LOGNORMAL CONTROL CHARTS FOR MOISTURE CONTENT OF KILN-DRIED LUMBER

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(Received August 2003)

ABSTRACT

This paper focuses on the application of statistical process control principles to monitor the lumber kiln-drying process through the use of innovative quality control charts. Three Lognormal control charts are proposed to monitor quality characteristics that follow a three-parameter Lognormal statistical distribution. The first two control charts, called the "scale chart" and the "chart for geometric means," monitor the central tendency of the process. The third chart, called the "shape chart," monitors the process variability. Practical procedures are presented for calculating center lines and control limits, and for plotting the data on the charts. A rationale is given for using geometric means rather than arithmetic means for assessing process' central tendency. The choice of parameters to be monitored on control charts, along with parameter estimation issues, are discussed. A succinct comparison with the customary "Normal" charts is also included. The methods presented were tested on a data set of Douglas-fir (*Pseudotsuga menziesii*) lumber collected from a production facility in British Columbia, Canada, for which the statistical distribution of moisture content measurements was determined to be well modeled by a three-parameter Lognormal distribution.

Keywords: Quality control charts, statistical process control, lumber moisture content, Lognormal distribution, Lognormal control charts.

INTRODUCTION

One of the most effective ways to improve consistency in kiln-drying is to monitor the variability of moisture content (MC) in lumber. It is known that moisture content of lumber varies considerably among boards in a kiln charge, and this can happen mainly because of natural variability in drying rate, initial moisture content, sapwood and heartwood, wet pockets in lumber, and variability of drying conditions in different parts of the kiln.

The variability of moisture content can be monitored using statistical process control (SPC). The essential idea of SPC is to continuously improve the quality of a process through the constant application of statistical methods to control that process. A major goal of SPC is to detect the occurrence of any assignable causes of disturbances in the process as soon as possible so that investigation of the process and corrective action can be taken before nonconforming products reach the final stage of the process. This may be done through the use of control charts (Shewhart 1931; Deming 1986; Young and Winistorfer 1999).

The parametric control charts used in SPC are based on the knowledge of the underlying statistical distribution of the data and the assumption is that the data are approximately Normally dis-

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tributed. If the process shows evidence of a significant departure from normality, then the control limits calculated may be inappropriate. The moisture content of kiln-dried lumber does not appear to be well modeled by a normal distribution (McMahon 1961). Some authors (McMahon 1961; Ristea and Maness 2003) suggest that the Lognormal distribution may offer a good model for the moisture content. Ristea and Maness (2003) described formal numerical methods for determining the distribution of moisture content in kiln-dried lumber. Lognormal control charts were discussed by Ferrell (1958), Morrison (1958), and Joffe and Sichel (1968).

Ferrell (1958) proposed a control chart for monitoring the "geometric midrange," which was defined as the Lognormal equivalent of the arithmetic midrange from a Normal distribution (half the sum of the largest and smallest data values). However, the author used an equivalent to the "3-sigma" limits for the geometric midrange without validating the distribution of the parameter. This control chart was developed only for the two-parameter Lognormal. No chart for monitoring process dispersion was discussed.

Morrison (1958) suggested two control charts for monitoring the sample mean and the sample ratio, respectively. However, the limits of these control charts apply only to the two-parameter Lognormal. Also, the measuring units of the control charts were not in the original scale of measurement of data, so the interpretation of these charts by mill personnel may be difficult.

Joffe and Sichel (1968) used the arithmetic mean to sequentially test hypotheses about the mean of a Lognormal distribution. The procedure was an acceptance sampling plan for attribute data, and was intended to be used for the inspection of the final product rather than for a real-time monitoring. Also, the procedure applies only to the two-parameter Lognormal.

Statistical process control in lumber drying

Quality control methods have been successfully applied in many industries. However, the application of statistical process control in the forest products industry, specifically in lumber drying, lags behind other industries. There are three primary reasons for this:

1. Technology development in the past has been based on achieving high production rates of low-valued products. Product quality was not a market driver.
2. Wood is a non-homogenous material, and even when it is kiln-dried under controlled conditions, the moisture content does not stabilize or become constant throughout the wood pieces. Due to this large variability, statistical process control (SPC) methods based on manually inspecting product do not work because of the large volume of product that must be inspected.
3. The mathematical distribution of the moisture content in dried wood is not well understood. Therefore, SPC methods borrowed from other industries do not work well.

Although process variability is often the most important parameter to be measured and controlled, many quality control methods suggested in literature are concerned only with the average moisture content alone, such as sample estimation of average moisture content for a charge (Fell and Hill 1980; Rasmussen 1988; Simpson 1991). Other methods are based on "go/no-go" decision criteria, such as acceptance sampling (Bramhall and Wellwood 1976; Bramhall and Warren 1977); and conformance tests (Cheung 1994). These quality control methods help lumber producers to comply with lumber standards, but they do not help to improve the consistency of drying processes.

Other methods for checking consistency, although concerned with the variation of moisture content, did not find practical applicability because they are based on the often incorrect assumption that moisture content has a Normal probability distribution (Pratt 1953, 1956; Bramhall 1975; Maki and Milota 1993). This assumption judges the asymmetry and significant skewness of the MC distribution as a "defect" or out-of-control situation, when in fact these are typical outcomes of the drying process.

McMahon (1961) proposed Lognormal probability plotting in conjunction with frequency distribution analysis as quality control methods. These techniques were used to estimate the average moisture content and the percentage of MC above or below given limits. The methods were only approximate, however, and the possible procedural errors were large. Also, the author did not consider the third parameter of the Lognormal distribution.

METHODS AND MATERIALS

The data used in this study were related to the data from Ristea (2001) and Ristea and Maness (2003). The distribution of moisture content was found to follow the three-parameter Lognormal distribution. Model validation of verification of the Lognormal distribution, as well as comparison of probability plots with Normal and Weibull, are shown in Ristea (2001) and Ristea and Maness (2003). Moisture content measurements of kiln-dried lumber were collected from a process that was believed to be operating in a state of statistical control. Specific control limits were proposed and quality control charts were developed for Lognormal distributed data, to monitor the process average and dispersion. Subsequent samples collected from later kiln charges were plotted on the proposed charts, and a comparison was made with conventional charts based on the normality assumption.

Data collection

The lumber used in this study was kiln-dried Douglas-fir with a nominal section size of 50.8 mm × 101.6 mm (2 in. × 4 in.), and a nominal length of 1.83 m (6 ft). The lumber came from two different kiln charges that had the same species, dimensions, and drying conditions. Lumber from the first charge was used to determine the parameters of control charts, and lumber from the second charge was used for plotting data on the control charts. The boards were cut in 0.3048-m (1-ft)-long specimens, and their moisture content was determined by oven-drying.

To determine the control limits and centerlines of the control charts, 20 samples were successively selected from the first kiln charge. Each sample contained 50 MC measurements, resulting in a total of 1000 measurements. The sampling procedure was "simple random sampling without replacement."

To plot moisture content data on the control charts, 20 samples were successively selected from the second kiln charge, each sample consisting of 50 MC measurements. The same sampling procedure was used.

Control charts for lognormal data

The methodology for estimating distribution parameters is presented for the three-parameter Lognormal distribution, and control charts for Lognormal data are proposed. The choice of variables to be monitored on control charts is also discussed.

Three-parameter lognormal distribution.—The lognormal distribution function has usually two parameters, scale and shape, and a lower bound of zero. Details on this distribution and its parameters are given in Aitchinson and Brown (1957), and Crow and Shimizu (1988). The probability density function for the two-parameter Lognormal distribution is:

$$f(x) = \frac{1}{x}\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{1}{2}\left(\frac{\ln(x)-\mu}{\sigma}\right)^2\right]\text{for } x>0 \quad (1)$$

where:

μ is the mean (scale parameter) of the normally distributed Y, that is, of logX ($-\infty < \mu < \infty$);

σ^2 is the variance (shape parameter) of the normally distributed Y, that is, of logX; $\sigma > 0$.

However, a third parameter—threshold or location—must be taken into consideration, because the distribution of moisture content in wood has a positive lower bound, always greater than zero. For kiln-dried lumber, this lower bound may be related to the lowest equilibrium moisture content (EMC), which depends on the relative humidity and temperature of the surrounding air in the kiln. The three-parameter Lognormal distribution has the probability density function

$$f(x) = \frac{1}{x-\theta}\frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{\ln(x-\theta)-\mu}{\sigma}\right)^2\right] \text{for } x > \theta > 0 \quad (2)$$

where:

θ is the threshold parameter,

μ is the scale parameter, ($-\infty < \mu < \infty$,)

σ is the shape parameter, $\sigma > 0$.

The cumulative distribution function for the three-parameter case is (for further details refer to Aitchinson and Brown 1957, and Crow and Shimizu 1988):

$$F(x) = \Phi\left(\frac{\ln(x-\theta)-\mu}{\sigma}\right) \text{for } x > \theta \quad (3)$$

with $\Phi(z)$ denoting the standard normal cumulative distribution function.

The three parameter Lognormal distribution has the mean (Aitchinson and Brown 1957):

$$\alpha = \theta + \exp\left(\mu + \frac{\sigma^2}{2}\right) \quad (4)$$

and variance:

$$\beta^2 = \exp\left(2 * \mu + \sigma^2\right) * \left(\exp\left(\sigma^2\right) - 1\right) \quad (5)$$

It is important to note here that a change in the value of the parameter θ affects only the location of the distribution (and its arithmetic mean), and it does not affect the variance or the shape (Johnson et al. 1994).

There is a very important connection between Lognormal and Normal distributions. The Lognormal distribution in its simplest form may be defined as the distribution of a variable whose natural logarithm obeys the normal law of probability. In other words, if a variable X is distributed Lognormal with a threshold parameter θ, a scale parameter μ, and a shape parameter σ, then the variable $Y = \ln(X - \theta)$ has a normal distribution with mean μ and standard deviation σ. This property of the Lognormal is very useful in quality control work, because the methods of statistical process control are well known and widely applied for Normally-distributed variables.

Although many of the properties of the Lognormal may immediately be derived from those of the Normal distribution, there are certain features of the former that differ from anything arising in Normal theory. One example is that the mean and the variance of the Lognormal distribution are not parameters of the distribution, contrasting with the Normal case, where the parameters of the distribution are the mean and the variance. Another example is that, when the threshold value is unknown and it has to be estimated from the sample, this complicates the estimation procedures developed for the two-parameter case (Aitchinson and Brown 1957).

What to monitor on control charts.—SPC control charts typically use the Normal distribution to monitor process average and process dispersion. The process average is checked with a control chart for sample means, and the dispersion is usually monitored with a control chart for either sample standard deviations or sample ranges. For Normally-distributed data, the mean and the standard deviation are parameters of the distribution, and are measures of central tendency and dispersion, respectively. In contrast, for Lognormal data, the mean, standard deviation, and variance are not parameters of the distribution. One question that arises for Lognormal variables is: what parameters should be monitored on the charts to control the central tendency and variability of the process?

One approach would be to monitor the average and the standard deviation of the moisture content, which is a standard practice in quality control applications. For large samples, conventional control charts for sample averages could be constructed regardless of the Lognormal assumption, because of the central limit theorem, which makes sample averages to be approximately Normally distributed with increasingly larger samples. However, the distributions of the standard deviation and the variance of a Lognormal variable are not well known, so conventional methods such as "3-sigma" or probability limits cannot be efficiently employed here.

This paper proposes another approach, which is to monitor directly the scale and shape parameters of the three-parameter Lognormal distribution. This consideration is based on the close relationship between the Lognormal and the Normal distributions. As it was discussed earlier, if a variable X is distributed Lognormal with a threshold parameter θ, a scale parameter μ, and a shape parameter σ; then the variable $Y = \ln(X - \theta)$ has a normal distribution with mean μ and standard deviation σ. Monitoring the mean and standard deviation of the Normal variable Y would actually control the scale and shape parameters of the Lognormal variable X. The threshold parameter would not be monitored directly with a specific chart. However, inferences could be made about a change in the threshold value from the other two charts. For example, if a shift in the scale parameter occurs, it could be an indication that the threshold value also changed, especially if the shape remains the same.

Suppose that the moisture content X is distributed Lognormal with known threshold, scale, and shape parameters. If $x_1, x_2, \ldots x_n$, is a sample of size n, we transform it to Normality, by subtracting the value of θ from all x_i and then taking the natural logarithm:

$$\ln(x_i - \theta) = y_i \tag{6}$$

According to the definition of the Lognormal distribution, the new variable Y is distributed Normal with mean μ and standard deviation σ. The average of the transformed sample is:

$$\bar{y} = \frac{y_1 + y_2 + \ldots + y_n}{n} \tag{7}$$

It is known that $\bar{y}$ is Normally distributed with mean μ and standard error $\frac{\sigma}{\sqrt{n}}$, for moderately large sample sizes (Montgomery 1997). The conventional "3-sigma" control limits and the center line for a control chart for sample means $\bar{y}$ are:

$$\begin{cases} \text{UCL}_\text{N} = \mu + 3 * \dfrac{\sigma}{\sqrt{n}} \\ \text{Center Line}_\text{N} = \mu \\ \text{LCL}_\text{N} = \mu - 3 * \dfrac{\sigma}{\sqrt{n}} \end{cases} \tag{8}$$

The subscript "N" in Eq. (8) refers to the fact that the control limits are developed for Normal distributed data.

In practice, the parameters of the Lognormal distribution θ, μ, and σ, are not known, *a priori.* Therefore, they must be estimated from preliminary samples taken when the process is thought to be in-control. The following section discusses parameter estimation issues for the three-parameter Lognormal distribution.

Parameter estimation.—The knowledge of the threshold parameter θ is critical for the choice of methods employed to estimate the other two Lognormal parameters. If the threshold is known, the estimation methods are well developed and straightforward. If the threshold is not known, and it has to be estimated from historical data, the methods involved in parameter estimation are much more complex.

The threshold is said to be "known" when it can be determined *a priori* by reference to the generating system. The minimum moisture content that the lumber can possibly have during drying may be related to the equilibrium moisture content (EMC) in the kiln, which is determined by the temperature and relative humidity of air inside the kiln. This assumes, of course, that all the lumber entering the kiln has moisture content greater than the initial EMC. However, in order to force water out of the wood, the drying schedules are maintained in a way that does not allow the wood to attain the EMC set by the schedule, so the threshold parameter may be indeed related to EMC, but this relationship is not known yet. Therefore, the threshold parameter cannot be determined solely by reference to the generating system, and it will have to be estimated from samples drawn from historical data.

The fact that θ is unknown creates complications in the estimation of Lognormal parameters. A great deal of research concerning the three-parameter Lognormal was published (Crow and Shimizu 1988). In general, global maximums of the likelihood function are used to estimate distribution parameters. However, Heyde (1963) demonstrated that the three-parameter Lognormal is not uniquely determined by its moments, and this raised questions about moment estimators. Also, Hill (1963) has shown that global maximum likelihood estimators can lead to inadmissible estimates regardless of the sample. Therefore, other estimators have to be used that at least produce reasonable estimates. As an alternative to global maximum likelihood estimators (MLE), local maximum likelihood estimators (LMLE) are generally accepted for the estimation of the threshold, scale, and shape parameters, especially for moderately large sample sizes (Cohen 1951; Cohen et al. 1985; and Crow and Shimizu 1988).

Given a sample of moisture content data, the first parameter that can be estimated is the threshold. For an ascending ordered sample x_1, $x_2, \ldots, x_n$, Cohen (1951) proved the following local maximum likelihood estimator for θ:

$$F(\theta) = \left[\sum_1^n \left(\frac{1}{x_i - \theta}\right)\right] * \left[n * \sum_i^n \ln(x_i - \theta) - n * \sum_1^n \ln^2(x_i - \theta) + \left(\sum_1^n \ln(x_i - \theta)\right)^2\right] - n^2 * \sum_1^n \frac{\ln(x_i - \theta)}{x_i - \theta} = 0 \quad (9)$$

A first approximation $\theta_1 < x_1$ is chosen and $F(\theta_1)$ is evaluated. If $F(\theta_1)$ is zero, then no further calculations are required. Otherwise it is continued until a pair of values θ_i and θ_j is found in a sufficiently narrow interval such that $F(\theta_i) > 0 > F(\theta_j)$ or $F(\theta_i) < 0 < F(\theta_j)$, and the final estimate $\hat{\theta}$ is found by interpolation. When solving Eq. (9) for $\hat{\theta}$, only values for which $0 < \hat{\theta} < x_1$ are accepted.

After obtaining the local maximum likelihood estimate $\hat{\theta}$ for the threshold, the estimates for the other two parameters, scale and shape, can be determined with the following relations (Cohen 1951):

$$\hat{\mu} = \frac{1}{n} * \sum_1^n \ln\left(x_i - \hat{\theta}\right) \quad (10)$$

$$\hat{\sigma}^2 = \frac{1}{n} * \sum_1^n \ln^2\left(x_i - \hat{\theta}\right) - \left[\frac{1}{n} * \sum_1^n \ln\left(x_i - \hat{\theta}\right)\right]^2 \quad (11)$$

It can be seen from Eq. (10) that the scale parameter μ is estimated by the sample mean of the logarithmic-transformed data, $\ln(x - \hat{\theta})$.

Sometimes this estimation technique does not yield a valid result for the threshold, because local maximums of the likelihood function do not always exist, especially in small samples. If this case occurs, the parameters of the distribution can be estimated from the sample by an alternative technique, based on modified moment estimators MME (Cohen et al. 1985).

In practical applications, once the threshold $\hat{\theta}$ is calculated for a kiln and a set of conditions, it will be assumed as constant until further proof that it has changed. The other two parameters are estimated from each sample using Eqs. (10) and (11).

Arithmetic mean vs. geometric mean.—It is known that the arithmetic mean quantifies the central tendency of Normal variables. When the observations are not distributed Normal (but their natural logarithms are), then the geometric mean is a better measure of central tendency. This is true for the two-parameter Lognormal distribution. For the three-parameter Lognormal, the central tendency is determined by the geometric mean minus the threshold. For the Lognormal variable x_i (i = 1,2,. . .,n), $y_i = \ln(x_i)$ is a Normal variable. With a few algebraic operations, the average of y_i (called $\bar{y}$) becomes:

$$\bar{y} = \frac{\sum_{i=1}^n y_i}{n} = \frac{1}{n}\sum_{i=1}^n \left(\ln x_i\right) = \frac{1}{n}\ln\left(\prod_{i=1}^n xi\right) = \ln\left(\prod_{i=1}^n xi\right)^{\frac{1}{n}} \quad (12)$$

It is known that

$$\left(\prod_{i=1}^{n} x_i\right)^{\frac{1}{n}} \tag{13}$$

is the geometric mean of x_i (further called "geomean(x_i)".)

For the three-parameter case, $y_i = \ln(x_i - \hat{\theta})$ is a Normal variable, and the average $\bar{y}$ is given by:

$$\bar{y} = \ln\left(geomean\left(x_i - \hat{\theta}\right)\right) \tag{14}$$

which is equivalent to:

$$\exp(\bar{y}) = geomean\left(x_i - \hat{\theta}\right) \tag{15}$$

This indicates that the geometric mean of the moisture content less the threshold should be actually the entity that is monitored on a chart for controlling central tendency. Once the parameters of the underlying distribution are estimated for the in-control process, the control limits and center lines of the control charts can be determined.

Control chart for the scale parameter (the "scale chart").—The sample scale parameter of the moisture content can be monitored with the "scale chart" described in this section. The scale parameter refers to the three-parameter Lognormal distribution, which it is assumed to be the underlying distribution of MC. It was shown earlier that the sample scale parameters of the MC data are equivalent to the sample averages of the transformed Normal data—the Y variable.

If m samples each of size n are used as preliminary data, and $\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_m$, are the averages of each Normal sample—identical with the scale parameter of each Lognormal sample calculated with Eq. (10)—then the scale parameter of the Lognormal distribution is given by

$$\bar{\bar{y}} = \left(\bar{y}_1 + \bar{y}_2 + \ldots + \bar{y}_m\right) / m \tag{16}$$

Similarly, if S_i is the standard deviation of the i^{th} sample, then

$$\bar{S} = \left(S_1 + S_2 + \ldots + S_m\right) / m \tag{17}$$

The relationship between the standard deviation (of the Normal variable Y) and the shape parameter (of the Lognormal variable X) follows from the biased maximum likelihood estimator of the variance σ^2 for the Normal variable Y (Johnson et al. 1994; Montgomery 1997):

$$S^2 = \frac{\sum_i^n \left(y_i - \bar{y}\right)^2}{n} = \frac{\sum_i^n y_i^2 - \frac{\left(\sum_i^n y_i\right)^2}{n}}{n} \tag{18}$$

If we substitute $\ln(x_i - \hat{\theta}) = y_i$ in (18) the equation becomes:

$$S^2 = \frac{1}{n} * \sum_1^n \left[\ln\left(x_i - \hat{\theta}\right)\right]^2 - \left[\frac{1}{n} * \sum_1^n \ln\left(x_i - \hat{\theta}\right)\right]^2 \tag{19}$$

which is identical in expression with Eq. (11). However, the standard deviation of the sample, S, is not an unbiased estimator of σ. To obtain an unbiased estimator of σ, S must be divided by c_4, a constant that depends on the sample size. For a detailed discussion about c_4 and its values see (Johnson et al. 1994; Montgomery 1997). Therefore, S/c_4 is to be used for estimating the standard deviation σ of the Norma1 population, which is equivalent to estimating the shape parameter σ of the Lognormal population.

Using the common notations for control charts, the center line and control limits for the "scale chart" for sample scale parameters are:

$$\begin{cases} \text{UCL}_\text{N} = \bar{\bar{y}} + 3 * \dfrac{\bar{S}}{c_4\sqrt{n}} \\ \text{Center Line}_\text{N} = \bar{\bar{y}} \\ \text{LCL}_\text{N} = \bar{\bar{y}} - 3 * \dfrac{\bar{S}}{c_4\sqrt{n}} \end{cases} \tag{20}$$

where:

n = the size of the samples that will be used for subsequent plotting;

$\bar{\bar{y}}$ = estimate of the Normal population mean, obtained from preliminary data, given by Eq. (16); it is also an estimate of the Lognormal population scale parameter;

$\bar{S}$ = an estimate of the standard deviation of the Normal population, given by Eq. (17); it is also an estimate of the Lognormal population shape parameter σ,

c_4 = a constant that depends on the sample size; this constant is introduced because $\bar{S}/c_4$, is an unbiased estimator of the population standard deviation.

Control chart for geometric means.—The chart described above could be used as it is. However, it will not make much sense to the ordinary user, because the values plotted don't make much sense in terms of moisture content, due to the logarithmic transformation. The "chart for geometric means" is proposed here to address the problem of plotting the data in the original scale of measurements. The control limits and center line for the original scale are obtained by taking the antilog by exponentiation and then adding the threshold. From $Y = \ln(X - \theta)$ it follows that $X = \theta + \exp(Y)$, and this relationship will be used to transform all the results back to the original scale of measurements.

$$\begin{cases} \text{UCL}_{\text{LN}} = \hat{\theta} + \exp(UCL_N) \\ \text{Center Line}_{\text{LN}} = \hat{\theta} + \exp(CenterLine_N) \\ \text{LCL}_{\text{LN}} = \hat{\theta} + \exp(LCL_N) \end{cases} \quad (21)$$

The subscript "LN" in Eq. (21)refers to the fact that the control limits are developed for the Lognormal distributed data. By substituting (20) in (21) the center line and control limits of the "chart for geometric means" become:

$$\begin{cases} \text{UCL}_{\text{LN}} = \hat{\theta} + \exp\left(\bar{\bar{y}} + 3 * \dfrac{\bar{S}}{c_4\sqrt{n}}\right) \\ \text{Center Line}_{\text{LN}} = \hat{\theta} + \exp\left(\bar{\bar{y}}\right) \\ \text{LCL}_{\text{LN}} = \hat{\theta} + \exp\left(\bar{\bar{y}} - 3 * \dfrac{\bar{S}}{c_4\sqrt{n}}\right) \end{cases} \quad (22)$$

Once these control limits are established, they can be used to monitor subsequent kiln charges that are obtained under the same drying conditions, for the same type of material and dimensions. To plot a point on the "chart for geometric means," a sample x_i (i = 1,2,. . .,n) of MC measurements is taken, then x_i is transformed to a normal variable $y_i = \ln(x_i - \hat{\theta})$. The sample average of y_i, which is $\bar{y}$, is then plotted on the "scale chart." To plot the corresponding point on the "chart for geometric means," $\bar{y}$ needs to be transformed to the original scale of measurements, with:

$$anti\log(\bar{y}) = \hat{\theta} + \exp(\bar{y}) \quad (23)$$

At the first sight, plotting *anti*log($\bar{y}$) on the "chart for geometric means" seems to be just an algebraic manipulation. However, it was demonstrated earlier that exp($\bar{y}$) is in fact the geometric mean of $(x_i - \theta)$.

From (15) it follows that the point plotted on the "chart for geometric means" is:

$$anti\log(\bar{y}) = \hat{\theta} + geomean\left(x_i - \hat{\theta}\right) \quad (24)$$

A note should be made here about the proposed name of this control chart, which may incorrectly suggest to the user that the chart monitors the actual geometric mean of the moisture content. The entity that is monitored on the "chart for geometric means" is in fact the geometric mean of, moisture content *less* the threshold. The threshold is added back later just to give the right scale to the chart.

It is important to note that, besides the scale of the plotted data, there are practically no differences between the "scale chart" and the "chart for geometric means," and either one could be used in practice with equivalent results.

The following is a procedure for constructing the "chart for geometric means":

1. For each kiln and set of conditions (species, type of lumber, stacking method, drying schedule), collect m random samples of n moisture content measurements, from when the process is thought to be in-control, that is, operating as consistently as it can. It is recommended that m ≥ 20, and n can be as low as 5 (Montgomery 1997), but considering that the number of boards in a kiln is so large, a reasonably larger sample size is needed.
2. Determine if the moisture content is distributed Lognormal, using the methods presented in Ristea and Maness (2003). If the three-parameter Lognormal is not a good

model for the data, then the methods presented here may not be appropriate. This procedure assumes hereafter that the MC is distributed Lognormal.

3. From the preliminary data estimate the threshold parameter using Eq. (9). The threshold should be estimated from the entirety of preliminary data, instead as average of sample estimates, because of estimation problems explained above. With $\hat{\theta}$ known, the other two parameters are estimated from the transformed samples, using (16) and (17).
4. Calculate the center lines and the control limits for both charts using relations (20) and (22). While rational sampling and subgrouping of the data are not discussed here, care should be exercised to not underestimate total variation, which can lead to too tight limits on the scale chart. Equations from (20) define the "scale chart," which is used to monitor the sample scale parameter. Equations from (22) describe the corresponding chart in the original scale of measurements, the "chart for geometric means."
5. To plot the $\bar{y}$ data for a subsequent sample of size n, first normalize x_i by subtracting $\hat{\theta}$ from each value, and then taking the natural logarithm, $\ln(x_i - \hat{\theta})$. Calculate the mean of the transformed sample, $\bar{y}$, which is plotted on the "scale chart."
6. To plot the corresponding value on the "chart for geometric means," the result needs to be brought back to the original scale of measurements: *anti*$\log(\bar{y}) = \hat{\theta} + \exp(\bar{y})$.

Control chart for the shape parameter (the "shape chart").—The third control chart proposed in this paper monitors the sample standard deviation of transformed data. This chart is just an adaptation of the customary Normal chart for standard deviations, and probability limits are proposed to be used in the construction of this control chart. Because the shape parameter of the Lognormal distribution is equivalent to the standard deviation of the transformed data, this chart will be called the "shape chart." The construction of this chart is based on the fact that the variance of the Normal variable Y follows a "chi-square" statistical distribution:

$$\text{If } Y \sim N(\mu, \sigma^2), \text{ then } \frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1} \quad (25)$$

where:

Y is the normal variable with mean μ and variance σ^2,

S^2 is the sample variance, and

χ^2_{n-1} is the chi-square distribution with n-l degrees of freedom.

A 100(l – α)% two-sided confidence interval on the variance is:

$$\frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,n-1}} \quad (26)$$

where:

$\chi^2_{1-\alpha/2,n-1}$ denotes the percentage point of the chi-square distribution such that $P\{\chi^2_{n-1} \ge \chi^2_{\alpha/2,n-1}\} = \alpha/2$.

The control limits for the "shape chart" are calculated as follows (Montgomery 1997):

$$\begin{cases} \mathrm{UCL_N} = \dfrac{\bar{S}}{c_4}\sqrt{\dfrac{\chi^2_{\alpha/2,n-1}}{n-1}} \\ \text{Center Line}_\mathrm{N} = \dfrac{\bar{S}}{c_4} \\ \mathrm{LCL_N} = \dfrac{\bar{S}}{c_4}\sqrt{\dfrac{\chi^2_{1-\alpha/2,n-1}}{n-1}} \end{cases} \quad (27)$$

where:

$\bar{S}$ is an estimate of the standard deviation of the Normal population, given by (17).

The following is a procedure for constructing the control chart for Lognormal shape parameter, the "shape chart":

1. Using the results from steps 1 to 4 of the procedure above, calculate the center lines and the control limits using (27).

TABLE 1. *Parameters of preliminary data for control charts based on the Normal distribution.*

Parameter	Symbol	Value
Estimate of the process average	$\bar{\bar{x}}$	14.484
Biased estimate of process standard deviation	$\bar{s}$	1.624

2. To plot a subsequent sample of size *n,* calculate the standard deviation S of the transformed sample, $\ln(x_i - \hat{\theta})$, and then plot this value on the "shape chart."

RESULTS AND DISCUSSION

The charts proposed in this paper are based on the assumption that the moisture content reasonably follows a three-parameter Lognormal distribution. On the other hand, in many practical applications the moisture content is assumed to have a Normal distribution. To demonstrate the difference between these two approaches, the control charts are constructed first by assuming a Normal distribution, and then by applying the correct Lognormal distribution.

The preliminary data, consisting of 20 samples of 50 measurements each, was used to estimate the parameters of the Normal distribution, given in Table 1.

The control limits for the customary control charts based on the Normality assumption were calculated with established methods—not presented here—and are given in Table 2. Three-sigma limits were employed for the "X bar chart Normal," and probability limits were used for the "S chart Normal"; the charts are shown in Fig. 1 and Fig. 2. The data plotted on the two charts came from 20 samples randomly selected from a subsequent kiln charge with the same species and drying conditions (see Table 5).

TABLE 2. *Control limits and center lines for control charts based on Normal assumption.*

	X Bar Chart Normal *[%]*	*S* Chart Normal *[%]*
UCL	15.18	2.12
Center Line	14.48	1.63
LCL	13.79	1.17

The "X bar chart Normal" in Fig. l shows one point above the upper control limit, the 4th data point. In the "S chart Normal" (Fig. 2) the 5th data point is also outside the control limits. If a practitioner would analyze these charts, without other knowledge about how the process is performing, would draw the conclusion that 2 out-of-control situations have occurred, and further investigation is necessary. However, the process from which the data were collected was believed to be in a state of statistical control without any process shifts or assignable causes, although a formal investigation into the state of control was not performed. The out-of-control signals given by these charts are likely to be just "false alarms," or type I errors of the control charts (concluding the process is out of control when it is really in control).

The distribution of the experimental data was found to be well modeled by the three-parameter Lognormal with parameters summarized in Table 3.

The control limits for the Lognormal charts presented in Table 4 were calculated using procedures described in this paper, and Lognormal charts were constructed. Figure 3 shows the "scale chart," Fig. 4 shows the "chart for geometric means," and the "shape chart" is shown in Fig. 5. For a valid comparison between the 2 types of charts (respectively based on Normality and Lognormality assumptions), the data plotted on the Lognormal charts came from the same samples chosen for the Normality assumption.

As expected, the "scale chart" and the "chart for geometric means" have practically identical plots, although the scale of measurement is different. This suggests that the analysis of patterns and sensitizing rules that are found to be valid for the "scale chart," could also be applied to the corresponding "chart for geometric means." Practitioners may find it convenient to use the "chart for geometric means" because it shows the control limits and plotted data in the original scale of measurements–percentages of moisture content.

The "chart for geometric means" does not show any out-of-control signals, in contrast with what the "X bar chart Normal" reveals. It contains

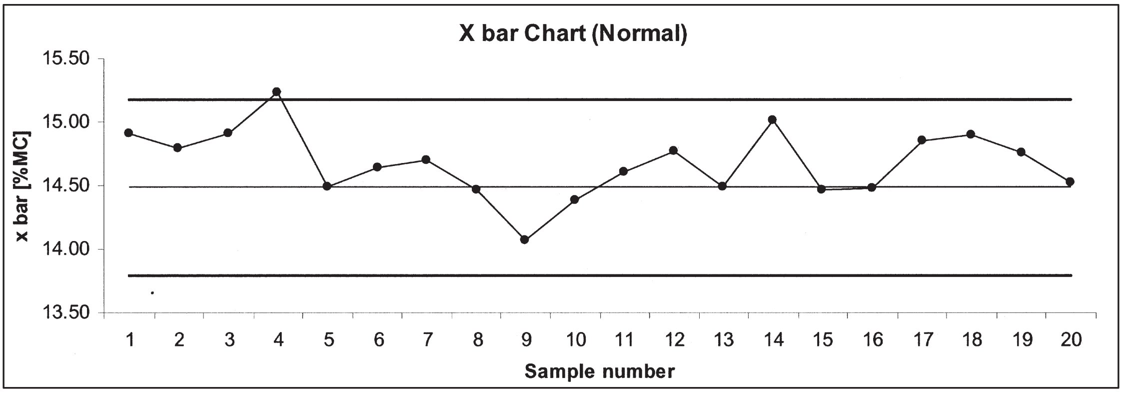

FIG. 1. "X Bar Chart" for sample averages based on Normality assumption.

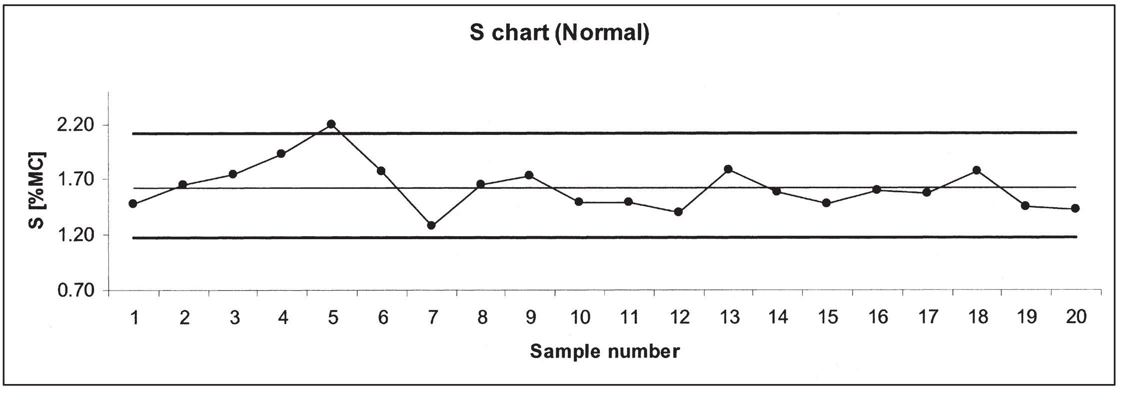

FIG. 2. "S Chart" for sample standard deviations based on Normality assumption.

TABLE 3. *Parameters of preliminary data for control charts based on the Lognormal distribution.*

Parameter	Symbol	Value
Lognormal variable X		
Threshold	$\hat{\theta}$	7.48
Scale	$\hat{\mu}$	1.92
Shape	$\hat{\sigma}$	0.23
Number of Samples	m	20
Sample size	n	50
Normal variable *Y*		
Estimate of the mean of the transformed data	$\bar{\bar{y}}$	1.92
Biased estimate of standard deviation of the transformed data	$\bar{s}$	0.23

other out-of-control indicators, such as four out of five consecutive points beyond one-sigma, and eight consecutive points are above centerline. While it is true that the 5th point on the "chart for geometric means" is very close to the upper limit, it is still within the prescribed limits, and will not produce an out-of-control signal. Except for the 5th data point, these two charts seem to plot the data in a similar manner. This was anticipated because the sample averages, which are monitored on the charts, tend to follow a Normal distribution, according to the Central Limit Theorem, for increasingly large samples.

A larger dissimilarity can be seen between the "shape chart" and the "S chart Normal." The "shape chart," based on the Lognormal assumption, does not show any out-of-control situations, suggesting what was expected, that the process is in a state of statistical control. In contrast, the "S chart Normal" not only shows an out-of-control signal, but almost all the other data points are closer to the upper control limit. It is apparent that the "S chart Normal" might have a tendency to overestimate the variability inherent in the data, likely due to the Normality assumption. Moisture content observations in the upper tail of the distribution are a typical occurrence in Lognormal data, and a properly chosen control chart should consider this accordingly. The control limits of the "shape chart" are constructed in such a way as to allow for larger variations of MC, which are due to the inherent positive skewness of the Lognormal data. On the other hand, the "S chart Normal" may falsely signal out-of-control conditions, generating type I errors.

TABLE 4. *Control limits and center lines for control charts based on Lognormal distribution.*

	Scale Chart *[unitless]*	Chart for Geometric Means *[%]*	Shape Chart *[unitless]*
UCL	2.02	15.00	0.29
Center Line	1.92	14.30	0.23
LCL	1.82	13.67	0.16

The mathematical calculations presented in this research may seem cumbersome for a practical application of these control charts in a mill. However, as more and more quality control personnel receive training in basic Statistical Process Control tools and the use of computers, these calculations can be performed routinely by simple spreadsheet formulations. It is also noted that, while ideally a control chart would allow immediate feedback on the drying process, applying SPC on the freshly-dried lumber is after the fact and too late to prevent non-conformances in the kiln.

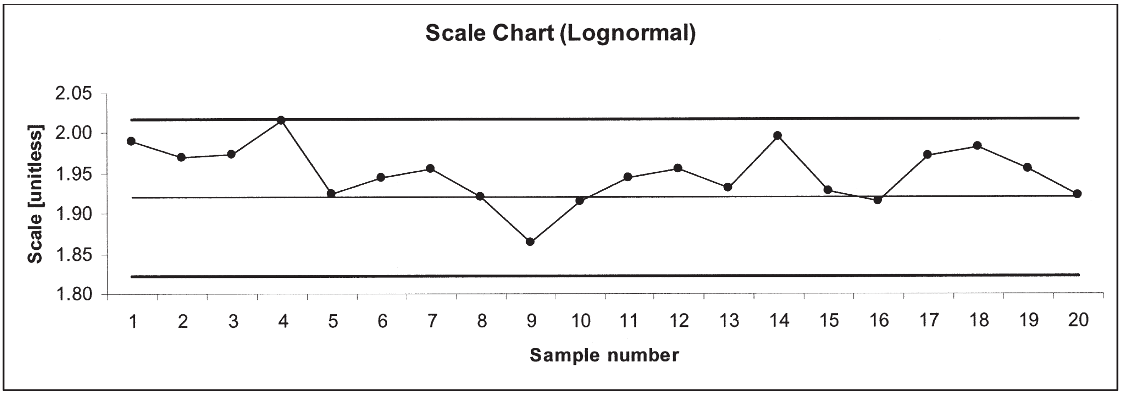

FIG. 3. "Scale Chart" for sample scale parameters based on Lognormality.

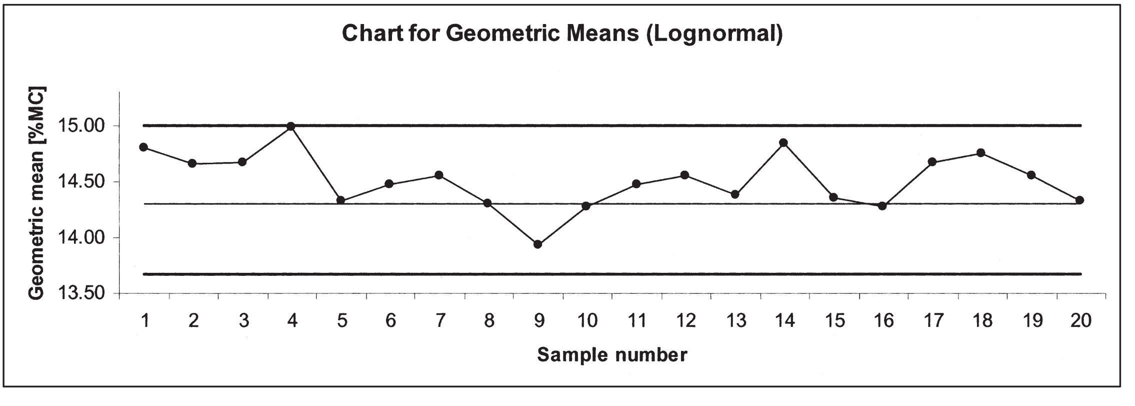

FIG. 4. "Chart for Geometric Means" based on Lognormality.

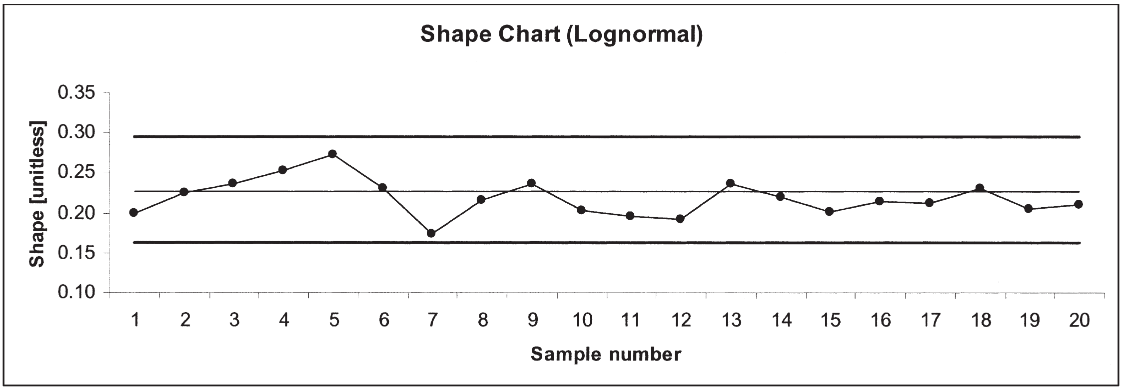

FIG. 5. "Shape Chart" for sample shape parameter based on Lognormality.

TABLE 5: Data points plotted on control charts.

Sample No.	*X* Bar Chart Normal [%]	*S* Chart Normal [%]	Scale Chart [unitless]	Chart for Geometric Means [%]	Shape Chart [unitless]
1	14.91	1.48	1.99	14.80	0.20
2	14.79	1.65	1.97	14.65	0.23
3	14.91	1.74	1.97	14.68	0.24
4	15.23	1.94	2.02	14.99	0.25
5	14.49	2.20	1.92	14.33	0.27
6	14.64	1.78	1.94	14.48	0.23
7	14.70	1.29	1.95	14.54	0.17
8	14.47	1.65	1.92	14.31	0.22
9	14.08	1.73	1.86	13.94	0.24
10	14.38	1.50	1.92	14.27	0.20
11	14.61	1.49	1.94	14.47	0.20
12	14.77	1.41	1.96	14.55	0.19
13	14.49	1.78	1.93	14.38	0.24
14	15.01	1.59	2.00	14.84	0.22
15	14.47	1.48	1.93	14.36	0.20
16	14.48	1.60	1.92	14.27	0.21
17	14.85	1.57	1.97	14.67	0.21
18	14.89	1.77	1.98	14.74	0.23
19	14.76	1.45	1.96	14.55	0.21
20	14.52	1.42	1.92	14.32	0.21

The control charts presented here can be still useful for root cause analysis.

LIMITATIONS OF THE STUDY

The specific numerical data and results of the study can be applied to Douglas-fir lumber from the interior of British Columbia, Canada, for comparable kiln types, drying conditions, and lumber sizes. However, the methods for constructing control charts can be applied to many practical situations. For each kiln and set of conditions (species, lumber type and size, drying conditions), the distribution of moisture content should be first assessed. If the distribution is proved to be well modeled by the Lognormal, control limits could then be established using the calculated distribution parameters, and the charts proposed here could be used to monitor the process average and dispersion.

CONCLUSIONS

This paper proposed three Lognormal control charts, two of which are equivalent, to monitor process central tendency and variability, with application to the moisture content of kiln-dried lumber. A rationale was given for using geometric means rather than arithmetic means for assessing process' central tendency. Instead of monitoring sample averages and sample standard deviations, it was proposed to monitor the scale and shape parameters of the three-parameter Lognormal distribution. Moisture content data were collected from a kiln-drying process that operated in a state of statistical control, and it was found to follow a three-parameter Lognormal distribution. Control limits and center lines of Lognormal charts were calculated with the methods proposed in this paper. The data plotted on the Lognormal control charts came from a different kiln charge, which had however the same species, dimensions and drying conditions, and which was also known to have been operating in a state of statistical control. When these Lognormal charts were compared to their customary counterparts based on the Normal distribution, it was found that the latter indicated that the process is out-of-control when it was really in control. Control charts that are based on the Lognormal assumption are more appropriate to use in cases when the moisture content has such a skewed distribution, and especially when monitoring process variability.

A practical implementation of the proposed charts is recommended to begin with the assessment of the underlying distribution of moisture content. If it is concluded that a three-parameter Lognormal is a proper fit, then the central tendency and the variability of the drying process can be monitored with the charts proposed in this paper. The central tendency would be monitored with the "scale chart" or with its equivalent "chart for geometric means," which plots the data in the original scale of measurements. The variability of the process would be monitored with the proposed "shape chart."

ACKNOWLEDGMENTS

The authors would like to thank Wagner Electronics Inc. for providing financial and technical support on this research, especially Ed Wagner whose vision made this project possible, and also Jim Felsheim and Ron Smith, who made everything easier.

REFERENCES

AITCHINSON, J., AND J.A.C. BROWN. 1957. The Lognormal distribution. Cambridge University Press, London, UK.

BRAMHALL, G. 1975. Meeting new kiln drying standards. Can. Forest Ind. 95(9):33–35.

———, AND W. G. WARREN. 1977. Moisture content control in drying dimension lumber. Forest Prod. J. 27(7):26–28.

———, AND R. W. WELLWOOD. 1976. Kiln drying of western Canadian lumber. Canadian Forestry Service, Western Forest Products Laboratory, Vancouver, Canada.

CHEUNG, K. C. 1994. Hand-held moisture meters in lumber production, *in* ASTM Hand-held Moisture Meter Workshop. Forest Products Society, Madison, WI.

COHEN, A. C. 1951. Estimating parameters of logarithmic-normal distributions by maximum likelihood. J. Am Statis. Assoc. 46(254):206–212.

———, B. J. WHITTEN, AND Y. DING. 1985. Modified moment estimation for the three-parameter lognormal distribution. J. Quality Technol. 17(2):92–99.

CROW, E. L., AND K. SHIMIZU. 1988. Lognormal distributions, theory and applications. Marcel Dekker, New York, NY.

DEMING, W. E. 1986. Out of the crisis. Massachusetts Institute of Technology, Center for Advanced Engineering Study. Cambridge, Mass. 507 pp.

FELL, J. D., AND J. L. HILL. 1980. Sampling levels for hardwood kiln-drying control. Forest Prod. J. 30(3):32–36.

FERRELL, E. B. 1958. Control charts for log-normal universes. Ind. Quality Cont. l5(2):4–6.

HEYDE, C. C. 1963. On a property of the lognormal distribution. J. Royal Statist. Soc. Series B (Methodological) 25(2):392–393

HILL, B. M. 1963. The Three-parameter lognormal distribution and bayesian analysis of a point source epidemic. J. Am. Statist. Assoc. 58(301):72–84.

JOFFE, A. D., AND H. S. SICHEL. 1968. A chart for sequentially testing observed arithmetic means from lognormal populations against a given standard. Technometrics 10(3):605–612.

JOHNSON, N. L., S. KOTZ, AND N. BALAKRISHNAN. 1994. Continuous univariate distributions. Vol. 1, 2nd ed., John Wiley and Sons, New York, NY.

MAKI, R. G., AND M. R. MILOTA. 1993. Statistical quality control applied to lumber drying. Quality Progress 26(12):75–80.

McMahon, E. P. 1961. Moisture control during kiln drying. Forest Prod. J. 11(3):133–138.

Montgomery, D. C. 1997. Introduction to statistical quality control. 3rd ed., John Wiley and Sons, New York, NY.

Morrison, J. 1958. The lognormal distribution in quality control. Applied Statistics 7(3):160–172.

Pratt, W. E. 1953. Some applications of statistical quality control to the drying of lumber. J. FPRS 3(5):28–31.

———. 1956. Estimating the moisture content of lumber during the drying process. Forest Prod. J. 6(9):333–337.

Rasmussen, E. F. 1988. Dry kiln operator's manual. Forest Products Laboratory, Hardwood Research Council, Memphis, TN.

Ristea, C. 2001. Quality control methods for monitoring the variability of moisture content in kiln-dried lumber. MSc Thesis, the University of British Columbia, Vancouver, BC, Canada.

———, and T. C. Maness. 2003. Methods for analyzing the distribution of moisture content in kiln-dried lumber. Accepted for publication the J. of the IWSc.

Shewhart, W. A. 1931. Economic control of quality of manufactured product. D. Van Nostrand Company Inc. New York, NY. 501 pp.

Simpson, W. T. 1991. Dry kiln operator's manual. USDA Forest Prod. Lab. Forest Prod. Society, Madison, WI.

Young, T. M., and P. M. Winistorfer. 1999. Statistical process control and the forest products industry. Forest Prod. J. 49(3):10–17.