x) = 0.000633526041\dot{6}$$

$$\mathrm{Cov}(z, y) = 0.00096314\dot{6}$$

$$\mathrm{Cov}(z, P) = 0.001558791\dot{6}$$

Since the variances of x, y, P, and z have already been determined, the correlation matrix is:

$$\begin{array}{c|cccc|}
 & x & y & P & z \\
\hline
x & 1.0 & 0.09052280 & 0.33103926 & 0.69234252 \\
y & 0.09052280 & 1.0 & 0.16678903 & 0.55664176 \\
P & 0.33103926 & 0.16678903 & 1.0 & 0.78468276 \\
z & 0.62934252 & 0.55664176 & 0.78468276 & 1.0
\end{array}$$

The coefficient of determination of overall specific gravity, z, on the proportion of latewood, P, is:

$$r^2_{zP} = 0.78468276^2 = 0.6157.$$

Likewise

$$r^2_{zx} = 0.69234252^2 = 0.4793.$$

$$r^2_{zy} = 0.55664176^2 = 0.3099.$$

Thus, if one were to use a forward selection procedure for including variables in a multiple linear regression, the first to enter would be the proportion of latewood, which, with these illustrative data would account for approximately 62% of the variation in overall specific gravity. Smith (1973), in a study of the influence of nitrogen fertilization on young Douglas-fir trees, has reported coefficients of determination from 52.4% to 73.0% for the average specific gravity of annual rings estimated from percentage latewood. The example thus agrees well with his results.

By application of the formulae for partial and multiple correlation viz.

$$r_{zx\cdot y} = \frac{r_{zx} - r_{zy}r_{xy}}{\sqrt{(1 - r^2_{zy})(1 - r^2_{xy})}}$$

$$r_{zx\cdot yP} = \frac{r_{zx\cdot y} - r_{zP\cdot y}r_{xP\cdot y}}{\sqrt{(1 - r^2_{zP\cdot y})(1 - r^2_{xP\cdot y})}} = \frac{r_{zx\cdot P} - r_{zy\cdot P}r_{xy\cdot P}}{\sqrt{(1 - r^2_{zy\cdot P})(1 - r^2_{xy\cdot P})}}$$

$$1 - r^2_{z\cdot xP} = (1 - r^2_{zP})(1 - r^2_{zx\cdot P})$$

$$1 - r^2_{z\cdot xyP} = (1 - r^2_{zP})(1 - r^2_{zx\cdot P})(1 - r^2_{zy\cdot xP})$$

etc. (see, e.g. Kendall and Stuart 1967, chapter 27), the coefficients of determination for any combination of the "independent" variables can be readily determined. Hence

$r^2_{z\cdot xP} = 0.8259$, $r^2_{z\cdot yP} = 0.8022$, $r^2_{z\cdot xy} = 0.7254$ and $r^2_{z\cdot xyP} = 0.9979$.

Thus, in this example, around 80% of the variance in overall specific gravity can be accounted for by linear regression on the proportion of latewood and either one of earlywood or latewood specific gravity. Regression on earlywood and latewood specific gravity (with the proportion of latewood excluded) accounts for somewhat less of the overall variance.

The most interesting feature, however, is the coefficient of determination of 99.79% for the regression on all three independent variables. This clearly demonstrates that a high value of the coefficient of determination cannot be taken as evidence of the validity of an additive linear model, since it is known that the true model here is, exactly,

$$z \equiv x(1 - P) + yP.$$

Thus although

$$z = a + bx + cy + dP$$

here provides a remarkably good tracking of the data, it is a purely empirical relationship with no physical meaning ascribable to the parameters a, b, c, d.

DISCUSSION

It has been demonstrated that there is no simple answer to the question of which component has the greatest influence on overall specific gravity. With some typical data, the covariance of the earlywood specific gravity and the latewood proportion, weighted as dictated by the form of the identity, is shown to be one of the more important components of the variance of the overall specific gravity. This can be regarded as confirmation that the simplistic partial differentiation approach, which suggests the dominance of the earlywood specific gravity, is unrealistic. The latter method can apply only if one has the means to vary one component while holding the others fixed, something that does not appear to occur in nature and would be very difficult, if not impossible, to achieve by artificial means.

The exact analysis also has been used to demonstrate why, with a typical data set, regression on the proportion of latewood will account for a respectable proportion of the variability in overall specific gravity and, as a single variable, appears to dominate either earlywood or latewood specific gravity. However, the addition of either earlywood or latewood specific gravity results in a coefficient of determination of about 80%, which must be regarded as a quite good representation of the data.

If one were given all three components, one should, of course, make use of the identity

$$z \equiv x(1 - P) + yP = x + P(y - x).$$

However, with the data set used, multiple linear regression on x, y, and P, yields a coefficient of determination in excess of 99%. This provides a clear warning against ascribing physical meaning to estimated regression coefficients, even when the multiple linear regression provides an excellent representation of the data.

Finally, in her regression study, Diana Smith (1956) obtained a coefficient of determination of 89.06% for total ring specific gravity on the proportion of latewood and the ring width, a value negligibly greater than the 88.94% she obtained on latewood proportion alone. Nevertheless, Smith et al. (1976) express the opinion that "ring density can be estimated from component widths and percentage latewood," although their simple correlations between ring density and ring width are of the same order of magnitude as those given by Diana Smith. (Component widths, of course, provide the same information as total width and latewood proportion, i.e. the one pair is derivable from the other.) Smith et al. (1976) also state, after observing the high correlation between ring density and percentage latewood, that "Adjustment for number of rings from pith and ring width could further improve the use of percentage latewood as a basis for estimating ring density."

Clearly a system for estimating ring density based on width measurements, if reasonably accurate, is operationally preferable to one based on both width and density measurements. Further study of this aspect is, however, outside the scope of the present paper which has been concerned with the ramifications surrounding the use of the three properties, earlywood and latewood specific gravities and the proportion of latewood, from which, via the basic identity, the overall specific gravity can be obtained deterministically.

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