MODULUS OF ELASTICITY OF WOOD COMPOSITE PANELS WITH A UNIFORM VERTICAL DENSITY PROFILE: A MODEL[1]

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ABSTRACT

An analytical model was developed to understand and predict the development of modulus of elasticity (MOE) of wood composite panels with a uniform vertical density profile. The model analysis was based on the assumption of elasticity and all particles being bonded together. The simulation showed that MOE of particleboard decreased as average out-of-plane orientation angle of particles increased, but was not influenced by particle size. The simulation also showed that high density wood species resulted in higher MOE when used to manufacture composites at the same compaction ratios as low density wood species, but resulted in lower MOE at the same board density levels.

In single layer oriented strandboard (OSB), MOE in the orientation direction increased continuously as percent alignment increased; MOE across the orientation direction decreased and then leveled off after the percent alignment exceeded approximately 60%.

Keywords: Model, modulus of elasticity, Monte Carlo simulation, orientation, oriented strandboard, particleboard, tension, wood composites.

INTRODUCTION

MOE is an important material property of both interior and exterior wood composite panels. There are many experimental studies on the subject of MOE in relation to processing variables. A literature review showed: 1) MOE increases as board density increases; 2) MOE increases as resin content increases; 3) MOE increases as particle size increases; and 4) MOE increases in the orientation direction as orientation level of particles increases (Geimer 1976; Kelly 1977).

However, experimental studies did not lead to a general understanding of the mechanism of MOE development. Although wood composite panels usually possess a vertical density profile that influences MOE, our efforts to model MOE started from a simpler structure—a uniform vertical density profile. This paper presents an MOE model for such a simplified wood composite to understand the development of MOE in relation to various manufacturing parameters. An MOE model of real composite panels will be developed and verified in the future by combining the present model with the laminate theory to include the vertical density profile as a structural input.

THE MOE MODEL

Modulus of elasticity of wood composites with a uniform vertical density profile would be the same whether it is determined in a bending test or in a tension test. Our modeling

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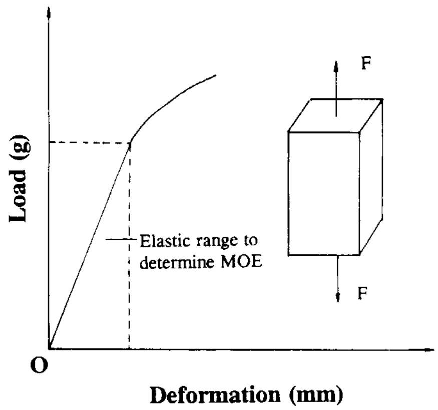

FIG. 1. A schematic of load-deformation curve in tension parallel to the board surface of wood composites.

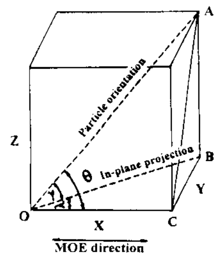

FIG. 2. A schematic showing the definition of out-of-plane angle (γ), angle between in-plane projection of a particle and loading direction (λ), and angle between a particle and loading direction (θ).

development of MOE used the tensile loading mode. Figure 1 shows schematically a composite specimen in tension and the corresponding load-deformation diagram. MOE is determined within the elastic range of the load-deformation curve.

Within the range of elasticity, the work (W) applied to the specimen by the external force is given by (Bodig and Jayne 1982)

$$\begin{aligned} W &= 1/2 \cdot F \cdot \Delta L = 1/2 \cdot \sigma \cdot A \cdot \Delta L = \\ &= 1/2 \cdot A \cdot L \cdot \sigma \cdot \Delta L/L = \\ &= 1/2 \cdot V \cdot \sigma \cdot \epsilon = 1/2 \cdot V \cdot E \cdot \epsilon^2 \end{aligned} \tag{1}$$

where F is the tensile force (g), ΔL is the deformation of the specimen (mm), $\sigma = F/A$ is the stress (MPa), A is the cross-sectional area of the specimen (mm^2), L is the length of the specimen (mm), $V = A \cdot L$ is the volume of the specimen (mm^3), $\epsilon = \Delta L/L$ is the strain (%), and E (MPa) is the MOE of the specimen to be determined.

This external work must be stored in the specimen as internal energy. Furthermore, this energy must be elastic since MOE is determined in the elastic range. Let us assume that the strain (ϵ) is uniformly distributed in the direction of loading within the specimen, the elastic energy (ϕ_i) of an individual particle is given by (Xu and Suchsland 1997)

$$\phi_i = 1/2 \cdot V_i \cdot E_i \cdot \epsilon^2 \tag{2}$$

where E_i and V_i are respectively the MOE and volume (mm^3) of wood particles in the direction of loading.

The total energy ($\varnothing$) due to external tensile loading in the whole composite system is therefore given by:

$$\varnothing = \frac{1}{2} \sum V_i \cdot E_i \cdot \epsilon^2 \tag{3}$$

As each particle in a composite may be oriented at a different angle (Fig. 2), E_i can be replaced by E_θ and Eq. (3) can be expressed as

$$\varnothing = \frac{1}{2} \sum V_i \cdot E_\theta \epsilon^2 \tag{4}$$

in which θ denotes the angle of the longitudinal direction of individual particles in relation to the loading direction (Fig. 2), while E_θ can be expressed by the Hankinson formula (Forest Products Laboratory 1987) as:

$$E_\theta = E_1 E_2/(E_1 \sin^2\theta + E_2 \cos^2\theta) \tag{5}$$

where E_1 and E_2 are, respectively, the longitudinal MOE and transverse MOE.

The external work and internal energy must be equal; therefore

$$\frac{1}{2} \cdot V \cdot E \cdot \epsilon^2 = \frac{1}{2} \sum V_i \cdot E_\theta \epsilon^2 \tag{6}$$

Solving Eq. (6), we have

$$E = \sum V_i \cdot E_\theta / V \tag{7}$$

Three-dimensional orientation of particles

Particles in a composite system can assume a three-dimensional orientation. Let us assume that the out-of-plane angle (angle between the longitudinal direction of a particle and its in-plane projection) is γ (degrees), the angle between the in-plane projection of a particle and the loading direction is λ, and the angle between the longitudinal direction of a particle and the direction of loading is θ (degrees) (Fig. 2). The following trigonometric relationships exist for the three random angles (Xu and Suchsland 1998):

$$\cos^2\theta = \cos^2\gamma \cos^2\lambda \tag{8}$$

$$\sin^2\theta = 1 - \cos^2\gamma \cos^2\lambda \tag{9}$$

Equations (8) and (9) are needed to use Eqs. (5) and (7) to model MOE of wood composites if particles are oriented in a three-dimensional fashion.

GENERALITY OF THE MODEL

Equations (5), (8), (9), together with Eq. (7), will be used later in this paper to simulate the development of MOE in relation to particle size, board density, compaction ratio (CR), wood species, and directional orientation of particles. However, the following discussion shows that they can also be used to understand or analyze other important processing parameters.

Adhesive type and resin content

A literature review showed that, similarly to many other board properties, MOE increases as resin content increases from zero percent to approximately 4–10 percent, depending on specific product and adhesive type, and then increases very slowly beyond those levels (Kelly 1977). This response of MOE to resin content can be fully explained by Eq. (7). As resin content increases initially, more and more particles would be bonded to the system, which increases the summation of $\Sigma\, V_i \cdot E_\theta$ and therefore MOE. When all the particles in the volume (V) are connected to the system, further increase of resin content would only increase MOE through modification of E_θ by adhesives. Therefore, a much slower increase of MOE follows.

Modification of E_θ and V_i due to manufacturing treatments

Manufacturing processes of wood composites can conceivably reduce particle volume (V_i) (during mat consolidation) and modify E_θ (Shaler 1986). Flaking can introduce flaws in the wood that may reduce E_θ (Price and Lehmann 1979); indentation of particles during densification may also reduce E_θ (Price 1976). On the other hand, densification itself might increase E_θ (Price 1976) and thermal treatment during drying, and press cycle might "repair" to some extent the damages caused to the particles and recover E_θ (Geimer et al. 1985). If a relationship among manufacturing processes, particle volume (V_i), and E_θ can be established, modified inputs of V_i and E_θ can be used for the model simulation.

SIMULATION PROCEDURE

Simulation analysis of MOE in this paper assumes: 1) a uniform vertical density profile exists, 2) all particles are bonded to the composite system, and 3) possible modification of E_θ and V_i by the manufacturing processes is minimal. Simulation procedures under these conditions are similar to those of modeling linear expansion outlined in a previous publication (Xu and Suchsland 1997). Specifically, they are:

1. Define the volume of a composite specimen (V = 38.1 cm × 5.08 cm × 1.27 cm was arbitrarily chosen).

2. Obtain the angular distribution of particle orientation and the particle size distribution. For a panel of random in-plane orientation (e.g., particleboard), a uniform distribution between −90 (degrees) and 90 (degrees) was assumed for λ. For a panel made of perfectly in-plane oriented particles, λ was assumed to be zero. For a panel with imperfect

TABLE 1. *Modulus of elasticity of quaking aspen and loblolly pine for the simulation analysis of particle size and out-of-plane orientation effects.*

Species	E_1 (MPa)	E_1/E_2
Quaking aspen	9,190	16
Loblolly pine	14,150	16

TABLE 2. *Distribution type and distribution parameters of particle size (volume) for the simulation analysis and simulation results.*

Type of distribution	Mean (mm^3)	Standard deviation (mm^3)	MOE (MPa)
Uniform	25	0	2,730.8
Uniform	25	5	2,734.0
Uniform	25	10	2,733.7
Uniform	50	5	2,753.8
Uniform	50	10	2,749.7
Lognormal	25	5	2,746.7
Lognormal	50	10	2,726.9

in-plane orientation (OSB), the Von Mises distribution was used to describe λ between -90 (degrees) and 90 (degrees) (Harris and Johnson 1982). For all the panel types, out-of-plane orientation angle (γ) varies from 0 (degrees) to 90 (degrees) according to the distributions provided later in this paper.

3. Randomly select an angle from each of the angular distributions (λ, γ) and a particle size (V_i) from the size distribution; calculate the off-axis MOE (Eqs. (5), (8), and (9)). The calculated E_θ and the selected V_i were then used as the inputs in Eq. (7).

4. Calculate the compaction ratio: CR = $\Sigma V_i/V$.

5. Repeat steps 3 and 4 until a predetermined value of CR is obtained.

6. Calculate MOE with Eq. (7).

MODEL SIMULATION

Particle size (volume)

Quaking aspen (*Populus tremuloides*) (Table 1) was used to simulate how particle size and its distribution might influence MOE of particleboard. Two types of distribution (uniform and lognormal) and three sets of distribution parameters (mean and standard deviation) were selected for the particle size (Table 2). A CR of 1.2 was used, and an out-of-plane orientation angle of zero degrees was assumed. Simulation result (Table 2) clearly shows that MOE of particleboard is independent of particle size and its variation (a moisture content of 7% of wood and wood composites was used throughout this paper).

However, literature review showed that MOE increases as particle size, especially particle length, increases. We believe the independence of MOE to particle size in this simulation is the result of our assumption of zero degrees in out-of-plane orientation of particles. This assumption may not be valid in real wood composites. In real composite panels, small particles might have a greater tendency of out-of-plane orientation than large particles (Xu and Suchsland 1998); diminished MOE reported in the literature using small particles might have arisen from the effect of out-of-plane orientation rather than from the particle size effect itself. Because of this inherent interaction, the real influence of particle size on board properties can not be determined experimentally. This limitation of inherent interaction of preferential directional orientation and particle size, however, is overcome with the simulation analysis. The influence of out-of-plane orientation of particles on MOE is discussed in the next section.

Since our model simulation of MOE is independent of particle size and its variation, a constant particle size of 25 mm^3 (10 mm $\times$ 5 mm $\times$ 0.5 mm) was used for subsequent model analysis.

Influence of out-of-plane orientation on MOE

Quaking aspen (*Populus tremuloides*) and loblolly pine (*Pinus taeda*) (Table 1) were used to simulate the influence of out-of-plane orientation of particles on MOE of particleboard. Because there is no prior knowledge about this structural characteristics of out-of-plane orientation of particles in various commercial wood composites, three types of dis-

TABLE 3. *Distribution type and standard deviation assigned to the out-of-plane orientation angle for the simulation analysis.*

Scenario	Type of distribution	Standard deviation (degrees)
1	Uniform	0
2	Uniform	2.89
3	Uniform	5.77
4	Normal	2.89
5	Normal	5.77
6	Lognormal	2.89
7	Lognormal	5.77

tribution (uniform, normal and lognormal) and three sets of standard deviation were arbitrarily selected for this orientation parameter (Table 3); the mean (average out-of-plane orientation angle) changes from 0 (degrees) to 90 (degrees). However, we believe out-of-plane orientation under these conditions will cover a wide range of scenarios, which allow us to investigate their influence on MOE with the simulation technique (Xu and Suchsland 1998). A CR of 1.2 was also used. The simulation results are shown in Fig. 3. As expected, MOE of particleboard from both species decreased as average out-of-plane orientation angle of particles increased. This simulation result agrees with the well-known fact that the extruded particleboard has much lower MOE than the platen pressed particleboard because of the vertical alignment of particles. We are also aware of one experimental study that attributed the exceptionally low MOE of black tupelo (*Nyssa sylvatica*) flakeboard to the cross-grained flakes associated with the species (Hse 1975); the effect of cross-grain in flakes would be similar to that of flakes oriented out-of-plane.

The simulation also showed that the reduction of MOE was most sensitive when average out-of-plane orientation angle was less than 45 degrees, after which this decrease leveled off. This leveling-off characteristic reflects the Hankinson formula used to describe the influence of the slope of the grain on MOE. The simulation further revealed that neither distribution type nor standard deviation of the out-of-plane orientation angle had any significant influence on MOE (Fig. 3). This characteristic of out-of-plane orientation was also found for linear expansion of particleboard (Xu and Suchsland 1998). This means that only average out-of-plane orientation angle needs to be considered in future studies for its influence on MOE.

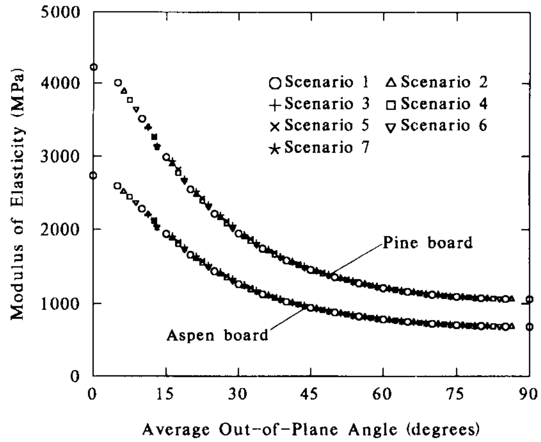

FIG. 3. Simulated influence of out-of-plane orientation of particles on MOE of particleboard.

Out-of-plane orientation of particles has never been measured in particleboard. We are aware of only one attempt to measure this out-of-plane orientation of fibers in fiberboard (Suchsland and MacMillin 1983). It is not known to what extent the out-of-plane orientation of particles exists in various wood composites, how it is controlled by particle size, and how much loss of MOE resulted in commercial wood composite panels due to this orientation component.

Board density and compaction ratio

Quaking aspen (*Populus tremuloides*) (Table 1) was used to simulate the development of MOE in relation to board density (or CR). An average out-of-plane orientation angle of zero degrees was used. The simulation showed that MOE increases linearly (r = linear correlation coefficient) with the increase of board density or (CR). Values in parentheses along the X-axis of Fig. 4 are the corresponding

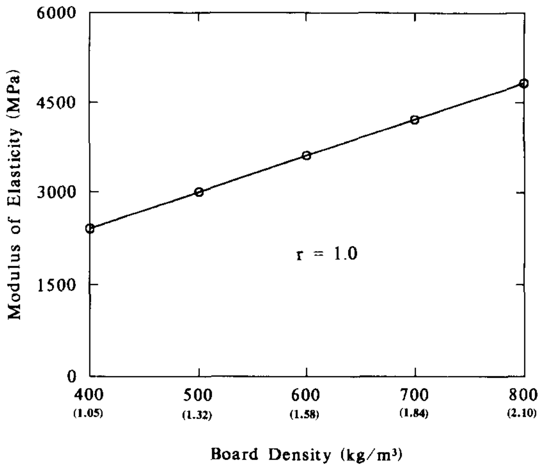

FIG. 4. Simulated influence of board density and CR on MOE (values in parentheses are the corresponding compaction ratios; species: quaking aspen).

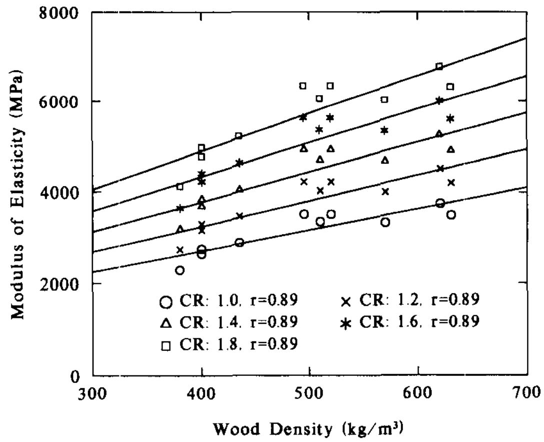

FIG. 5. Simulated influence of wood density on MOE of particleboard at various CRs.

CRs. The linearity of the development of MOE in relation to board density (or CR) is strikingly in agreement with experimental studies of MOE (Hse 1975; Stewart and Lehmann 1973; Suchsland and Woodson 1974; Vital et al. 1974).

Wood species

Ten species (Table 4) were used to simulate how wood species might influence MOE of particleboard at the same CRs and at the same board density levels. Again, an average out-of-plane orientation angle of zero degrees was used. The simulation results are shown in Figs. 5 and 6, respectively. In general, higher MOE resulted from high density wood species when particleboards were "manufactured" to the same CRs; when particleboards were "manufactured" to the same density levels, lower MOE values resulted with high density wood species. These simulation results agree well with the conclusions drawn from experimental studies (Carll 1994; Kelly 1977; Price and Lehmann 1979).

Single layer oriented strandboard

The Von Mises distribution and the measure of percent alignment (%) have been used to

TABLE 4. *Species and associated properties for the model simulation.**

Species	Density (g/cm^3)	E_1 (MPa)	E_1/E_2
Black walnut (*Juglans nigra*)	0.570	12,359	13.6
Douglas-fir (*Pseudotsuga menziesii*)	0.495	14,758	17.4
Eastern cottonwood (*Populus deltoides*)	0.400	10,623	16
Loblolly pine (*Pinus taeda*)	0.520	14,150	16
Quaking aspen (*Populus tremuloides*)	0.380	9,190	16
Red oak (*Quercus rubra*)	0.630	14,079	16
Sitka spruce (*Picea sitchensis*)	0.400	11,745	18
Sweetgum (*Liquidamber Spp.*)	0.510	12,754	14.3
Yellow birch (*Betula alleghaniensis*)	0.620	15,281	16.4
Yellow-poplar (*Liridendron tulipfera*)	0.435	12,035	17.1

* The values are from the Wood Handbook (Forest Products Laboratory 1987). Density was based on oven-dry weight and volume at 7% moisture content. Modulus of elasticity was adjusted to be at 7% moisture content based on the equation from the Wood Handbook. E_1/E_2 was assumed to be 16 for red oak, loblolly pine, eastern cottonwood, and quaking aspen.

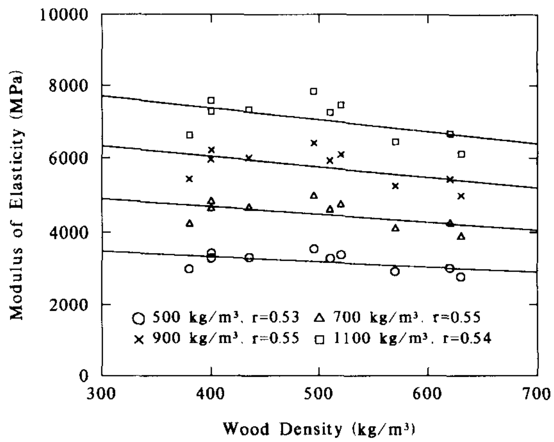

FIG. 6. Simulated influence of wood density on MOE of particleboard at various board density levels (The legends in the figure are board densities).

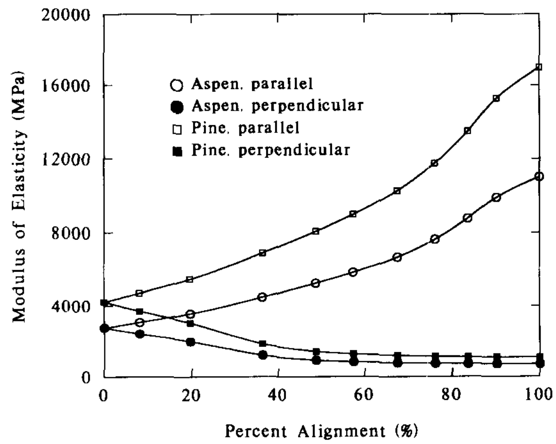

FIG. 7. Simulated influence of percent alignment on MOE of single layer OSB in the orientation and across the orientation directions (CR = 1.2).

characterize the in-plane orientation of particles in OSB (Geimer 1976; Harris and Johnson 1982). As in our previous publication (Xu and Suchsland 1997), the Von Mises distribution was used for the actual simulation and the percent alignment was used to report the simulation results.

Quaking aspen (*Populus tremuloides*) and loblolly pine (*Pinus taeda*) were used to simulate how in-plane orientation might influence MOE in the orientation and across the orientation directions of single layer OSB. A CR of 1.2 and an average out-of-plane orientation angle of zero degrees were used.

Figure 7 shows the simulation results for both species in the two principal directions. As expected, loblolly pine single layer OSB had larger MOE in both directions than aspen single layer OSB when the two boards were manufactured to the same CR. The simulation analysis showed that MOE in the orientation direction increased continuously as percent alignment increased for both products. MOE of single layer OSB exceeded MOE of the corresponding wood when percent alignment was approaching 100%; this was because the board density is higher than the wood density (A CR of 1.2 was used). The simulation also showed that the decrease of MOE across the orientation direction leveled off after the percent alignment reached 50–60%. Percent alignment level in the face of current industrial OSB panels is around 45–65% (Xu 1997). This simulation suggests that orientation level in commercial OSB can be further increased to the maximum potential without worrying about MOE loss in the perpendicular direction. (A future publication will discuss the development of MOE in three-layer OSB.)

CONCLUSIONS

The model presented in this paper adds to the understanding of the mechanism of MOE in relation to various manufacturing parameters. The simplicity of the model allows practitioners to perform necessary simulations so that proper selection or adjustment of processing variables can be made for the control of MOE. The following simulation results using the model were obtained:

1. Particle size does not influence MOE;
2. Out-of-plane orientation of particles diminishes MOE;
3. MOE increases linearly with the increase of either board density or CR;
4. High density wood species result in higher MOE than low density species at the same CRs, but result in lower MOE at the same density levels.

5. In-plane orientation improves MOE in the orientation direction but reduces MOE across the orientation direction; the decrease of MOE across the orientation direction levels off after percent alignment reaches approximately 50–60%.

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