

A NEW METHOD FOR SEPARATING DIFFUSION COEFFICIENT AND SURFACE EMISSION COEFFICIENT

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ABSTRACT

An analytical procedure has been developed to separate the diffusion and surface emission coefficients in Newman's solution of the unsteady-state diffusion equation from a single lumber-drying curve. Previous methods required two to four drying curves corresponding to as many specimen thicknesses to achieve the same purpose. The new procedure reduces the experimental effort by 50% and more. The separation of the two coefficients will make it possible to study the dependence of diffusion coefficient on moisture concentration and that of surface emission coefficient on viscosity and velocity of airflow in any specified drying environment. Numerical examples are included to demonstrate the application of the new technique.

Keywords: Diffusion coefficient, diffusion equation, lumber-drying curve, moisture concentration, moisture in wood, surface emission coefficient.

INTRODUCTION

In this study we separated the diffusion coefficient, D , and the surface emission coefficient, S , in Newman's solution (Newman 1931) of the unsteady-state diffusion equation (Fick's second law) from a lumber-drying curve. The diffusion coefficient describes the rate of internal moisture movement, whereas the surface emission coefficient describes the rate at which moisture is lost from the surface. The absolute and relative values of D and S are important in determining the drying rate of lumber. A lumber-drying curve shows the average moisture concentration in wood at any time during drying.

In Newman's solution, both D and S are assumed to be constant. This is not true for wood, especially when it is above fiber saturation (Skaar 1954). In the hygroscopic range, the value of D , evaluated at the time when the fraction of average moisture in wood E equals 0.5, can be used to determine the dependency of D on moisture concentration C (Crank 1975); the value of S can be used to evaluate the effects of viscosity and velocity of airflow on lumber-drying rate through correlation with the Schmidt number and the Reynolds number (Rohsenow and Choi 1961). Therefore, the separation of D and S in Newman's solution is of practical importance in lumber-drying research.

Skaar (1954) discussed a procedure to determine both S and D when the moisture distributions in a board at various times during the drying period are known.

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Choong and Skaar (1969) were the first to use the following approximate equation for Newman's solution (1931) to obtain both S and D:

$$t_{0.5}/0.2a = a/D + 3.5/S$$

where $t_{0.5}$ is the half-drying time (the time when the fraction of average moisture in wood $E = 0.5$), and a is half of the specimen thickness. In their tests, Choong and Skaar used yellow-poplar sapwood and heartwood specimens with moisture movement confined to either the tangential or the radial direction. By using specimens of two different thicknesses for each case, they obtained two equations in which D and S are the unknowns and can be readily evaluated.

Later, Choong and Skaar (1972) performed tests with sweetgum and redwood specimens with moisture moving in either the longitudinal or the tangential direction. For each case, they used specimens of four thicknesses and drew a straight line through the four data points in a plot of $t_{0.5}/0.2a$ versus a . According to the aforementioned equation, the slope of the line should give the value of $1/D$ and the intercept of it the value of $3.5/S$. Choong and Skaar found that S must be considered in predicting the lumber-drying rate and that S is a function of wood density, among other things. These results were confirmed by Rosen (1982a).

Rosen (1978) used the same equation to analyze moisture adsorption rates in specimens of black walnut and silver maple in the radial and the longitudinal directions. Three thicknesses and six air velocities were used for each case. He found that the relative importance of the ratio S/D increased as specimen thickness and/or air velocity decreased and was greater for the longitudinal than for the radial direction.

With the importance of surface emission coefficient in lumber drying clearly established, the experimental data by Biggerstaff (1965), Choong and Fogg (1968), Comstock (1963), and McNamara and Hart (1971) may need to be reanalyzed because these studies disregarded surface effects. As reported by Rosen (1978), the true value of D can be significantly larger than its apparent value with $S = \infty$.

The present study describes a method to estimate the diffusion and surface emission coefficients from just one lumber-drying curve. This will eliminate the necessity of using specimens of several thicknesses or measuring moisture distribution in a specimen to achieve the same purpose. The saving in experimental effort thus realized can be significant.

BASIC EQUATIONS AND ANALYSIS

Moisture diffusion in wood has generally been analyzed using Fick's second law, which is also called the diffusion equation:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial X} \left(D \frac{\partial C}{\partial X} \right) \quad (1)$$

where C is moisture concentration, t time, D diffusion coefficient, and X space coordinate measured from the center of a symmetrical specimen in the direction of moisture flow.

Under the following boundary and initial conditions

$$C = C_0, \quad -a < X < a, \quad t = 0 \quad (2)$$

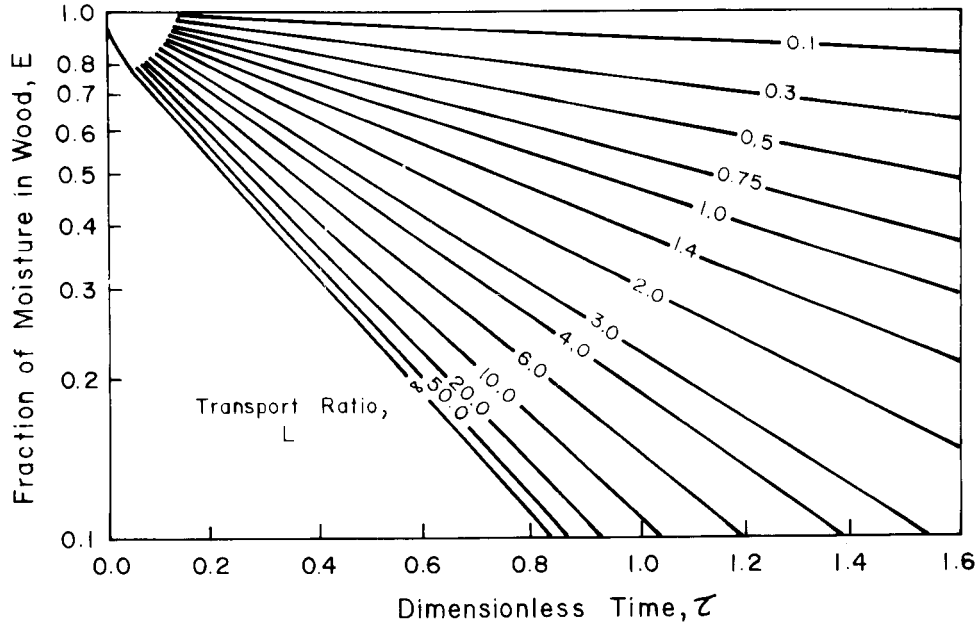


FIG. 1. Fraction of moisture in wood E versus dimensionless time τ for various values of transport ratio L (from Skaar 1954). (ML88 5381)

and

$$\pm D \frac{\partial C}{\partial X} = S(C_a - C_e), \quad X = \pm a, \quad t \geq 0 \quad (3)$$

where C_a is the moisture concentration in equilibrium with the vapor pressure in the airflow remote from the surface, C_e is the actual moisture concentration in the surface at any time, and a is half of the specimen thickness, the solution to Eq. (1) (Crank 1975) becomes

$$E = \frac{\bar{C} - C_a}{C_0 - C_a} = 2L^2 \sum_{n=1}^{\infty} \frac{\exp(-\beta_n^2 \tau)}{\beta_n^2(\beta_n^2 + L^2 + L)} \quad (4)$$

where $\bar{C}$ is the average value of C across the specimen thickness, β_n are the positive roots of

$$\beta_n \tan \beta_n = L \quad (5)$$

$$L = Sa/D \quad (6)$$

and

$$\tau = \frac{Dt}{a^2} \quad (7)$$

The solution of Eq. (4) for various values of the transport ratio L has been obtained by Newman (1931) and Skaar (1954) and is plotted in Fig. 1. From this, Choong and Skaar (1969) obtained the following relationship between $\tau_{0.5}$ and L :

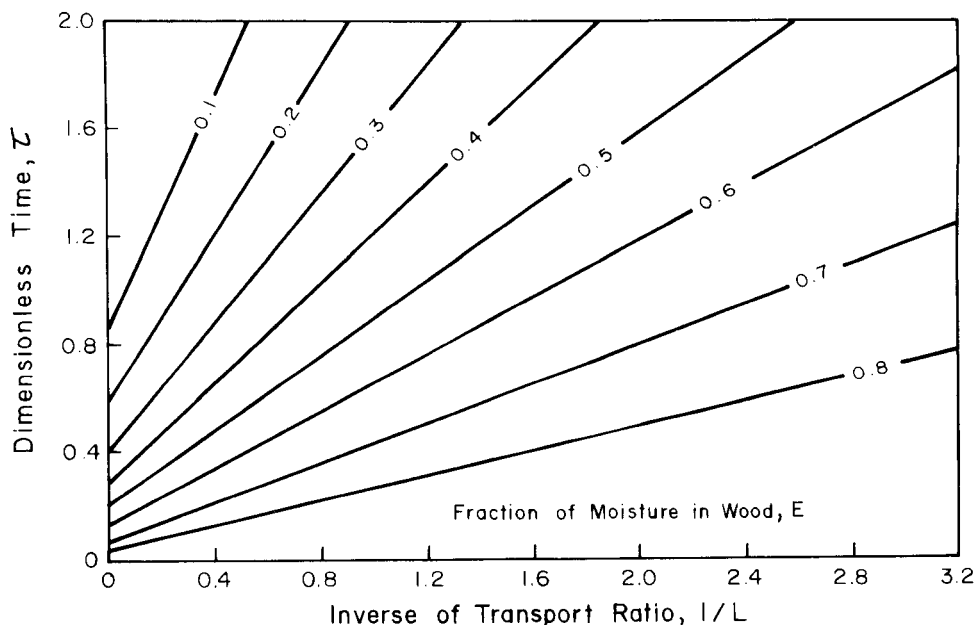


FIG. 2. Dimensionless time τ versus inverse of transport ratio $1/L$ for various values of moisture fraction in wood E . (ML88 5382)

$$\tau_{0.5} = 0.2 + 0.7/L \quad (8)$$

where $\tau_{0.5}$ is the value of τ when $E = 0.5$. Replacing Eq. (3) by $C = C_e$ at $X = \pm a$ and $t \geq 0$, we obtain the simplest form of the solution to Eq. (1) as

$$E = 1 - 2 \left(\frac{Dt}{\pi a^2} \right)^{1/2} \quad (9)$$

which is valid for small times only (Crank 1975). We note that by assuming no external resistance ($L = \infty$) in Eq. (8) and $E = 0.5$ in Eq. (9), the two equations are approximately the same.

According to Crank (1975), the value of D in Eq. (9) is some mean value $\bar{D}$ when $E = 0.5$, and

$$\bar{D} = \frac{1}{C_0} \int_0^{C_0} D(C) dC \quad (10)$$

which provided the basis to obtain some functional relationship between D and C . It is therefore necessary to separate D and S in L of Eq. (8) for $L \neq \infty$ to study the relationship between D and C and that between S and the viscosity and velocity of the airflow (Rohsenow and Choi 1961).

Figure 1 shows that for any specified value of E , the relationship between τ and $1/L$ is linear. This is plotted in Fig. 2. Therefore, Eq. (8) can be put in the general form

$$\tau_E = A + \frac{B}{L} \quad (11)$$

TABLE 1. Values of A and B for eight values of E in Eq. (11).¹

Con- stants	Values of E							
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
A	0.8448	0.5680	0.4030	0.2900	0.1963	0.1256	0.0706	0.0314
B	2.1512	1.5684	1.1980	0.9311	0.7010	0.5250	0.3720	0.2390

¹ E = average fraction of moisture in wood.

where A and B are constants for any specified value of E . From Fig. 2, A and B in Eq. (11) for each line can be obtained. These are shown in Table 1, which can be used to separate S and D contained implicitly in Eq. (8) from one drying curve.

Let a typical drying curve with fraction of moisture in wood E against time t be plotted in Fig. 3. E_1 and E_2 are two values on the curve with corresponding times t_1 and t_2 . Substitute Eq. (7) for τ in Eq. (11), and let A and B be subscripted likewise. We obtain

$$\frac{Dt_1}{a^2} = A_1 + \frac{B_1}{L} \quad (12)$$

$$\frac{Dt_2}{a^2} = A_2 + \frac{B_2}{L} \quad (13)$$

where D , L , and a are constants.

Eliminating L from Eqs. (12) and (13) yields

$$\frac{D}{a^2} = \frac{A_1/B_1 - A_2/B_2}{t_1/B_1 - t_2/B_2} \quad (14)$$

Since A , B , and t all vary with E as shown in Figs. 3 and 4, Eq. (14) can be put in the following form:

$$\frac{D}{a^2} = \left. \frac{d(A/B)/dE}{d(t/B)/dE} \right|_{E=0.5} \quad (15)$$

which is evaluated at $E = 0.5$. From now on D is understood to correspond to $E = 0.5$ for $L \neq \infty$. Equation (15) can be further simplified to

$$\frac{D}{a^2} = \left. \frac{B dA/dE - A dB/dE}{B dt/dE - t dB/dE} \right|_{E=0.5} \quad (16)$$

From Table 1 and Fig. 4, Eq. (16) becomes

$$\frac{D}{a^2} = \left. \frac{-0.1654}{0.701 dt/dE + 2.05 t} \right|_{E=0.5} \quad (17)$$

where t and dt/dE are obtained from the drying curve with $E = 0.5$.

Some authors (Biggerstaff 1965; Choong and Skaar 1969) plotted their drying curves as E versus $t^{1/2}$ rather than E versus t . When that is the case, the following relationship may be conveniently used in Eq. (17):

$$\frac{dt}{dE} = 2t^{1/2} \frac{dt^{1/2}}{dE} \quad (18)$$

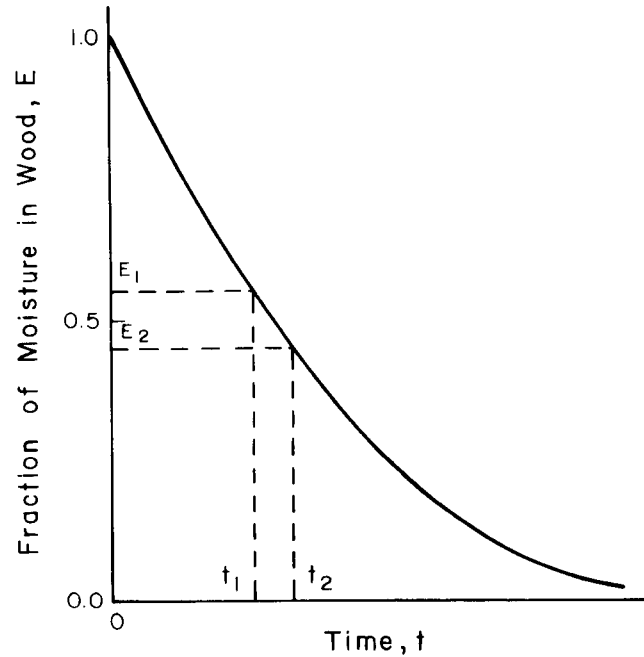


FIG. 3. Typical drying curve showing fraction of moisture in wood E versus time t . (ML88 5383)

As mentioned earlier, several authors disregarded the surface effects in their studies and used Eq. (9) to evaluate D . It would be of interest to compare the D value so evaluated with that obtained from Eq. (17) (Skaar 1987). Let D in Eq. (9) be denoted by $\bar{D}$ for $E = 0.5$. We obtain from Eq. (9)

$$t = 0.1963 \frac{a^2}{\bar{D}} \quad (19)$$

and

$$\frac{dt}{dE} = -0.7854 \frac{a^2}{\bar{D}} \quad (20)$$

Substituting these into Eq. (17) yields

$$D = 1.116\bar{D} \quad (21)$$

Note that Eq. (9) is true for small times and $L = \infty$. Therefore, Eq. (21), which is based on Eq. (9) through Eqs. (19) and (20), can only be considered as approximately correct. Once D is obtained, L can be obtained from either Eq. (12) or Eq. (13) with $E = 0.5$ and S from Eq. (6).

NUMERICAL EXAMPLES

Choong and Skaar (1969) used Eq. (11) with $E = 0.5$ to calculate D and S from drying curves for yellow-poplar sapwood and heartwood specimens in either the tangential or the radial direction. They used two specimen thicknesses for each case. The environment of the drying chamber was controlled at 32 ± 0.3 C and

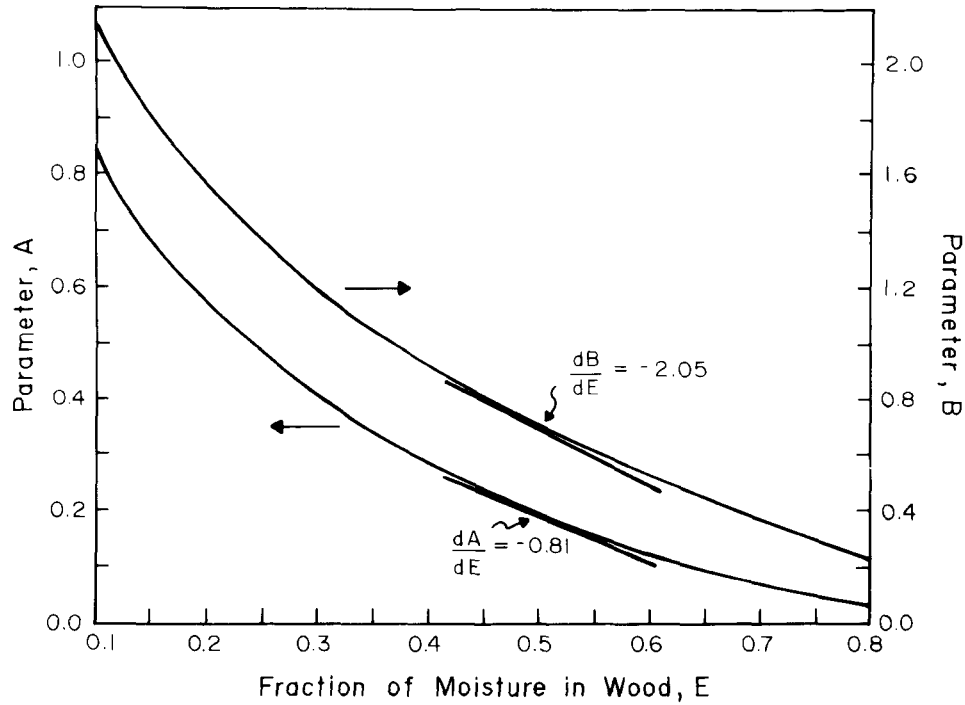


FIG. 4. Parameters A and B versus fraction of moisture in wood E in Eq. (11) (slopes evaluated at $E = 0.5$). (ML88 5384)

40% relative humidity with an air velocity of 3.3 m/sec. Their results for sapwood in the tangential direction were $D = 0.75 \times 10^{-5} \text{ cm}^2/\text{sec}$ and $S = 15.2 \times 10^{-5} \text{ cm}/\text{sec}$. These results will be compared with results from the present study.

The following data were obtained from Fig. 1 of Choong and Skaar (1969) and Eq. (18):

- (a) For $a = 1.431 \text{ cm}$, $t = 1,024 \text{ min}$ and $dt/dE = -4,090 \text{ min}$ at $E = 0.5$, Eq. (17) then gives

$$\begin{aligned} \frac{D}{a^2} &= \frac{-0.1654}{-0.701 \times 4,090 \times 60 + 1,024 \times 60 \times 2.05} \\ &= 3.5899 \times 10^{-6} \frac{1}{\text{sec}} \end{aligned}$$

and

$$D = 7.3513 \times 10^{-6} \text{ cm}^2/\text{sec}$$

Substituting these into either Eq. (12) or Eq. (13) for $E = 0.5$ and with L expressed by Eq. (6), we find $S = 1.4842 \times 10^{-4} \text{ cm}/\text{sec}$.

- (b) For $a = 0.479 \text{ cm}$, we obtained likewise $t = 139 \text{ min}$ and $dt/dE = -533 \text{ min}$ at $E = 0.5$, and

$$\frac{D}{a^2} = \frac{-0.1654}{-5.3184 \times 10^3} = 3.1100 \times 10^{-5} \frac{1}{\text{sec}}$$

$$D = 7.1356 \times 10^{-6} \text{ cm}^2/\text{sec}$$

and

$$S = 1.6556 \times 10^{-4} \text{ cm/sec}$$

These results are very close to those from Choong and Skaar (1969). In their approach, the physical properties of the specimens were assumed to be identical so that both D and S were not affected by specimen thickness.

Another example is taken from Biggerstaff (1965), who did not separate D from S because S was disregarded, and D was estimated by considering the linear relationship between E and $t^{1/2}$ for small times in Eq. (9). The specimens were made from flat-sawn eastern hemlock sapwood with $a = 0.3175$ cm in the radial direction. The drying chamber consisted of a forced-convection oven in which temperature could be varied. The environment of the drying chamber was not described in detail. Figure 1 of Biggerstaff (1965) presented some test data for specimens preheated to oven temperature of 50 C before drying. Using the procedure suggested therein, the calculated diffusion coefficient corresponding to $E = 0.575$ and $t = 37.2$ min is $D = 0.6405 \times 10^{-5} \text{ cm}^2/\text{sec}$. The time at $E = 0.5$ in the figure is 49 min. Using Eq. (9), we then obtain $\bar{D} = 0.6732 \times 10^{-5} \text{ cm}^2/\text{sec}$. Thus, from Eq. (21)

$$D = 0.7513 \times 10^{-5} \text{ cm}^2/\text{sec}$$

and

$$S = 0.7270 \times 10^{-3} \text{ cm/sec}$$

DISCUSSION AND CONCLUSIONS

Newman's (1931) solution of the unsteady-state diffusion equation with constant diffusion coefficient D and surface emission coefficient S can be closely approximated by Eq. (11) with A and B equal to some constants for any specified value for E . This relationship makes it possible to separate D and S from one drying curve using Eqs. (17) and (11) with $E = 0.5$.

When the boundary conditions specify no resistance to moisture movement ($S = \infty$), Eq. (11) can be reduced to Eq. (9) for small times. Under such conditions, the value of D obtained from Eq. (9) is also the true value of D . According to Crank (1975), the value of D so obtained from Eq. (9) for $E = 0.5$ is some mean value of D within a specified range of moisture concentration C . This seems to provide the justification that the separation of D and S should be performed at $E = 0.5$.

In the present study we found that the curve of E versus t does not always change very smoothly in the vicinity of $E = 0.5$, which indicates that data in that range must be carefully evaluated to obtain an accurate slope estimate. If any uncertainty should exist, it might be useful to apply the analytical method of Rosen (1982b) to determine the shape of the curve or use Eqs. (9) and (21) to calculate D .

Successful separation of the diffusion and surface emission coefficients will pave

the way for determining the diffusion coefficient as a function of moisture concentration and the surface emission coefficient as a function of viscosity and velocity of airflow. The present method has reduced the experimental effort needed to achieve that separation.

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